PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXXVII.

PART I.

LONDON:

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MDCCCXXXVII.

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body,

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upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

A List of Public Institutions and Individuals, entitled to receive a copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

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The Geological Society of France.
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The Observatory at Armagh.
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The Observatory at St. Helena.
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The Royal Society, Edinburgh.
The Astronomical Institution, Edinburgh.
The President of the Royal Society.
The Lowndes's Professor of Astronomy, Cambridge.
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The Royal Academy of Sciences at Paris.
The Imperial Academy of Sciences at St. Petersburgh.
The Royal Academy of Sciences at Stockholm.
The Royal Society of Sciences at Upsal.
The Board of Longitude of France.
The University of Göttingen.
The University of Leyden.
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The Observatory at Wilna.
Professor Bessel, of Königsberg.
Dr. William Olbers, of Bremen.
The Dépôt de la Marine, Paris.
The Bowden College, United States.
The Waterville College, United States.

ROYAL MEDALS.

HIS MAJESTY KING WILLIAM THE FOURTH, in restoring the Foundation of the Royal Medals, graciously commanded a Letter, of which the following is an extract, to be addressed to the Royal Society, through His Royal Highness the DUKE OF SUSSEX, K.G., President:

"Windsor Castle, March 25, 1833.

"It is HIS MAJESTY'S wish,—

"First, That the Two Gold Medals, value of Fifty Guineas each, shall "henceforth be awarded on the day of the Anniversary Meeting of the "Royal Society, on each ensuing year, for the most important discoveries "in any one principal subject or branch of knowledge.

"Secondly, That the subject matter of inquiry shall be previously settled "and propounded by the Council of the Royal Society, three years pre-"ceding the day of such award.

"Thirdly, That Literary Men of all nations shall be invited to afford the "aid of their talents and research: and,

"Fourthly, That for the ensuing three successive years, the said Two "Medals shall be awarded to such important discoveries, or series of in-"vestigations, as shall be sufficiently established, or completed to the "satisfaction of the Council, within the last five years of the days of award, "for the years 1834 and 1835, including the present year, and for which "the Author shall not have previously received an honorary reward.

(Signed) "H. TAYLOR."

The Royal Medals for the year 1833 were awarded to

SIR JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Paper on the Investigation of the Orbits of Revolving Double Stars; and to

PROFESSOR AUGUSTE PYRAME DE CANDOLLE, of Geneva, Foreign Member of the Royal Society,

for his Discoveries and Investigations in Vegetable Physiology.

Those for 1834 were awarded to

JOHN WILLIAM LUBBOCK, ESQ., V.P. & TREAS. R.S.,

for his Papers on the Tides published in the Philosophical Transactions; and to

CHARLES LYELL, ESQ., F.R.S.,

for his Work entitled "Principles of Geology."

Those for 1835 were awarded to

MICHAEL FARADAY, D.C.L., F.R.S.,

for his Investigations and Discoveries contained in the Series of Experimental Researches in Electricity, published in the Philosophical Transactions, and more particularly for the Seventh Series, relating to the definite nature of electro-chemical action; and to

SIR WILLIAM ROWAN HAMILTON, Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland,

for the Papers published by him in the 16th and 17th volumes of the Transactions of the Royal Irish Academy, entitled "Supplement to an Essay on the Theory of "Systems of Rays," and more particularly for those Investigations at the conclusion of the third and last Supplement, which relate to the discovery of Conical Refraction.

The Council propose to give one of the Royal Medals in the year 1836, to the most important unpublished paper in Astronomy, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council also propose to give one of the Royal Medals in the year 1836 to the

most important unpublished paper in Animal Physiology, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council propose to give one of the Royal Medals in the year 1837 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in their Transactions, after the present date (November 27th, 1834,) and prior to the month of June in that year.

The Council also propose to give one of the Royal Medals in the year 1837 to the author of the best paper, to be entitled "Contributions towards a System of Geo-"logical Chronology founded on an examination of fossil remains, and their attendant "phenomena," such paper to be communicated to the Royal Society after the present date (December 1st, 1834,) and prior to the month of June 1837:—but in case no paper is presented to the Society fulfilling the conditions implied by the above Resolution, or possessing sufficient merit, the Council propose to give one of the Royal Medals in the year 1837 to the author of the best paper in Geology and Mineralogy, communicated to the Royal Society for insertion in their Transactions after the present date and prior to the month of June in that year.

The Council propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper on Chemistry, communicated to the Royal Society for insertion in their Transactions, after the present date (November 12th, 1835,) and prior to the month of June 1838.

The Council also propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in the Philosophical Transactions, after the present date (November 19, 1835,) and prior to the month of June 1838.

Those for 1836 were awarded to

SIR JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Papers on Nebulæ and Clusters of Stars, published in the Philosophical Transactions for 1833; and to

GEORGE NEWPORT, ESQ.,

for his Series of Investigations on the Anatomy and Physiology of Insects, contained

in his two Papers published in the Philosophical Transactions within the last three years.

The Council propose to give one of the Royal Medals in the year 1839 to the most important unpublished Paper in Astronomy, communicated for insertion in their Transactions after the present date, (November 30th, 1836,) and prior to the termination of the Session in June 1839.

The Council also propose to give one of the Royal Medals in the year 1839 to the most important unpublished Paper in Physiology, communicated for insertion in their Transactions after the present date, (November 30th, 1836,) and prior to the termination of the Session in June 1839.

The Council have appointed Mr. H. F. Talbot's Paper, entitled "Further Obser-"vations on the Optical Phenomena of Crystals," the Bakerian Lecture for the present year.

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PHILOSOPHICAL TRANSACTIONS.

I. *Researches in the Integral Calculus.—Part* II. *By* H. F. Talbot, *Esq. F.R.S.*

Received October 26,—Read November 17, 1836.

§ 1.

HAVING explained a general method of finding the sums of integrals, I propose to apply it to discover the properties of different transcendents, beginning with those of the simplest nature.

In the first place, therefore, I will show its application to the arcs of the circle and conic sections.

As there will be frequent occasion to make use of cubic equations, I shall suppose their general form to be

$$x^3 - p\,x^2 + q\,x - r = 0.$$

When therefore the letters $p\ q\ r$ occur without explanation, it will be understood that they represent these coefficients.

§ 2. *Application to the Circle.*

Let us take the integral $\int \frac{dx}{1+x^3}$, and suppose nothing to be previously known concerning the properties of the function which it represents.

Let us put, in the first place,

$$1 + x^2 = v\,x \quad \therefore x^2 - v\,x + 1 = 0.$$

The two variables $x\ y$ will be roots of this equation, so that they must satisfy the condition $x\,y = 1$. Also

$$\frac{dx}{1+x^3} + \frac{dy}{1+y^3} = \frac{dx}{v\,x} + \frac{dy}{v\,y} = \frac{1}{v}\,\mathrm{S}\,\frac{dx}{x} = 0,$$

because $\mathrm{S}\,\frac{dx}{x} = 0$ in any equation whose last term is constant.

$$\therefore \int \frac{dx}{1+x^3} + \int \frac{dy}{1+y^3} = \text{const.}$$

We thus obtain a characteristic property of the function

$$\int \frac{dx}{1+x^3} = f.x,$$

namely, that if $xy = 1$,

$$fx + fy = \text{const.}$$

The truth of which is otherwise evident, for if $x = \tan\theta$, then $y = \operatorname{cotan}\theta$

$$fx = \theta \text{ and } fy = 90° - \theta.$$

$$\therefore fx + fy = \text{const.}$$

Next let us investigate such a relation between *three* of these integrals that they may have an algebraic sum.

Assume $\frac{1}{1+x^3} = \frac{v+x}{ax}$, whence

$$x^3 + vx^2 + (1-a)x + v = 0,$$

where a is any constant quantity.

The three variables $x\ y\ z$ must be roots of this equation, which however gives only one necessary relation between them, viz.

$$x + y + z = xyz.$$

We have

$$\frac{a}{1+x^3} = \frac{v}{x} + 1$$

$$\therefore a\,\mathrm{S}\frac{dx}{1+x^3} = v\,\mathrm{S}\frac{dx}{x} + \mathrm{S}\,dx.$$

But $\mathrm{S}\frac{dx}{x} = \frac{dv}{v}$, and $\mathrm{S}\,dx = -dv$,

$$\therefore a\,\mathrm{S}\frac{dx}{1+x^3} = dv - dv = 0,$$

$$\therefore \mathrm{S}\int\frac{dx}{1+x^3} = \text{const.}$$

whence we obtain this well-known theorem in trigonometry.

If the sum of three tangents equals their product, the sum of the arcs is constant. The constant $= 180°$.

Next let us suppose

$$\frac{1}{1+x^3} = \frac{v+x}{ax^3}$$

$$\therefore x^3 + (v-a)x^2 + x + v = 0.$$

This gives only one necessary relation between the roots, viz.

$$q = xy + xz + yz = 1.$$

For the two other coefficients $(v-a)$ and v, may be made to agree with any two arbitrary quantities. Since we have

$$\frac{a}{1+x^3} = \frac{v}{x^3} + \frac{1}{x}$$

$$\therefore a\,\mathrm{S}\frac{dx}{1+x^3} = v\,\mathrm{S}\frac{dx}{x^3} + \mathrm{S}\frac{dx}{x}.$$

But $\mathrm{S}\frac{1}{x} = \frac{q}{r}$, and here

$$q = 1 \text{ and } r = -v$$

$$\therefore \mathrm{S}\frac{1}{x} = -\frac{1}{v},$$

whence

$$\mathrm{S}\frac{dx}{x^3} = -\frac{dv}{v^3}$$

and

$$v\,\mathrm{S}\frac{dx}{x^3} = -\frac{dv}{v}.$$

Also

$$\mathrm{S}\frac{dx}{x} = \frac{dv}{v}.$$

Therefore

$$a\,\mathrm{S}\frac{dx}{1+x^3} = -\frac{dv}{v} + \frac{dv}{v} = 0$$

$$\therefore \mathrm{S}\int\frac{dx}{1+x^3} = \text{const.},$$

which furnishes this other well-known theorem, viz. *If three tangents are such that the sum of their products* = 1, *then the sum of the arcs is constant.* The constant in this case = 90°.

The same theorem results from the supposition

$$\frac{1}{1+x^3} = v + ax;$$

for this gives

$$x^3 + \frac{v}{a}x^2 + x + \frac{v-1}{a} = 0,$$

and $q = 1$ is the only necessary relation between the roots. Also

$$\mathrm{S}\frac{dx}{1+x^3} = v\,\mathrm{S}\,dx + a\,\mathrm{S}\,x\,dx.$$

But

$$\mathrm{S}\,x^2 = p^2 - 2q = \frac{v^2}{a^3} - 2,$$

whence

$$\mathrm{S}\,x\,dx = \frac{v\,dv}{a^3}.$$

Also

$$\mathrm{S}\,dx = -\frac{dv}{a}.$$

Therefore

$$\mathrm{S}\frac{dx}{1+x^3} = -\frac{v\,dv}{a} + \frac{v\,dv}{a} = 0. \qquad \textit{Q.E.D.}$$

These theorems, and the analogous ones which exist between any number of tangents, are well known. But when we apply the method to the integral $\int \frac{dx}{\sqrt{1-x^3}}$, we obtain relations between circular arcs which appear to be of a more novel description, and perhaps have not hitherto been noticed. Of which I will proceed to give an example. Let us suppose

$$\frac{1}{\sqrt{1-x^3}} = vx + 1,$$

whence

$$x^3 + \frac{2}{v}x^2 + \left(\frac{1}{v^2} - 1\right)x - \frac{2}{v} = 0.$$

In this instance the symmetrical $v = \frac{2}{r}$, and therefore making this substitution, we have

$$x^3 + rx^2 + \left(\frac{r^3}{4} - 1\right)x - r = 0.$$

There are therefore two necessary relations between the three roots, viz.

$$p = -r \qquad q = \frac{r^3}{4} - 1.$$

And since

$$\frac{1}{\sqrt{1-x^3}} = vx + 1$$

$$\mathrm{S}\frac{dx}{\sqrt{1-x^3}} = v\,\mathrm{S}\,x\,dx + \mathrm{S}\,dx.$$

But

$$\mathrm{S}\,x^2 = p^2 - 2q = r^2 - \left(\frac{r^3}{2} - 2\right) = \frac{r^3}{2} + 2$$

$$\therefore \mathrm{S}\,x\,dx = \frac{r\,dr}{2}$$

$$v\,\mathrm{S}\,x\,dx = \frac{2}{r} \cdot \frac{r\,dr}{2} = dr.$$

Also

$$\mathrm{S}\,dx = -dr$$

$$\therefore \mathrm{S}\frac{dx}{\sqrt{1-x^3}} = dr - dr = 0;$$

whence this theorem:

If the sines of three circular arcs are roots of the equation

$$x^3 + rx^2 + \left(\frac{r^3}{4} - 1\right)x - r = 0,$$

the sum of the arcs is constant.

I will give a numerical example of this theorem.

The value of r is arbitrary. Suppose it to be

$$= 3 - \sqrt{12} = -0{\cdot}4641016.$$

The roots of the equation then have the following values:

$$\begin{aligned} x &= \quad 0{\cdot}5 \quad\quad = \sin 30^\circ \quad\quad = \sin \theta \\ y &= \quad 0{\cdot}94565 = \sin 71^\circ\, 1' \quad = \sin \theta' \\ z &= -\,0{\cdot}98154 = \sin -(78^\circ\, 59') = \sin \theta''; \end{aligned}$$

and the theorem gives the sum of the arcs, or $S\,\theta = \text{const.}$ The word *sum* is used in an algebraic sense, as including the case where one or more of the arcs are to be taken *negatively*, or its definition is

$$S\,\theta = \pm\theta \pm \theta' \pm \theta''.$$

The same ambiguity in the signs pervades the whole of this class of formulæ. In the present instance

$$\begin{aligned} S\,\theta &= \theta + \theta' - \theta'' \\ &= 30^\circ + 71^\circ\, 1' + 78^\circ\, 59' = 180^\circ \end{aligned}$$

$\therefore$ the constant is a semicircle.

Ex. 2. Let $r = 0$.

$$\therefore x^3 - x = 0;$$

and the roots are

$$\begin{aligned} x &= 0 \quad = \sin 0^\circ \\ y &= 1 \quad = \sin 90^\circ \\ z &= -\,1 = \sin -90^\circ \end{aligned}$$

$$\therefore \theta = 0^\circ \quad \theta' = 90^\circ \quad \theta'' = -\,90^\circ;$$

and the same formula gives, as before,

$$\theta + \theta' - \theta'' = 180^\circ.$$

A very extensive class of formulæ respecting the arcs of the circle may be obtained in a similar manner, by applying the method more generally. Thus, if we make the supposition

$$[1.] \qquad \frac{1}{\sqrt{1 - x^2}} = a_0 + a_1 x + \ldots\ldots + a_{n-1} x^{n-1},$$

where $a_0, a_1, \ldots\ldots a_{n-1}$ are constants, or any entire rational functions whatever of the variable v, we have an equation of $2n$ dimensions, of which x is a root.

If $x = \sin \theta_1$, and the other roots are $\sin \theta_2$, $\sin \theta_3, \ldots\ldots\ldots \sin \theta_{2n}$, then

$$\int \frac{dx}{\sqrt{1 - x^2}} = \theta_1,$$

and the other integrals $= \theta_2, \theta_3 \ldots \theta_{2n}$. And by a direct process we obtain the final equation

$$S\,\theta, \text{ or } \theta_1 + \theta_2 + \ldots\ldots + \theta_{2n} = f.v + \text{const.},$$

$f.v$ being an entire rational function of v.

But since it is generally admitted that no combination of circular arcs can be equal to an algebraic quantity, I conclude that we have generally

$$f.v = 0.$$

If we consider the generality of the supposition [1.], which admits any number of arbitrary quantities, it certainly appears remarkable that this equation $f.v = 0$ should be always verified.

§. 3. *Application to the Parabola.*

If the tangent at the vertex of a parabola be taken for the axis of abscissæ, and the semiparameter = 1, then if x be the abscissa, the equation of the curve will be

$$2y = x^2,$$

and the arc, which may be designated as arc x,

$$= \int dx \sqrt{1+x^2}.$$

The known value of this is

$$[2.] \qquad \text{Arc } x = \tfrac{1}{2} x \sqrt{1+x^2} + \tfrac{1}{2} \log (x + \sqrt{1+x^2}).$$

This is a function of x, the properties of which appear to have been hitherto little examined. I will establish two theorems concerning it, which are of considerable simplicity.

Theorem I.—*If three abscissæ are the roots of the equation*

$$x^3 - r x^2 + \left(\frac{r^2}{4} + 1\right) x - r = 0,$$

the sum of the arcs equals the sum of the abscissæ.

Since each arc is greater than its corresponding abscissa, it is evident that the word *sum* is to be understood in an algebraic sense, or that at least one of the arcs must be taken negatively.

Theorem II.—*If three abscissæ are the roots of the equation*

$$x^3 - a x^2 + \left(\frac{a^2}{4} - a b - \frac{a^2 b^2}{4}\right) x + \frac{a^2 b}{2} = 0,$$

the sum of the arcs equals the product of the abscissæ.

This theorem is remarkable for its simplicity, when it is considered that it contains two arbitrary quantities, a and b, which, as it appears, may have any values.

Demonstration of Theorem I.

Put $\sqrt{1+x^2} = x^2 + v x + 1$: whence

$$[1.] \qquad x^3 + 2 v x^2 + (v^2 + 1) x + 2 v = 0.$$

Also

$$\mathrm{S}\, dx \sqrt{1+x^2} = \mathrm{S}\, x^2 dx + v\, \mathrm{S}\, x\, dx + \mathrm{S}\, dx$$

$$\therefore \mathrm{S} \int dx \sqrt{1+x^2} = \frac{\mathrm{S}\, x^3}{3} + \int v\, \mathrm{S}\, x\, dx + \mathrm{S}\, x.$$

The first term $\frac{\mathrm{S}\, x^3}{3} = \frac{p^3}{3} - p q + r$ in all equations. Here $p = -2v \quad q = v^2 + 1 \quad r = -2v$.

$$\therefore \frac{\mathrm{S}\,x^3}{3} = \frac{-8\,v^3}{3} + 2\,v\,(v^2+1) - 2\,v = -\frac{2\,v^3}{3}.$$

To find the value of the second term $\int v\,\mathrm{S}\,x\,dx$, we have

$$\mathrm{S}\,x^2 = p^2 - 2\,q = 4\,v^2 - (2\,v^2+2) = 2\,v^2 - 2$$
$$\therefore \mathrm{S}\,x\,dx = 2\,v\,dv$$

and

$$\int v\,\mathrm{S}\,x\,dx = \int 2\,v^2\,dv = \frac{2\,v^3}{3}.$$

Therefore these two terms destroy each other. Consequently we have simply

$$\mathrm{S}\int dx\,\sqrt{1+x^2} = \mathrm{S}\,x + \mathrm{C}.$$

It appears by trial that $\mathrm{C} = 0$, and the equation between the roots [1.] becomes, by writing for v its value $\frac{-r}{2}$,

$$x^3 - r\,x^2 + \left(\frac{r^2}{4}+1\right)x - r = 0$$

$\therefore$ the sum of three arcs $= \mathrm{S}\,x = r$. *Q.E.D.*

Example.—Let us suppose $r = 4 + 2\,\sqrt{2}$
$= 6{\cdot}828427.$

The three roots will be

$$x = 1$$
$$y = 4{\cdot}2042580$$
$$z = 1{\cdot}6241690.$$

Calculating the arcs accurately by the formula [2.], we have

$$\text{Arc } x = 1{\cdot}147793$$
$$\text{Arc } y = 10{\cdot}156004$$
$$\text{Arc } z = 2{\cdot}179773$$

In forming the sum we must notice that arc x and arc z are to be accounted negative. Consequently we find *by subtraction*,

$$\text{Arc } y = 10{\cdot}156004$$
$$\text{Arc } x + \text{Arc } z = 3{\cdot}327566$$
$$\text{Sum} = 6{\cdot}828438$$
$$r = 6{\cdot}828427$$
$$\text{Error of calculation} = 0{\cdot}000011$$

Thus the calculation verifies the theorem with considerable exactness, and shows that no constant is required to be added to the integral.

Since the sum of these three arcs is algebraic, and that each contains a logarithmic part, the sum of these three logarithms must be $= 0$: for if not, it must be an alge-

braic quantity, which is considered to be impossible. This is verified by calculation; for

$$2 \text{ arc } x = x\sqrt{1+x^2} + \log(x + \sqrt{1+x^2}).$$

Calling $\log(x + \sqrt{1+x^2}) = f.x$, we have

$fx = 0{\cdot}881372$	$f.y = 2{\cdot}143099$
$fz = 1{\cdot}261722$	$fx + fz = 2{\cdot}143094$
$2{\cdot}143094$	sum $= 0{\cdot}000005$

This sum approaches zero very nearly. The quantities fx, fz are subtractive, being parts of 2 arc x and 2 arc z, which have been already shown to be so.

Demonstration of Theorem II.

Let $v\sqrt{1+x^2} = nx^2 + x + v$, where n is a constant,

$$\therefore x^3 + \frac{2}{n}x^2 + \frac{1 + 2vn - v^2}{n^2}x + \frac{2v}{n^2} = 0,$$

and

$$v\,\mathrm{S}\sqrt{1+x^2}.dx = n\,\mathrm{S}\,x^2\,dx + \mathrm{S}\,x\,dx,$$

the term $v\,\mathrm{S}\,dx$ being omitted; because, since $\mathrm{S}\,x = \frac{-2}{n}$ is constant, the factor

$$\mathrm{S}\,dx = 0.$$

The formula $\mathrm{S}\,x^3 = p^3 - 3pq + 3r$ gives

$$\frac{\mathrm{S}\,x^3}{3} = \frac{p^3}{3} - pq + r,$$

$\therefore$ (observing that p is constant and $= -\frac{2}{n}$) $\mathrm{S}\,x^2\,dx = \frac{2}{n}\,dq + dr$. Therefore the first term, or

$$n\,\mathrm{S}\,x^2\,dx = 2\,dq + n\,dr.$$

The formula $\mathrm{S}\,x^2 = p^2 - 2q$ gives the second term, or

$$\mathrm{S}\,x\,dx = -\,dq$$

$$\therefore n\,\mathrm{S}\,x^2\,dx + \mathrm{S}\,x\,dx = dq + n\,dr,$$

or

$$v\,\mathrm{S}\sqrt{1+x^2}.dx = dq + n\,dr.$$

Now we have (omitting constants),

$$q = \frac{2}{n}v - \frac{v^2}{n^2}$$

$$\therefore dq = \frac{2}{n}\,dv - \frac{2v\,dv}{n^2},$$

and

$$n\,dr = -\frac{2}{n}\,dv$$

$$\therefore dq + n\,dr = -\frac{2v\,dv}{n^2}.$$

Therefore

$$S\sqrt{1+x^2}\,.\,dx = -\frac{2\,dv}{n^2}$$

$$S\int\sqrt{1+x^2}\,.\,dx = -\frac{2v}{n^3} = r.$$

Now writing $n = -\frac{2}{a}$, $v = b$, we have the equation in the form given above, viz.

$$x^3 - a\,x^2 + \left(\frac{a^2}{4} - a\,b - \frac{a^3\,b^3}{4}\right)x + \frac{a^3\,b}{2} = 0;$$

and therefore the theorem is demonstrated.

Examples.

Ex. 1. Let $a = 2 + \sqrt{2}$, $b = 1$, the roots of the equation are

$$x = 1$$

$$y = \tfrac{1}{2}\cdot\frac{\sqrt{5}+1}{\sqrt{2}-1}$$

$$-z = \tfrac{1}{2}\cdot\frac{\sqrt{5}-1}{\sqrt{2}-1},$$

or

$$x = 1 \qquad \therefore \text{arc } x = 1{\cdot}147793$$
$$y = 3{\cdot}906278 \quad \therefore \text{arc } y = 8{\cdot}911399$$
$$z = -1{\cdot}492065 \quad \therefore \text{arc } z = 1{\cdot}935186$$

$$\therefore S\,x = 3{\cdot}414213 = 2 + \sqrt{2} = a$$

and

$$x\,y\,z = -(3 + 2\sqrt{2}) = -5{\cdot}828426.$$

Now we have

$$\text{Arc } y = 8{\cdot}911399 = (1.)$$
$$\text{Arc } x + \text{Arc } z = 3{\cdot}082979 = (2.)$$

$$\text{Sum } (\textit{subtractive}) = -5{\cdot}828420 = (2.) - (1.)$$
$$x\,y\,z = -5{\cdot}828426$$

$$\text{Error} = 0{\cdot}000006$$

The quantity which we previously called $f\,x = \log\,(x + \sqrt{1+x^2})$ has the following values:

$$f\,x = 0{\cdot}881372$$
$$f\,y = 2{\cdot}071728$$
$$f\,z = 1{\cdot}190354$$

$$\therefore \text{we have } f\,x + f\,z = 2{\cdot}071726$$
$$f\,y = 2{\cdot}071728$$

$$\text{Error} = 0{\cdot}000002$$

Thus it is seen that the logarithmic parts destroy each other as in the first theorem.

Ex. 2. Let $x = \frac{3}{4}$ $y = \frac{4}{3}$ be *assumed* for two roots of the equation; in which case we find $a = -\frac{5}{6}$, $b = \frac{42}{5}$, and the third root $z = -\frac{35}{12}$.

Since $xy = 1$ in this example, the theorem gives the sum of three arcs $= xyz = z$, which we propose to verify.

Now the formula

$$2 \text{ arc } x = x\sqrt{1+x^2} + \log(x + \sqrt{1+x^2})$$

gives

$$2 \text{ arc } \left(\tfrac{3}{4}\right) = \frac{15}{16} + \log 2$$

$$2 \text{ arc } \left(\tfrac{4}{3}\right) = \frac{20}{9} + \log 3$$

$$\text{the sum of which two} = \frac{455}{144} + \log 6$$

$$\text{and } 2 \text{ arc } \left(\tfrac{35}{12}\right) = \frac{1295}{144} + \log 6.$$

Therefore the sum (*subtractive*)

$$= -\frac{840}{144} = -\frac{70}{12}$$

$$\therefore \text{arc } x + \text{arc } y - \text{arc } z = -\frac{35}{12}$$

But on the other hand we have

$$z = -\frac{35}{12}.$$

Therefore the sum of the arcs $= z$: which was to be shown.

§ 4. *Analogous Properties of the Circle and Parabola.*

There is a manifest analogy between the *area* of the circle and the *arc* of the parabola, the former being expressed by $\int dx\sqrt{1-x^2}$, the latter by $\int dx\sqrt{1+x^2}$, which only differ in the sign. The same analogy is seen in the theorems which may be deduced respecting these integrals. Thus, for instance, the Theorem II., which we have demonstrated in the parabola, may be applied, with a slight modification, to the circle. If we put

$$v\sqrt{1 \pm x^2} = nx^2 + x + v,$$

we find the sum of three integrals of the form

$$\int dx\sqrt{1 \pm x^2} = \pm r,$$

the constant being $= 0$. The upper sign applies to the parabola, the lower to the circle. The demonstration of the latter case is omitted for brevity, being exactly similar to that of the former.

The three variables $x\,y\,z$ are roots of

$$[1.] \qquad x^3 - a\,x^2 + \left(\frac{a^2}{4} - a\,b \mp \frac{a^2\,b^2}{4}\right) x + \frac{a^2\,b}{2} = 0,$$

the upper sign applying to the parabola; a and b being two arbitrary quantities.

To exemplify this theorem in the circle.—Since

$$2\int d\,x\,\sqrt{1-x^2} = x\,\sqrt{1-x^2} + \text{arc sin}\,x,$$

the theorem gives

$$-2\,r = \text{S}\,x\,\sqrt{1-x^2} + \text{S arc sin}\,x.$$

The latter term, being the sum of three circular arcs, cannot form any part of the quantity $-2\,r$: therefore we must have this other equation,

$$\text{S arc sin}\,x = 0,$$

which we propose to verify.

Ex. 1. Suppose $a = \frac{6}{5}$, $b = \frac{2}{3}$, the equation [1.] becomes

$$x^3 - \frac{6}{5}x^2 - \frac{7}{25}x + \frac{12}{25} = 0,$$

and its roots are

$$x = \frac{4}{5}$$

$$y = 1$$

$$z = -\frac{3}{5}$$

$$\therefore \text{arc sin}\,x = 53^\circ\,8' = \theta$$

$$\text{arc sin}\,y = 90^\circ = \theta'$$

$$\text{arc sin}\,z = -143^\circ\,8' = \theta''$$

$$\therefore \theta + \theta' + \theta'' = 0,$$

in accordance with the theorem.

We may assume two of the arcs arbitrarily, and thence determine the third, so as to satisfy the theorem.

Ex. 2. Thus, let $x = \frac{4}{5}$, $y = \frac{12}{13}$, we find $a = \frac{56}{65}$, $b = \frac{12}{7}$. Here it happens that $x\,y = \frac{a\,b}{2} = \frac{48}{65}$: therefore, dividing the equation

$$x\,y\,z = -\frac{a^2\,b}{2}$$

by the equation

$$x\,y = \frac{a\,b}{2},$$

we find the third root

$$z = -a.$$

Now these three values satisfy the theorem; for we have

$$\text{arc sin } \frac{4}{5} = 53^\circ\ 8' = \theta$$

$$\text{arc sin } \frac{12}{13} = 67^\circ\ 23' = \theta'$$

$$\text{arc sin } -\frac{56}{65} = -\ 120^\circ\ 31' = \theta''$$

and

$$\theta + \theta' + \theta'' = 0.$$

§ 5. *Application to the Ellipse.*

In order to obtain a relation between three elliptic integrals, the simplest supposition which we can make appears to be

$$\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = v x + 1,$$

whence

$$x^3 + \frac{2}{v} x^2 + \frac{1 - e^2 - v^2}{v^2} . x - \frac{2}{v} = 0.$$

This determines the value of the symmetrical v to be $= \frac{2}{r}$: and therefore making this substitution we have

$$x^3 + r x^2 + \left(\frac{1 - e^2}{4} . r^2 - 1\right) x - r = 0$$

and

$$\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = \frac{2}{r} x + 1,$$

whence

$$\mathrm{S}\, dx \sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = \frac{2}{r} \mathrm{S}\, x\, dx + \mathrm{S}\, dx.$$

But since

$$\mathrm{S}\, x^2 = p^2 - 2q = r^2 - \left(\frac{1 - e^2}{2} . r^2 - 2\right) = \frac{1 + e^2}{2} . r^2 + 2$$

$$\therefore \mathrm{S}\, x\, dx = \frac{1 + e^2}{2} r\, dr$$

$$\therefore \frac{2}{r} \mathrm{S}\, x\, dx = (1 + e^2)\, dr.$$

Also

$$\mathrm{S}\, dx = -\, dr$$

$$\therefore \frac{2}{r} \mathrm{S}\, x\, dx + \mathrm{S}\, dx = e^2\, dr$$

$$\therefore \mathrm{S}\, dx \sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = e^2\, dr$$

$$\therefore \mathrm{S} \int dx \sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = e^2 r + \mathrm{C}.$$

Or, if we suppose the radical to have a negative sign,

$$S\int dx\sqrt{\frac{1-e^2x^2}{1-x^2}} = C - e^2 r;$$

whence the following theorem: *If three abscissæ of an ellipse are roots of the equation*

$$x^3 + r x^2 + \left(\frac{1-e^2}{4}.r^2 - 1\right)x - r = 0,$$

the sum of the arcs $= 2Q - e^2 r$, Q *being the quadrant of the ellipse.*

Ex. 1. Let $e = 0$, or the ellipse be a circle; the theorem then assumes this form: *If three abscissæ of a circle are roots of the equation*

$$x^3 + r x^2 + \left(\frac{r^2}{4} - 1\right)x - r = 0,$$

the sum of the arcs is a semicircle; the truth of which has been demonstrated previously (vide page 4.).

Fagnani's theorem becomes illusory when $e = 0$: it is therefore interesting to observe that the present theorem, on the contrary, has a real application to the circle.

Ex. 2. Let e have any value, and $r = 0$; then the roots are

$$\begin{aligned} x &= 0 \quad \therefore \text{arc } x = 0 = (1.) \\ y &= 1 \quad\quad \text{arc } y = Q = (2.) \\ z &= -1 \quad \text{arc } z = -Q = (3.) \end{aligned}$$

and the sum, viz.

$$(1.) + (2.) - (3.) = 2Q,$$

which is therefore the value of the constant.

Ex. 3. When x is not actually $= 0$, as in the last example, but has an indefinitely small value $= \omega$, it will be found that the values of y and z differ from 1 and -1 by a quantity of the order of ω^2. But nevertheless the arcs which subtend these abscissæ differ from a quadrant of the ellipse by a quantity of the order of ω. This arises from the direction of the arc at the extremities of the axis being perpendicular to the abscissa, so that its increment is infinitely greater than that of the latter. It will be well to show the truth of the theorem in this case. When $x = \omega$ we have (putting $1 - e^2 = b^2$)

$$\begin{aligned} y &= 1 - \frac{b^2\omega^2}{8} \\ z &= -y. \end{aligned}$$

For from these values we deduce $y + z = 0$, and thence (neglecting quantities of the order ω^3)

$$\begin{aligned} &x + y + z = x = \omega \\ &xy + xz + yz = yz = -y^2 = \frac{b^2\omega^2}{4} - 1 \\ &xyz = -\omega. \end{aligned}$$

So that $x\,y\,z$ are roots of

$$x^3 - \omega x^2 + \left(\frac{1-e^2}{4}\omega^2 - 1\right)x + \omega = 0,$$

which agrees with the given form by putting $r = -\omega$.

We have now to find the sum of the arcs.

The arc subtending the abscissa ($x = \omega$) may be considered as equal to it.

The arc subtending the abscissa y differs from the elliptic quadrant by an arc which may be considered equal to the *ordinate* which corresponds to y. And the same with respect to z.

Let y' be the ordinate corresponding to y. The equation of the curve gives

$$y' = b\sqrt{1-y^2};$$

but since

$$y^2 = 1 - \frac{b^2\omega^2}{4} \therefore \sqrt{1-y^2} = \frac{b\omega}{2} \quad \therefore y' = \frac{b^2\omega}{2},$$

and the arc subtending $y = \mathrm{Q} - \frac{b^2\omega}{2}$.

The arc z has the same value. Therefore

$$\text{arc } y + \text{arc } z = 2\,\mathrm{Q} - b^2\omega;$$

adding arc $x = \omega$, we have

$$\text{Sum of arcs} = 2\,\mathrm{Q} + e^2\omega,$$

(or, since $\omega = -r$)

$$= 2\,\mathrm{Q} - e^2 r,$$

in accordance with the theorem.

Ex. 4. Let $1 - e^2 = \frac{1}{3}$.

And also let $r = 9 - 3\sqrt{10} = -0{\cdot}4868331$;

the roots of the equation

$$x^3 + r\,x^2 + \left(\frac{r^2}{12} - 1\right)x - r = 0,$$

are

$$\begin{aligned} x &= 0{\cdot}5 = \sin 30^\circ \\ y &= 0{\cdot}98019 = \sin 78^\circ\,34' \\ z &= -0{\cdot}99336 = \sin 83^\circ\,24' \end{aligned}$$

Entering LEGENDRE's Table IX. with modulus $e = \sqrt{\frac{2}{3}} = \sin 54^\circ\,44'$ and these amplitudes, we find

$$\begin{aligned} \text{arc } x &= 0{\cdot}5081 \\ \text{arc } y &= 1{\cdot}1446 \\ \text{arc } z &= 1{\cdot}1944 \\ \hline \text{Sum} &= 2{\cdot}8471 \end{aligned}$$

On the other hand we have

$$\begin{aligned} 2\,\mathrm{Q} = 2 \text{ arc } (90^\circ) &= 2{\cdot}5224 \\ -\,e^2 r &= 0{\cdot}3246 \\ \hline 2\,\mathrm{Q} - e^2 r &= 2{\cdot}8470 \\ \text{Sum of arcs} &= 2{\cdot}8471 \\ \hline \text{Error} &= 0{\cdot}0001 \end{aligned}$$

I will now indicate two other theorems respecting the sum of *three elliptic arcs.*

I. We may put the integral $\int dx \sqrt{\frac{1-e^2x^2}{1-x^3}}$ in the form

$$\int (1+ex)\,dx \sqrt{\frac{1-ex}{(1+ex)(1-x^3)}},$$

and assume $\frac{1-ex}{(1+ex)(1-x^3)}$ to be a symmetrical $= \frac{1}{v}$. This gives

$$x^3 + \frac{1}{e}x^2 - (v+1)\,x + \frac{v-1}{e} = 0,$$

and the result which I find is, that if three abscissæ are the roots of this equation, the sum of the corresponding arcs $= 2\,e\sqrt{v} + \mathrm{C}$.

II. We may put the integral in the form

$$\int \frac{dx}{1+x} \sqrt{\frac{(1+x)(1-e^3x^3)}{1-x}},$$

and assume $\frac{(1+x)(1-e^3x^3)}{1-x} = v$, whence

$$x^3 + x^2 - \frac{v+1}{e^3}x + \frac{v-1}{e^3} = 0.$$

The result which I find is, that if three abscissæ are the roots of this equation, the sum of the arcs $= 2\sqrt{v} + \mathrm{C}$.

These theorems respecting the sums of elliptic arcs appear to be some of the simplest which exist; but an unlimited number of theorems of a higher order and more complicated nature are obtainable, the discussion of which would lead too far at present.

Thus if we assume

$$\sqrt{\frac{1-e^2x^2}{1-x^3}} = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \&c.$$

where the coefficients are constants, or entire rational functions of v, we have an equation of $2n$ dimensions, which gives the sum of $2n$ elliptic arcs in terms of v.

There is no difficulty, beyond the length of the operation, in deducing these theorems, as they are all obtainable by an uniform method. But it will be of importance to show the relation between them and the previously received doctrines respecting elliptic integrals as established by LEGENDRE and others, the connexion between them not being at first sight very evident.

§ 6. *Application to the Equilateral Hyperbola.*

In order to obtain a relation between three values of the integral $\int \frac{dx}{x^3}\sqrt{1+x^4}$, which expresses the arc of the equilateral hyperbola, we may put

$$\sqrt{1+x^4} = vx + 1,$$

whence

$$x^3 - v^2 x - 2v = 0,$$

we have therefore $2v = r$, and making this substitution,

$$x^3 - \frac{r^2}{4}x - r = 0.$$

Also

$$\frac{\sqrt{1+x^4}}{x^3} = \frac{r}{2x} + \frac{1}{x^3}$$

$$\therefore \mathrm{S}\frac{\sqrt{1+x^4}}{x^3}.dx = \frac{r}{2}\mathrm{S}\frac{dx}{x} + \mathrm{S}\frac{dx}{x^3}.$$

Now we have

$$\frac{r}{2}\mathrm{S}\frac{dx}{x} = \frac{r}{2}.\frac{dr}{r} = \frac{dr}{2},$$

and

$$\mathrm{S}\frac{1}{x} = \frac{q}{r} = -\frac{r}{4}$$

$$\therefore \mathrm{S}\frac{dx}{x^3} = \frac{dr}{4}$$

$$\therefore \mathrm{S}\frac{\sqrt{1+x^4}}{x^3}.dx = \frac{dr}{2} + \frac{dr}{4} = \frac{3\,dr}{4}$$

$$\therefore \mathrm{S}\int\frac{\sqrt{1+x^4}}{x^3}.dx = \frac{3}{4}r + \mathrm{C};$$

so that if three abscissæ of the equilateral hyperbola are roots of the equation

$$x^3 - \frac{r^2}{4}x - r = 0,$$

the sum of the arcs $= \frac{3}{4}r + \mathrm{C}$, which is the theorem which I originally met with concerning the hyperbolic arc*.

It will be seen how very simply and directly we are conducted to it by the present method of investigation. Next let us suppose

$$-\sqrt{1+x^4} = vx^3 + 1,$$

whence

$$x^3 - \frac{x}{v^2} + \frac{2}{v} = 0.$$

Put $v = -\frac{2}{r}$,

* Philosophical Transactions, 1836, Part I. p. 185.

$$\therefore x^3 - \frac{r^2}{4} x - r = 0,$$

and

$$-\frac{\sqrt{1+x^4}}{x^2} = -\frac{2x}{r} + \frac{1}{x^2}.$$

Therefore

$$-\mathrm{S}\frac{\sqrt{1+x^4}}{x^2}.dx = -\frac{2}{r}\mathrm{S}\,x\,dx + \mathrm{S}\frac{dx}{x^2},$$

but

$$\mathrm{S}\frac{dx}{x^2} = \frac{dr}{4},$$

as in the last example; and

$$\mathrm{S}\,x^2 = \frac{r^3}{2} \quad \therefore -\frac{2}{r}\mathrm{S}\,x\,dx = -\,dr$$

$$\therefore -\mathrm{S}\frac{\sqrt{1+x^4}}{x^2}.dx = \frac{dr}{4} - dr = -\frac{3}{4}dr$$

$$\therefore \mathrm{S}\int\frac{\sqrt{1+x^4}}{x^2}.dx = \frac{3}{4}r + \mathrm{C}.$$

This result therefore agrees with the last example, and gives the same theorem, but it supplies a different demonstration of it.

We will now suppose

$$\frac{\sqrt{1+x^4}}{x^2} = 1 + \frac{a}{x} + \frac{v}{x^2},$$

a being a constant. This gives

$$x^3 + \frac{a^2+2v}{2a}x^2 + vx + \frac{v^2-1}{2a} = 0,$$

and I find this result, that if three abscissæ are roots of this equation, which may be written

$$x^3 - px^2 + qx - r = 0,$$

then the sum of the arcs

$$= p - \frac{v^2}{r} + \text{const.} = \phi v + \mathrm{C}.$$

This sum is therefore constant if ϕv is so.

Let $v = k$, $v = k'$, be two values of v, which give the same value to ϕv, or $p - \frac{v^2}{r}$.

Let the three hyperbolic arcs in the first case be $\alpha\ \alpha'\ \alpha''$, and in the second case $\beta\ \beta'\ \beta''$, then

$$\alpha + \alpha' + \alpha'' = \beta + \beta' + \beta''.$$

All the abscissæ have the same origin at the centre of the curve, therefore the arcs have the same origin, and therefore can be subtracted from one another. Therefore putting $\alpha - \beta = \gamma$, $\alpha' - \beta' = \gamma'$, $\alpha'' - \beta'' = \gamma''$, we have

$$\gamma + \gamma' + \gamma'' = 0.$$

This appears to me to show the possibility of finding three arcs such that (neglecting

their signs) the sum of two of them shall be equal to the third (though not superposable in any part). I believe that it has been hitherto held that this equality is impossible in the ellipse and hyperbola, without the addition of *some algebraic quantity*. I should have wished therefore to have added some numerical illustration of such a result, but the length of the calculation has hitherto prevented me from doing so.

II. *Researches towards establishing a Theory of the Dispersion of Light. No. III.*

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Introductory Remarks.

IN two former portions of researches on the subject of dispersion *, I have discussed all the observed refractive indices for definite rays, in different media, which had come to my knowledge; consisting of those for ten media determined by FRAUNHOFER, and those in ten other cases by M. RUDBERG, comparing them with the calculated results of the theory of M. CAUCHY; and the agreement is sufficiently close. In those papers, and elsewhere, I have remarked the importance of extending the inquiry, especially to media of higher dispersive power; in which cases (as appears from the nature of the formula) the theory would be put to a more precise test.

In the former instances the work of determining the indices was done to my hands, and I could proceed to the theoretical computations with the most perfect confidence in the accuracy of experimental data, furnished from the labours of observers so well known for precision and skill, and obtained, too, before the formula of theory had been deduced.

In any comparison of theory with experiment, it is, in all points of view, far more satisfactory that such comparison should be made with the observations of others rather than those of the theoretical computer himself. In the present instance, however, this desirable condition has not been fulfilled. Though the importance of obtaining a series of indices for the standard rays in different media had been long since pointed out and acknowledged by the most eminent philosophers, yet no observer was found to undertake the task of carrying on the work which FRAUNHOFER and RUDBERG had so successfully begun. I was thus left to make an attempt myself to supply the deficiency; and my observations were communicated, and printed copies distributed, to the Physical section of the British Association at the Bristol meeting in August 1836†.

From the remarks prefixed to those results, the scientific reader will, I trust, be sufficiently enabled to judge of the nature and degree of accuracy of the observations.

* Philosophical Transactions, 1835, Part I.; and 1836, Part I.

† This tract now forms one of the series of memoirs published by the Oxford Ashmolean Society.

I conceive, in general, the indices deduced may be relied on as exact, to at least three places of decimals. Some exceptions are noticed in the tract as less certain, viz. the oils of Angelica, Cummin, and Pimento, and the balsam of Peru. Such as they are, however, these results form (as far as I am aware) the only existing data for pursuing the comparison with theory. But I trust they may not be thought insufficient, when we consider that in the present stage of the inquiry the object to be aimed at seems chiefly such a general comparison as may enable us to see whether the main principle of the undulatory explanation of the dispersion is applicable, with a sufficient approach to precision, to encourage us to pursue the theory, or whether it must be abandoned, and some new principle sought.

With respect to media of low dispersive power, little doubt can exist. I have therefore not thought it worth while to go through the calculations for many of this class, but have confined my examination in the present instance chiefly to the higher cases to which my observations have extended.

Method of Calculation.

It may be necessary to premise a notice of the method of calculation adopted in the present communication, as it differs from that employed in my two former papers. That method consisted in finding, in the first instance, by a tentative process, (virtually equivalent to assuming the two extreme indices from observation,) the fundamental arc θ, from which the others were derived on dividing by λ for each ray, so as to fulfill the conditions of the approximate formula

$$\frac{1}{\mu} = \mathrm{H}\left\{\frac{\sin\left(\frac{\theta}{\lambda}\right)}{\left(\frac{\theta}{\lambda}\right)}\right\}.$$

It also appears from the investigations given in several consecutive papers in the London and Edinburgh Journal of Science, &c., that this formula is in fact obtained by supposing the sum of a series of analogous terms collected into one, with a common constant coefficient H. This simplified hypothesis (from the accordances already obtained) is evidently very near the truth for the whole range of media hitherto examined.

Professor Sir W. R. Hamilton afterwards pointed out (besides a direct process for performing this approximate calculation) a method of investigating the exact expression when we do not allow the above assumption as to the coefficients, viz.

$$\left(\frac{1}{\mu}\right)^2 = \mathrm{S}\left\{\mathrm{H}^2\left\{\frac{\sin\left(\frac{\theta}{\lambda}\right)}{\left(\frac{\theta}{\lambda}\right)}\right\}^2\right\}.$$

This is explained at large in the journal just named*; and in a subsequent number†

* March 1836. † August 1836.

I have stated some further particulars illustrative of the process. It is this method which I shall use in the present investigation, and therefore here merely quote the resulting formula from the last-mentioned paper. It is as follows;

$$\mu_l - \mu_\gamma = a_l(\mu_\gamma - \mu_\alpha) + b_l(\mu_\beta - 2\mu_\gamma + \mu_\alpha):$$

where the three indices of refraction are assumed from observation for the respective rays B, F, and H, in the particular medium; while μ_l is the index sought, any other of the four remaining of the seven standard rays, corresponding to which a_l and b_l are constants for the ray, independent of the particular medium, and which have been found from the values of λ in the paper last referred to, as follows:

$$\log a_c = \bar{1}\cdot 95433 \quad \log b_c = \bar{2}\cdot 65253$$

$$\log a_D = \bar{1}\cdot 80441 \quad \log b_D = \bar{1}\cdot 06281$$

$$\log a_E = \bar{1}\cdot 49646 \quad \log b_E = \bar{1}\cdot 03196$$

$$\log a_G = \bar{1}\cdot 74027^* \quad \log b_G = \bar{1}\cdot 62954^*.$$

The nature of the process of computation is obvious from an inspection of the formula; and the data here stated will suffice for those who may wish to verify the calculations. I proceed to give the results in a tabular form, and then to offer such general conclusions as I think may be safely derived from them.

Comparison of Refractive Indices from CAUCHY'S *Theory and from observation.*

1. Nitric Acid.				3. Sulphuric Acid.			
Ray.	Index observed.	Index calculated.	Difference.	Ray.	Index observed.	Index calculated.	Difference.
B	1·3988			B	1·4321		
C	1·3998	1·3998	·0000	C	1·4329	1·4329	·0000
D	1·4026	1·4024	− ·0002	D	1·4351	1·4349	− ·0002
E	1·4062	1·4058	− ·0004	E	1·4380	1·4375	− ·0005
F	1·4092			F	1·4400		
G	1·4155	1·4156	+ ·0001	G	1·4440	1·4448	+ ·0008
H	1·4211			H	1·4468		
2. Muriatic Acid.				**4. Oil Angelica.**			
B	1·4050			B	1·4836		
C	1·4065	1·4061	− ·0004	C	1·4863	1·4849	− ·0014
D	1·4095	1·4089	− ·0006	D	1·4887	1·4882	− ·0005
E	1·4130	1·4125	− ·0005	E	1·4932	1·4924	− ·0008
F	1·4160			F	1·4963		
G	1·4217	1·4222	+ ·0005	G	1·5049	1·5035	− ·0014
H	1·4265			H	1·5099		

* The two last numbers include the correction of an error which I detected in those given in the paper referred to.

5. Oil Cummin.

Ray.	Index observed.	Index calculated.	Difference.
B	1·5023		
C	1·5043	1·5039	− ·0004
D	1·5075	1·5079	+ ·0004
E	1·5130	1·5136	+ ·0006
F	1·5196		
G	1·5326	1·5317	− ·0009
H	1·5432		

6. Oil Sassafras.

B	1·5252		
C	1·5267	1·5268	− ·0001
D	1·5312	1·5315	+ ·0003
E	1·5380	1·5376	− ·0004
F	1·5441		
G	1·5568	1·5569	+ ·0001
H	1·5687		

7. Oil Pimento.

B	1·5281		
C	1·5317	1·5300	− ·0017
D	1·5347	1·5348	+ ·0001
E	1·5422	1·5412	− ·0010
F	1·5478		
G	1·5599	1·5605	+ ·0006
H	1·5719		

8. Kreosote.

B	1·5327		
C	1·5341	1·5344	+ ·0003
D	1·5387	1·5392	+ ·0005
E	1·5457	1·5455	− ·0002
F	1·5520		
G	1·5645	1·5646	+ ·0001
H	1·5762		

9. Balsam of Peru.

B	1·5846		
C	1·5869	1·5869	·0000
D	1·5931	1·5935	+ ·0004
E	1·6029	1·6028	− ·0001
F	1·6130		
G	1·6336	1·6336	·0000
H	1·6533		

10. Oil of Anise. Temp. 21°·2.

B	1·5452		
C	1·5473	1·5473	·0000
D	1·5533	1·5533	·0000
E	1·5623	1·5615	− ·0008
F	1·5705		
G	1·5883	1·5887	+ ·0004
H	1·6061		

11. Oil of Anise. General Mean.

Ray.	Index observed.	Index calculated.	Difference.
B	1·5469		
C	1·5489	1·5490	+ ·0001
D	1·5553	1·5551	− ·0002
E	1·5642	1·5635	− ·0007
F	1·5726		
G	1·5904	1·5909	+ ·0005
H	1·6082		

12. Oil of Anise. Temp. 15°·2.

B	1·5486		
C	1·5506	1·5509	+ ·0003
D	1·5574	1·5570	− ·0004
E	1·5661	1·5656	− ·0005
F	1·5747		
G	1·5926	1·5932	+ ·0006
H	1·6103		

13. Sulphuret of Carbon. Temp. 22°.

B	1·6145		
C	1·6176	1·6178	+ ·0002
D	1·6272	1·6269	− ·0003
E	1·6405	1·6392	− ·0013
F	1·6521		
G	1·6763	1·6776	+ ·0013
H	1·7009		

14. Sulphuret of Carbon. Temp. 12°.

B	1·6234		
C	1·6258	1·6268	+ ·0010
D	1·6369	1·6361	− ·0008
E	1·6496	1·6485	− ·0011
F	1·6616		
G	1·6864	1·6870	+ ·0006
H	1·7103		

15. Oil of Cassia.

B	1·5885		
C	1·5918	1·5913	− ·0005
D	1·6017	1·6000	− ·0017
E	1·6155	1·6135	− ·0020
F	1·6295		
G	1·6607	1·6646	+ ·0039
H	1·7002		

General Conclusions.

With regard to some of the first media above compared, the agreement of observation and theory is sufficiently near; but they are low in the scale of dispersion. The cases before mentioned as being somewhat uncertain do not allow of a close comparison. The balsam of Peru (though only from approximate data) and the kreosote, present higher numbers, and the agreement is therefore the most satisfactory. This is also the case with oil of anise seed, though perhaps not to the same degree; but the differences are probably not greater than what may reasonably be allowed.

In all these cases, however, we trace some uniformity in the character of the differences; theory being always in defect for the ray E, and in excess for G. When we proceed to the sulphuret of carbon this becomes more apparent; and the differences increase for the more refrangible rays. And lastly, in oil of cassia the same regular order of deviation is more marked, and the differences much greater, especially in the rays E and G. This regularity of character, as well as increase in the amount of the discrepancy, at once shows that it is at least partly due to some other cause than errors of observation. Yet we must remark, that even in these cases there is a sort of general accordance preserved between observation and theory, the agreement being still accurate to the second place of decimals.

The following considerations, then, suggest themselves:

1. It is to be observed that in all these cases closer accordances might be obtained if we took slightly different values for the assumed indices of B, F, and H; such as would be consistent with the probable errors of observation. In oil of cassia, however, I have found that the errors, even when thus distributed among all the indices, are still too great.

2. The constants a and b are derived solely from the values of λ for each ray, taken from the well-known determinations of Fraunhofer from interference. It is doubtless possible that these determinations may be affected by errors. In computing, by the method here used, some of Fraunhofer's indices, in which small discrepancies were found, Sir W. R. Hamilton undertook to investigate what amount of alteration in the values of λ would account for those differences. He communicated his researches in a letter to myself, with the values of λ, and consequently those of a and b, thus altered. I have repeated the calculation for oil of cassia, but find this change quite insufficient to remove the discrepancy; and that, in fact, a much larger alteration than could for a moment be allowed in Fraunhofer's data, must be supposed, in order to produce any sensible effect.

3. The entire method of computation here followed, though founded on the exclusion of those approximate suppositions which are allowed in the simpler formula, is yet dependent on the omission of terms in the series for μ beyond the three first*.

* See London and Edinburgh Journal of Science, &c., March 1836, eq. 9 and 13.

Since in this analysis we know nothing of the values of the coefficients of this series, but proceed solely by the elimination of them, it is still possible that in the neglected terms we may find the means of explaining the observed discrepancy. In a word, it remains an important subject for future research whether a further prosecution of the analysis, either by this method or by that which M. Cauchy has himself recently pointed out, or that of Mr. Kelland, may not lead to more successful results.

For the present I will only remark, by way of recapitulation, that upon the whole I conceive the formula, as already deduced from the undulatory theory, applies sufficiently well to the case of media whose dispersion is as high as that of oil of anise seed. It also represents, with a certain general approximation to the truth, the indices of some more highly dispersive bodies. It is therefore extremely probable that the essential principle of the theory has some real foundation in nature. While, looking at the regularity of the deviation, it seems likely that the formula only requires to receive some further development or extension, in order to make it apply accurately to the higher cases, while it shall still include the simpler form which so well accords with the lower.

October 10, 1836.

III. *On the Optical Phenomena of certain Crystals.* *By* H. F. Talbot, *Esq. F.R.S.**

Received April 20,—Read May 5, 1836.

SOME time ago I had the honour to communicate to the Royal Society an account of my invention of the polarizing microscope†. This instrument possesses so great a power of developing the internal structure of transparent bodies, even in their minutest visible particles, that I feel confident the employment of it will lead to many new and interesting results. At present I mean to confine myself to the description of a phenomenon which shows strikingly the beautiful order and regularity with which nature disposes the fabric of some of her minutest visible works.

The object I speak of is a kind of minute crystallization which may be obtained in peculiar circumstances, and I doubt not, in many different ways; but the manner in which it has presented itself to my observation is as follows.

A crystal of borax is placed in a drop of phosphoric acid, somewhat diluted, upon a plate of glass, and then moderately heated until the crystal dissolves in the acid. It is then set aside to crystallize. It is well to prepare a number of these plates at once, varying the relative proportion of the acid and salt, in order that the desired kind of crystallization may be found in one or other of them; for there is a considerable variety in the crystalline forms obtained by this method, some of which indeed are very singular. But when that kind of crystallization takes place which it is more particularly my intention to speak of, the field of view of the microscope is seen covered with minute circular spots, each of which is like a tuft of silk radiating from a centre, and is composed of a close assemblage of delicate acicular crystals forming a star. But besides these, are seen interspersed among them a number of circular transparent bodies, which are evidently modifications of the former, being, in fact, tufts or stars of acicular crystals in such close assemblage as to be in optical contact with each other and to produce the appearance of a single individual. Now let us suppose a group of these circles to be under examination with the polarizing microscope, and when the polarizers‡ are crossed, we observe the following phenomenon. The field

* This paper appeared in the Philosophical Magazine for October 1836, *after* it had been ordered by the Council to be printed in the second part of the Philosophical Transactions for that year. The circumstance arose entirely from a mistake of the person to whom the superintendence of the printing both of the Transactions and Magazine is entrusted, and neither Mr. Talbot, the Council, nor the Officers of the Royal Society had any cognizance of the error till the paper appeared in the latter publication.

† See London and Edinburgh Philosophical Magazine, vol. v. p. 321.

‡ By this term, for the sake of brevity, I here designate the polarizing and analysing prisms of single-image calcareous spar, or the plates of tourmaline which may be employed in their stead.

of view being dark, the little circles become luminous, and we see upon each of them a well-defined and dark cross, dividing the crystal into four equal parts. All these crosses are placed similarly, and are parallel to each other, and their direction remains unaltered when the crystals are turned round in their own plane by revolving the plate of glass upon which they stand. This beautiful appearance can be seen with a moderate magnifying power. I measured the diameter of some of the larger crystals, which I found to be from $\frac{1}{1\,1\,0}$ to $\frac{1}{2\,0\,0}$ of an inch. But there are many much smaller, and indeed they may be seen decreasing in size, until nothing remains visible of their structure but the four luminous quadrants, appearing like four minute dots of coloured light placed close together.

I proceeded to examine the circles with a high magnifying power, and under favourable circumstances of illumination, and I observed in them a very admirable structure.

Each circle has upon it one or more coloured rings arranged concentrically, but the number as well as the colour of these rings is different in different individuals.

The innermost ring is deeply coloured or black, and incloses a central space of white light, which is traversed by the arms of the cross intersecting in the centre. This part of the cross, which stands within the innermost ring, is beautifully well defined, and perfectly black. The general appearance resembles the figure 98, in Brewster's Optics, which is a representation of the rings seen in uniaxal crystals. It especially resembles it in the circumstance above mentioned, viz. the more defined outline of the part of the cross which is within the innermost ring.

We have hitherto supposed the polarizers to be crossed, but if we place them in a parallel position we shall see a phenomenon complementary to the above. The circle now presents four patches of coloured light, one in each quadrant; and we generally see near the centre four black or obscure spots, which correspond to the arms of the cross in the other position.

Such is an outline of the microscopic appearances presented by these little crystals, which are probably the minutest bodies in which so complicated an optical structure has hitherto been witnessed. I find that the smaller the circles are, the more perfect is their form and the brighter their colours.

These crystals, as I have already observed, probably consist of spicula diverging from a point, but which are in the closest possible contact, and in a state of complete mechanical cohesion. It seems to follow as a consequence from such a structure that their density must increase from their circumference towards their centre. Now it is worthy of remark, that Sir David Brewster has discovered very similar phenomena by polarized light in the crystalline lenses of certain fishes, which are known by direct experiment to increase in density towards the centre. Indeed the figure which he has given of the lens of the codfish in the Philosophical Transactions for 1816 (Plate XII. fig. 1.) is so like the appearance of one of the crystals which I have described, that it might be supposed to have been intended for a representation of it.

Having pointed out this resemblance, I may also mention another class of facts to which I think those I have described possess a considerable analogy. I mean the optical figures which BREWSTER has discovered in spheres of glass whose density was rendered variable by heating them.

He says* that, "if we take a cold sphere of glass and immerse it in a trough of hot oil, placed in a polarizing apparatus, we shall observe *a black cross with four sectors of polarized light*. If the sphere is turned round it will exhibit in every position the very same figure. If we now suppose the trough to be filled with such spheres they will exhibit the same phenomena in whatever direction the polarized light is transmitted through them, and even if they were in a state of motion. A fluid composed of such spherical particles would exhibit the same polarizing structure in every possible direction, and even if it were in a rapid state of gyration. If the particles possessed the structure that produces circular polarization the fluid would develop the phenomena exhibited by oil of turpentine, &c."

And again†, "The structure of the particles of a circularly polarizing fluid must be exactly the same along every one of its diameters; that is, the structure must be symmetrical round the centre of the particle, or analogous to that which takes place in common polarization when a sphere of glass has its density regularly increasing or regularly diminishing towards its centre."

I have quoted these remarkable passages at length, because it appears to me that what is there advanced merely as a hypothesis, acquires a considerable degree of probability from the facts which I have stated, since I have succeeded in rendering actually visible circular particles of excessive minuteness, in each of which the microscope detects the very structure imagined by BREWSTER, viz. the black cross and four sectors of light. So that it appears not improbable that the circular-polarizing properties of fluids may be owing to the presence of multitudes of particles similar to these, which they hold in solution.

* Library of Useful Knowledge, art. "Polarization of Light," p. 51. † Ibid. p. 45.

IV. *Further Observations on the Optical Phenomena of Crystals.*

By H. F. Talbot, *Esq. F.R.S.*

Received October 26,—Read December 15, 1836.

§ 1.

In my former paper on this subject I have described the remarkable circular mode of crystallization which sometimes occurs when borax crystallizes from a solution in phosphoric acid.

I have stated that when examined by the polarizing microscope, a black cross and four sectors of light are seen upon each crystal; and upon that kind which is most easily and frequently obtained, there are seen in addition one or more rings of vivid colour. Some deviations, however, from this usual form occur occasionally; one of which, being extremely beautiful as a microscopic object, deserves a separate mention. This variety of crystalline circles differs from the one first described in the following particulars.

1. The circles are much larger, attaining the diameter of $\frac{1}{10}$th of an inch; whereas those first observed did not exceed $\frac{1}{100}$dth of an inch in diameter.

2. They are flat, whereas the former ones were convex, and frequently I believe of a spherical form.

3. In consequence of which probably, they are seen to exhibit no coloured rings, but only a cross.

4. The cross is brightly coloured, red, green, blue, &c. upon a white ground (the polarizers being supposed to be parallel to each other). This has a beautiful appearance, especially when several circles seen at once have crosses of different colours.

5. When the polarizers are placed at right angles, the phenomenon complementary to the above is seen. For instance, the circle which presented a red cross upon a white ground now presents a black cross upon a green ground.

6. In an intermediate position of the polarizers, the circle just mentioned presents a red cross alternating with a green one, thus dividing the circle into eight sectors of coloured light.

Other circles present other colours, but they all follow the same analogy, and the crosses upon all the crystals are in a parallel position.

7. These crystals last longer than the former ones. I have found some of these retain their structure for two months; the former kind seldom last in perfection more than a day.

8. It sometimes happens that their circumference is imperfect, and presents a

notched or jagged outline. These have a very beautiful appearance, and have been almost universally compared by those who have looked at them to highly coloured flowers with four petals; the cross upon them being so dark as to have the appearance of being a division between the petals.

All the circles, when viewed by common light, appear transparent, white, and very uniform. If they are composed of acicular crystals diverging from a point, these latter must be exceedingly slender and numerous, and in perfect optical contact, since a high magnifying power does not render them separately visible.

§ 2.

With respect to the chemical nature of these crystals, it appears to me evident that they consist of boracic acid. They are obtained by dissolving borax in phosphoric acid; and it may be inferred that this latter substance unites with the alkali and isolates the boracic acid. In order to see if this supposition were correct, I dissolved boracic acid in alcohol, and I found that a drop of this solution evaporated on a plate of glass frequently yielded an abundant crop of the crystalline spherules. But these are generally exceedingly small, requiring a high power to display in them the cross and four sectors of light, and they speedily grow opake; for which reason they are not so well suited for observation as those prepared by the former method. They establish the fact, however, that this mode of crystallizing is a property of the boracic acid. It is highly improbable that it should be peculiar to that substance, but I have not yet met with it in any other.

§ 3. *Explanation of some of the Optical appearances.*

1. When any doubly refracting crystal is examined with the polarizing microscope, (the polarizers being transverse to each other, and the field of view consequently dark,) if it be turned round in one plane, it is seen to grow four times *luminous* and four times *dark* in the course of one revolution. This I have found to be universally the case with all the substances which I have tried, and it also is in accordance with theory.

2. In the case of an acicular crystal, one of the optical axes always coincided with the axis of figure, or length of the crystal; so that if a crystal of this sort appears unilluminated, all the others that are either parallel to it or perpendicular to it are likewise dark.

3. It results from the above that a circle composed of acicular crystals diverging from a point, must present the appearance of a black cross, and that the crosses on all the circles will be parallel.

4. With respect to the rings of colour, they are a consequence of the variable thickness of the crystalline circle at different distances from its centre. Their being visible, and indeed very conspicuous, upon a body of such small diameter, arises from the very energetic action of boracic acid upon polarized light.

§ 4.

I have remarked that the circular crystallization of boracic acid is frequently entirely superseded by other modes of crystalline formation; which circumstance appears to be chiefly owing to the presence or absence of combined water. Some of these variations deserve to be particularly specified.

1. Instead of circles there often occurs a formation of crystals resembling *two opposite sectors of a circle combined.* This form may be traced in different crystals, from its commencement when the angle of the sector is small, through all degrees of increase, until at length the opposite sectors unite and form a complete circle.

2. The crystals are frequently of a very irregular elongated shape, which does not approximate either to a prismatic or a cylindrical form. This stem, as it may be termed, subdivides itself *at both extremities* into an immense multitude of diverging fibres, giving it the appearance of a bundle of elastic filaments firmly held together in the central part, but with its extremities left at liberty to diverge*.

3. Another variety resembles in the same way irregular stems or branches, which, however, instead of being subdivided, are abruptly truncated at both extremities perpendicularly to the general line of their direction.

4. Sometimes, on the contrary, the ramification is much more developed, and then resembles two plumes united by a common stem.

5. Crystals of regular geometric form. These appear to require the presence of combined water.

Whichever of these formations occurs, it is for the most part seen in all the crystals at once, to the exclusion of any of the other forms.

§ 5.

These crystals generally undergo a spontaneous change in the course of one or two days after they have been formed. Those (No. 4) resembling plumes usually break up and resolve themselves into small rhombs and other geometric forms. The elongated crystals (Nos. 2, 3) undergo a remarkable change. They become traversed with innumerable fissures transverse to their length, and thus break up into thin plates, which either cohere loosely or separate entirely.

§ 6.

All these forms are very pleasing objects for the polarizing microscope. This arises from the very high depolarizing power of boracic acid, which enables its thinnest plates to exhibit colours of great variety and brilliancy, and causes even its dust or smallest particles to appear luminous. The more energetically any substance acts upon polarized light, the closer and more crowded are the bands and lines of

* This appearance is not very uncommon in the crystallization of other substances, though I believe it has not yet been described. The divergence of the filaments suggests the idea of electrical repulsion as being at least its primary cause.

colour which appear upon its crystals. These isochromatic lines, of which there are frequently many alternations, denote lines of equal thickness in the crystal. In the case of boracic acid, when anhydrous or nearly so, these lines are more crowded than in any other crystal that I have yet examined, insomuch that to exhibit them distinctly is as fine a test of the performance of a microscope as to resolve the more difficult lines on the scales of a butterfly's wing, or any other of the known test-objects. And in many cases the microscope only indicates the existence of a still more delicate structure, which, at least in its present state, it has not power distinctly to exhibit.

§ 7. *On Analytic Crystals.*

I now come to describe a property of crystals which I met with while employed in pursuing the above investigation. This is the power which certain crystals have of analysing polarized light in a manner analogous to the tourmaline; for which reason I shall propose for them the name of *Analytic Crystals.*

If I am not mistaken, this property has been hitherto confined to the tourmaline and a few other natural minerals; and it has not been known that their effects could be imitated, much less surpassed, by crystals artificially made. I trust, therefore, that it will be of some interest to describe a method of procuring such crystals.

In the following experiments it will be understood that the *analysing plate* of the microscope (or the polarizer next the eye) is removed.

1. A good example of this kind of crystal is obtained by dissolving the sulphate of chromium and potash in tartaric acid by the aid of heat. A drop of this solution placed on a plate of glass soon yields by evaporation filmy crystals, which very frequently have the characteristic property of tourmaline: that is to say, that if polarized light is transmitted through them, in one position they suffer it to pass freely, but if they are turned round 90°, they arrest and absorb it entirely.

When the experiment has been successfully conducted, the crystals will not in this position allow the smallest portion of light to pass.

If now we consider the extreme thinness of these crystalline films, it will appear how energetic must be their action upon light: since although white and transparent, they are able to produce an absorption equalling that of the best tourmalines, notwithstanding that the effect of the latter is aided by their natural dark colour.

But if these crystals are analogous to the tourmaline, they must have the power which that substance has of analysing the light that has been transmitted through other crystals.

Accordingly, if we place in the path of the polarized ray a plate of sulphate of lime of a proper thickness, the crystal, which before absorbed the light and appeared black, becomes splendidly coloured with that colour which the sulphate of lime produces, and which a tourmaline would show if it were employed as an analysing plate.

On reversing the polarization of the ray (or turning round the crystal), the complementary tint appears. The same results occur if the crystal is employed in the

first place to polarize the light, and tourmaline or calcareous spar is used to analyse it: so that the analogy or rather identity of effect with the tourmaline is complete.

I will now mention some other crystals which possess the analysing property, but not in such a high degree.

2. *Boracic acid.*—If dissolved in boiling water, it yields in cooling irregular crystals which have considerable analytic power. A crystal which in one position is so translucent as to be hardly distinguishable from the water in which it floats, is in the transverse position very strongly defined. It does not become dark all over, but *only in its outline.*

If now we employ it to analyse the tints of sulphate of lime, its *outline* becomes beautifully coloured. Nothing can exceed the delicacy of colouring which a number of these crystals exhibit when viewed together: those which lie in one direction appearing, for instance, *green*; those in a transverse direction *red*. The appearance is very unlike any other optical phenomenon that I know of, in consequence of two colours being seen in strong contrast, and without any intermediate tints; and also from the *outline* only of the objects being coloured, while their interior remains without colour. It is only when the crystals have a fibrous or striated structure that the tint extends over all their surface.

The boracic acid has the same analytic property and precisely the same appearance when it crystallizes from a solution of borax in phosphoric acid. The plumose crystals of it (No. 4. *suprà*) are very delicately coloured with the two opposite tints.

I obtained a very beautiful result by placing a drop of phosphoric acid upon a group of circular crystals. This caused a fresh deposition of boracic acid upon them as nuclei, which assumed the form of very delicate cilia, spreading in all directions as from a centre. These fringed circles showed the analytic property in an admirable manner, exhibiting four quadrants coloured alternately with complementary colours of great vivacity.

3. Another instance which is worthy of mention is the *oxalate af potash and chromium*, a salt whose optical properties have been investigated by Sir David Brewster*. If some gum arabic is added to a solution of this salt, and a drop of it put between two plates of glass, it abandons its usual mode of crystallization for another, which resembles a microscopic vegetation composed of minute prisms growing one out of another, and variously arranged in sprigs and branchlets; while in other places it assumes an undulating capillary form, much resembling in miniature the tufts or locks of a species of *conferva* which is seen growing in pools of water or in the sea. Now these objects are possessed of a high analytic power, insomuch that when a plate of sulphate of lime is placed beneath them they assume a colour of great intensity and splendour, which is changed for the complementary tint when the polarization of the incident ray is reversed.

4. *Nitre.*—If nitre and gum arabic are dissolved together in hot water, a drop of

* Philosophical Transactions for 1835, p. 91.

the solution put on a glass plate yields very good analytic crystals. These have a branched or plumose appearance, and assume beautiful colours in polarized light when a plate of sulphate of lime is placed beneath them. The microscope shows the colour to reside principally in the outline, but to the naked eye the whole film appears coloured. As these films may be obtained of large size, the phenomenon can be well seen by the unassisted eye.

A very interesting experiment, and one which throws much light upon the cause of these appearances, is to transmit a beam of polarized light *very obliquely* through a small prism of nitre immersed in gum, and viewed with the microscope. Its outline then generally exhibits two colours instead of one: for while the edge of the prism which is on that side from whence the ray of light comes is, for instance, of a red colour, the opposite edge will appear green. Reverse the polarization of the light, and these colours are exchanged one for the other. This observation enables us to explain the origin of the phenomenon in a satisfactory manner, and to show why it only occurs in crystals possessing strong double refraction, like nitre, in which the refractive indices of the two rays are materially different.

When a ray of common light is incident upon such a crystal, and therefore divides itself into two rays oppositely polarized, both rays are transmitted through the central parts of the crystal, which are bounded by parallel planes, or by planes approaching to parallelism. But when the bounding planes of the crystal are much inclined to each other, and therefore refract the light in the manner of a prism, the refractive indices of the rays may differ so much, that while one of them passes freely through such a prism, the other cannot pass at all, but suffers total internal reflexion, and is thereby dispersed; just as if the prism had a larger refracting angle with respect to that ray than to the other. Therefore if two oppositely polarized rays are presented successively to such a crystal, as in our experiment, one of them will be transmitted, and the other not. That this is the true explanation appears from this, that when the oblique planes are well formed and clearly defined by the microscope, the colour also is accurately limited by the same boundary: so that while this part analyses the tints of a plate of sulphate of lime, the rest of the crystal is inactive.

It may be inferred by analogy, that the same cause produces the analysing power of striated or fibrous surfaces, and of those in which the striæ are too minute to be discernible (as in No. 1. *suprà*, page 32): for it is not the property of all crystals with striated surfaces to have the analytic power, but only of such as are doubly refractive in a high degree.

I have said that the capillary crystals (No. 3.) possess the analytic property, although their diameter is often evanescent even with a microscope. An important inference may be drawn from this, viz. that a ray of light, *immediately* on entering one of these crystals, subdivides itself into two rays of different refractive indices, or at least that the thickness of crystal which is requisite to produce this effect is insensible to observation.

When nitre is made to crystallize in gum, it often shoots into prismatic crystals, which are very interesting objects, the more so, that they are of a permanent nature, and not liable to spontaneous change. When examined by polarized light, these prisms, in one position of their axis, frequently disappear completely. This arises from the refractive power of the gum being equal to that of one of the two rays in the crystal. Reverse the polarization of the ray, and the crystal appears, as it were, to start into existence, acquiring great strength and blackness of outline, and, not unfrequently, entire opacity. Again, when the sulphate of lime is interposed, this opacity disappears, and the crystal becomes brightly coloured.

Since it is probable that many better methods may be found of obtaining this kind of crystals than have hitherto presented themselves, I have hopes that it will be possible to obtain large and permanent artificial crystals, which may possess the advantages of the tourmaline without the inconvenience resulting from its dark colour.

V. *Observations on the Electro-chemical Influence of long-continued Electric Currents of Low Tension.* *By* Golding Bird, *F.L.S., F.G.S., Lecturer on Experimental Philosophy at Guy's Hospital, &c.* *Communicated by* Thomas Bell, *Esq., F.R.S.*

Received January 30,—Read February 2, 1837.

1. SCARCELY any branch of scientific investigation has yielded more important and interesting results than electro-chemistry, which, as is well known, in the hands of a Davy, demonstrated the existence of the alkaline metals, and formed a memorable and important epoch in the history of chemical science. Sir Humphry Davy availed himself of the energetic influence of voltaic currents in a high state of tension, and elicited by means of large batteries. To M. Becquerel, however, we are almost entirely indebted for our knowledge of the chemical agency of feeble currents in reducing certain refractory oxides to the metallic state, although it must not be forgotten that Dr. Edmund Davy applied the power of weak currents of electricity to the detection of the metallic poisons, by reducing them to the reguline state. Bucholz appears, in 1807, to have succeeded in obtaining crystals of metallic copper by the aid of a simple voltaic circle. But it is to our illustrious countryman Dr. Faraday that we owe our acquaintance with the interesting circumstance of the power possessed by a current of very low intensity (elicited by a single pair of platinum and zinc plates) in decomposing several saline combinations, as the iodide of potassium, sulphate of soda, &c., and isolating their respective constituents. In offering the following observations, I have not sufficient presumption to suppose them to be possessed of any very important or original value; but being, as they are, the results of carefully repeated experiments, and containing an account of what I believe to be some previously unobserved facts, I deem myself justified in submitting them to the notice of the Royal Society.

2. The facts recently pointed out by M. Becquerel of the energetic power exerted by weak electric currents in effecting the reduction not only of the oxides of copper, lead, or tin, but even of glucina, alumina, and silica, are probably very well known to every one. This philosopher obtained these interesting results by means of a single pair of plates, placing the solution of the metallic salt in a glass tube closed at one end by means of a plug of moistened clay, and immersed in a weak solution of common salt: on placing then a compound metallic arc formed of zinc and platinum in the solutions in such a manner that the platinum leg might be immersed in the tube containing the metallic solution, (to which M. Becquerel applies the general term of "negative tube,") whilst the zinc dips in the solution of salt, decomposition ensues, and after a lapse of time, varying from a few hours to some weeks, the metal is gene-

rally deposited from its solution on the platinum plate in a more or less crystalline form. M. Becquerel did not attribute the reduction of the metal to the electric current alone, but conceived that three distinct causes, at least, concurred in producing this effect. The decomposition of the water and of the common salt by the electric current set in motion, and the transference of hydrogen and soda through the clay diaphragm to the negative tube, where the alkali unites with the acid holding the metal in solution, causing the deposition of its oxide, which, while in its nascent state, is reduced by the hydrogen, and precipitated in its metallic form on the negative electrode; thus regarding the hydrogen furnished by the decomposition of the water as the actual reducing agent. In some cases, a fourth cause is supposed to be superadded to these, as when a body is used for the negative electrode, for which the metal in solution has a certain degree of affinity; a well-known example of which is found in the reduction of potassium from a solution of potassa when submitted to comparatively weak voltaic action in contact with mercury. Mercury is not the only metal applicable to this purpose, M. Becquerel having frequently used iron with success. He states that the solutions of the pure chlorides of zirconium, glucium, titanium, silicon, &c., refused to yield to the reducing action of weak electric currents, until after the addition of a small quantity of chloride of iron: this the current readily decomposed, precipitating the iron in a crystalline form on the platinum plate, (negative electrode,) which deposit speedily *induces* the commencement of the decomposition of the more refractory salts. This circumstance he attributes to the affinity of the iron for the other metal tending to the formation of an alloy, and expressly states, that when *perfectly pure* the above-mentioned chlorides *did not undergo the slightest decomposition*. I have ascertained, however, that if the electric current is *continuous*, notwithstanding its weak tension, this reduction and decomposition may be effected without the presence of iron, or indeed of any other metal in solution, excepting that which we are attempting to reduce. I must apologize for the above rapid sketch of some of M. Becquerel's researches, the introduction of which into this paper appeared to be necessary for the better understanding the results of my own more limited investigations.

3. On commencing my experiments on the chemical power of electric currents of weak tension, I soon found the want of an apparatus capable of affording an *equal* and *continuous* current of low intensity, which appeared to me to be absolutely necessary for the success of my experiments. In the common electromotor the currents are at first very energetic, but soon cease, or become so feeble as to be scarcely able to traverse a fluid medium unless fresh portions of the exciting fluid is added from time to time. Even Professor Daniell's very ingenious single-pair battery, although most excellent for electro-magnetic purposes, and affording considerable quantities of electricity, yet required the occasional addition of fresh acid and sulphate of copper, which was inconvenient when required to be kept in constant action for some weeks, besides occasioning an irregularity in the intensity of the current. The *quan-*

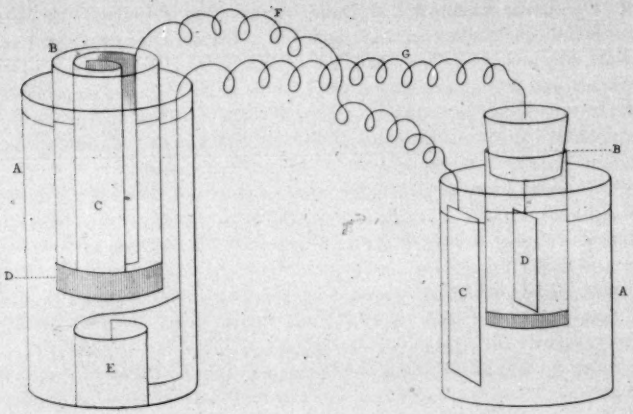


Fig. 1. Fig. 2.

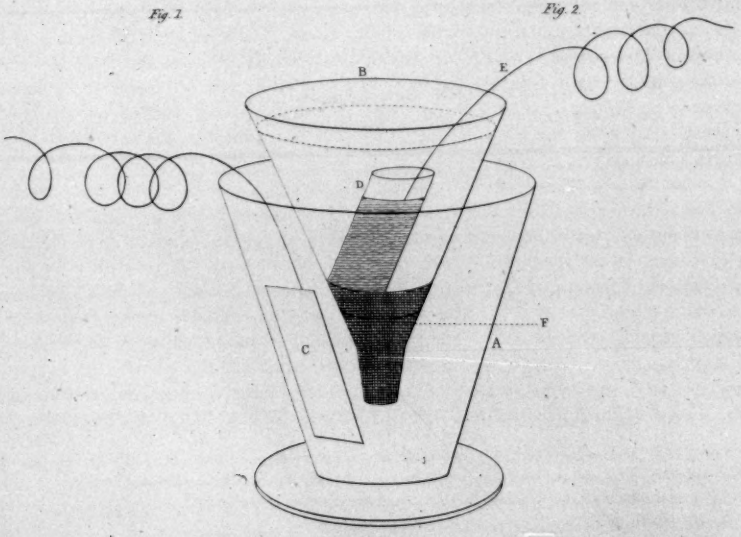


Fig. 3.

Fig. 1. The battery connected with the decomposing apparatus.

2. The decomposing apparatus.

3. A modification of Fig. 2. for obtaining the amalgams of the alcaline metals.

NB. The dotted line in Fig 3. shews the level of fluid in the funnel above the mercury.

tity of electricity appearing to be by no means so important as a *continuous and equable current*. For similar reasons Mr. Mullins's modification of Professor Daniell's battery was found equally objectionable; added to which the currents evolved are liable to be materially affected by the admixture of the exciting fluids through the membranous partition, which always takes place sooner or later by endosmosmic action.

4. After several experiments I was induced to prefer the following apparatus, (which after all is but a slight modification of Professor Daniell's,) in consequence of its affording a constant and regular current of electricity of very weak tension, continuing for several weeks or even longer without any fresh addition of exciting fluid. A glass cylinder, 1·5 inch in diameter and 4 inches in length, was closed at one end by means of a plug of plaster of Paris 0·7 inch in thickness: this cylinder was fixed by means of corks inside a cylindrical glass vessel about 8 inches deep and 2 inches in diameter. A piece of sheet copper, 6 inches long and 3 inches wide, (having a copper conducting wire soldered to it,) was loosely coiled up, and placed in the small cylinder with the plaster bottom: a piece of sheet zinc of equal size was also loosely coiled up, and placed in the larger external cylinder (being furnished like the copper plate with a conducting wire). The larger cylindrical glass being then nearly filled with weak brine, and the smaller with a saturated solution of sulphate of copper, the two fluids being prevented from mixing by the plaster of Paris diaphragm, the apparatus is complete*; and if care is taken that the fluids in the two cylinders are at the same level, will continue to afford a continuous current of electricity for some weeks, the sulphate of copper being very slowly decomposed. So feeble is the current evolved by an apparatus of this kind, that on connecting the two conducting wires with a common galvanometer, (having but one needle suspended on a pivot,) a deviation to only 10° or 12° took place: with Nobili's galvanometer, with nearly astatic needles, a deviation to 90° immediately ensued, as might be expected from the greater delicacy of the instrument. So small, indeed, is the quantity of electricity evolved by the apparatus I have described, as compared to that evolved by the ordinary electromotor, that I was unable to produce the simplest form of electromagnetic rotation by its aid. After it has been in action for some weeks, chloride of zinc is found in the external cylinder, and beautiful crystals of metallic copper, frequently mixed with the ruby protoxide (closely resembling the native ruby copper ore) and large crystals of sulphate of soda, are found adhering to the copper plate in the smaller cylinder, especially on that part where it touches the plaster diaphragm.

* So simple an apparatus scarcely requires an illustration: in the accompanying outline sketch I have, however, figured it, to prevent any error arising from the account in the text not being sufficiently explicit.

Fig 1. represents the battery connected with the apparatus described in paragraph 4.

A. The external cylinder.

B. The smaller one; with D. The plaster of Paris bottom.

C. The coil of copper in the cylinder B, having the conducting wire F soldered to it.

E. The coil of zinc with the wire G soldered to it.

5. If the two copper conducting wires belonging to the little apparatus just described are immersed in water acidulated with sulphuric acid, action soon commences, bubbles of hydrogen appearing at the wire connected with the zinc plate, whilst that connected with the copper plate became tarnished, oxydized, and at last partly dissolved, giving a blueish tint to the fluid; affording an approach to the decomposition of water by a single pair of plates. For the success of this experiment, it is, as might be expected, necessary that the *positive* electrode at least should be formed of a readily oxidizable metal; for when both wires were of platinum, no evidence of decomposing action ensued.

6. If, instead of acidulated water, the wires were immersed into a solution of nitrate or acetate of lead, no *immediate* action ensued, but in about fifteen minutes, or even less, some elegant and delicate feathers of metallic lead, which rapidly increased in size, appeared at the negative electrode. This effect did not occur when *both* conducting wires were of platinum; but when the *negative* electrode only was composed of that metal, the reduction of the lead continued with apparently increased energy. From these experiments, as well as many others of a similar kind which it is unnecessary to detail, it appears fair to presume, that in availing ourselves of the *reducing* agency of feeble currents, or at least of those elicited by a single pair of plates, it is necessary that the positive electrode should be composed of a readily oxidizable metal: thus using a kind of battery of *two* cells, in which the wires forming the electrodes, and the fluid submitted to experiment, form the contents of the second cell.

7. But few metallic solutions yield so readily as those of lead to the reducing agency of weak currents; and where a longer time and continuance of action is required to effect the reduction, the decomposing apparatus of M. Becquerel will be found a useful addition to the little battery (4.), with the substitution of a plug of plaster of Paris for one of clay. This piece of apparatus is, in fact, a counterpart of the battery itself, consisting, like it, of two glass cylinders, one within the other, the smaller one having a bottom or floor of plaster of Paris fixed into it: this smaller tube may be about half an inch wide and three inches in length, and is intended to hold the metallic solution submitted to experiment, the external tube in which it is immersed being filled with a weak solution of common salt *. Into the latter solution a slip of amalgamated zinc, (for the positive electrode,) soldered to the wire coming from the copper plate of the battery (4.), is immersed, whilst for the negative electrode a slip of platina foil, fixed to the wire from the zinc plate of the battery, passes through a

* Fig. 2. in the sketch represents this apparatus connected with the battery.

A. The larger tube.

B. The smaller one, with the plaster bottom.

C. The electrode of amalgamated zinc connected by the wire F with the copper plate C of the battery (Fig. 1.).

D. The negative electrode of platinum connected by the wire G with the zinc plate of the battery (Fig. 1.).

cork fixed in the mouth of the smaller tube, and dips into the metallic solution it contains.

8. When a solution of the chlorides or nitrates of iron, copper, tin, zinc, bismuth, antimony, lead, or silver, is placed in the smaller tube of the apparatus (7.), and connexion made with the battery * in the manner already described, action is almost instantly apparent, water is decomposed, and torrents of minute bubbles of hydrogen are evolved at the surface of the platinum plate, (negative electrode,) which continues for a short time, sometimes, indeed, lasting for hours; a circumstance depending apparently upon the degree of facility with which the metal under experiment is reduced. Thus with solutions of copper, scarcely a bubble appears, the metal being almost immediately reduced, all the hydrogen being probably employed from the instant of completing the circle, for that purpose: with solutions of lead, tin, or silver, the evolution of hydrogen continues for a short time only, and ceases as soon as the minutest portion of reduced metal appears on the platinum plate; but with solutions of iron and manganese the evolution of gas frequently continues for six, eight, or ten hours, or even longer; the evolution of hydrogen thus seeming to bear something like an inverse ratio to the ease with which metal is reduced. After the hydrogen has ceased to appear at the negative electrode, striæ of the reduced metal, which rapidly increase, are deposited on the surface of the platinum.

9. The metals thus reduced generally, but not invariably possess a perfectly metallic lustre, are always more or less crystalline, and often very beautifully so, affording a considerable contrast to the irregular soft spongy masses obtained from the same solutions by means of large batteries. The crystals of copper obtained by the process just detailed (8.), rival in hardness and malleability the finest specimens of native copper, which they much resemble in appearance. The crystallization of bismuth, lead, and silver by these means, is very beautiful, that of the former metal being lamellar, of a lustre approaching to that of iron, but with the reddish tint peculiar to this metal. Silver may be thus obtained of a snowy and indeed dazzling whiteness, usually under the form of needles.

10. The metallic solutions hitherto mentioned as yielding to the action of the little battery are, as is well known, equally acted on by larger voltaic batteries, consisting of a considerable number of alternations, the metal being reduced in a spongy form, often destitute of a metallic appearance. But there are some metals which are deposited from their solutions as oxides only, when acted on by currents from large batteries, and yet are deposited in a brilliant metallic form if submitted to the action of the currents from the little apparatus already described (4). Of these nickel is an example: a solution of its chloride or sulphate, when placed in the smaller tube of the decomposing apparatus (7.), yielding after some hours a crust of metallic nickel on the negative electrode, often of a silvery lustre on the surface immediately

* It may here be proper to remark, that by the word battery in the course of the following observations I always allude to the modification of Prof. Daniell's battery described in § 4.

applied to the platinum, that portion of the crust more in contact with the fluid being generally black, and frequently covered with a layer of the hydrated and gelatinous green oxide.

11. Finding that by means of this apparatus I could command a weak but continuous current capable of reducing even the more refractory metallic oxides, I was anxious to ascertain whether the current was sufficiently energetic to cause the reduction of those oxides which (as silica) do not yield to powerful batteries, and which M. Becquerel obtained only alloyed with iron.

12. The solution of silicon used by M. Becquerel was prepared by dissolving gelatinous silicic acid in hydrochloric acid of commerce, which always contains iron; this on being submitted to the action of a single pair of plates deposited an alloy of iron and silicon on the negative electrode. As this solution contains but a very small quantity of silicon, I substituted a solution of fluoride of silicon in alcohol obtained by passing a current of the gaseous fluoride into strong alcohol. On filling the smaller tube of the decomposing apparatus (7.) with this solution, and making the connexion with the battery in the manner already described, bubbles of hydrogen were copiously evolved at the surface of the platinum plate (negative electrode), which continued for eight or ten hours, when the platinum appeared to be tarnished, and in twenty-four hours a copious deposit of silicon had taken place on the platinum, to the surface of which it firmly adhered. Around the reduced silicon, and suspended in the fluid, was a dense gelatinous cloud of silicic acid. On quickly withdrawing the slip of platinum, dipping it in water, and then pressing it between folds of bibulous paper it was dried, and freed from any adhering solution. The silicon was nearly black and granular, under a lens, exhibiting a tendency to a crystalline form. It was not deposited on the platinum in a confused or irregular manner, but in longitudinal striæ, which appeared to follow the direction of certain lines of minute eminences on the surface of the piece of platinum, produced apparently by scouring it with fine sand and a piece of cork before being used for the construction of the negative electrode.

13. The silicon thus procured becomes of a snowy whiteness when ignited in the flame of a spirit lamp, and falls off the platinum in thin flakes, being in fact converted into silicic acid. It is not very easy to oxidate the whole, in consequence of the flakes of the acid forming an incrustation over the subjacent silicon, and protecting it from the oxidating influence of the air even at a red heat. A portion of the silicon *removed* from the platinum did not appear to dissolve in hydrochloric acid; but when the platinum itself with the firmly adhering silicon was immersed in the acid, slow action ensued, bubbles of hydrogen being evolved from the *exposed surface of platinum*, the silicon very slowly disappearing; the solution being probably occasioned by the formation of a simple voltaic circle, the silicon and platinum being the metals, and the acid the exciting fluid. When an aqueous solution of hydrofluosilicic acid is substituted for the fluoride of silicon, the metalloid is reduced, but slower and in

smaller quantity; differences arising in all probability from the smaller quantity of silicon present in the solution.

14. I have frequently had occasion to observe that when an aqueous solution of hydrofluosilicic acid has been submitted to the action of currents of low tension (from the battery already described (4.)) continued during two or three weeks, a considerable deposition of gelatinous silicic acid takes place around the reduced silicon; mixed with which, or precipitated in a zone on the sides of the tube, especially if of small diameter, or even upon the platinum electrode itself, frequently appear minute crystalline grains of sufficient hardness to scratch glass, and appearing translucent under the microscope. These minute crystals I have no hesitation in stating to be crystallized silicic acid, closely resembling its natural form of *quartzose sand*.

15. I next attempted to form potassium with the same apparatus, but failed, as I had anticipated, from the presence of water, which indeed would react on the potassium as soon as reduced. I therefore endeavoured to form its amalgam, well knowing that when dissolved in mercury a very weak electric current is sufficient to prevent the oxidating influence of water upon it; and by using a modification of the decomposing apparatus before described (7.) I succeeded perfectly. In place of the smaller tube containing the metallic solution, I used a small glass funnel*, the beak of which was carefully filled up with plaster of Paris: on this plaster floor I placed a piece of glass tube closed at one end, about 0·5 inch in length and 0·2 inch in diameter, and half filled with pure mercury; this tube was not placed vertically, but inclined so as to form an angle of about 40° with the plaster floor of the funnel, which with its contents was partly immersed in the weak brine contained in the larger cylinder of the decomposing apparatus. The external cylinder communicated as before with the copper plate of the battery, by means of a slip of amalgamated zinc dipping into the brine it contained. The funnel was then nearly filled with a solution of chloride of potassium, and a piece of platinum wire connected with the zinc plate of the battery being twisted into a flat spiral at one end so as to present a larger surface, was immersed in the mercury contained in the little tube submerged in the saline contents of the funnel. The circuit being thus completed, galvanic action soon became apparent, bubbles of hydrogen being evolved from the surface of the mercury (which now formed the negative electrode) in a very curious manner, not in confused and rapid streams, but in large and distinct bubbles, which very slowly appeared, and performed several gyratory movements on the surface of the fluid metal before they were evolved. Not unfrequently a single bubble only was seen, which continued

* This variety of the apparatus is shown in fig. 3. of the outline sketch.

A. The external vessel containing the salt and water.

B. The funnel containing the alkaline salt required, with its plaster floor F.

C. The positive electrode of amalgamated zinc communicating with the copper plate of the battery (fig. 1.).

D. The little tube containing mercury immersed in the alkaline solution with a platinum wire E, connected with the zinc plate of the battery (fig. 1.) dipping into it.

playing on the surface of the mercury for half an hour, or even longer, before it rose to the surface of the fluid. In about eight or ten hours the mercury had swollen to double its former bulk, and part of it had actually crept* up the platinum wire to the height of 0·3 inch above the level of the other portion, adhering to the wire like so much tenacious mucilage. On dipping a piece of turmeric paper into the contents of the funnel it turned brown, demonstrating the presence of an alkali. The mercury was removed from the little tube as quickly as possible, and poured into distilled water, which acted upon it, causing the evolution of hydrogen gas from its whole surface, and became alkaline from the formation and solution of the oxide of potassium or potass. The film of mercury adhering to the platinum wire remained on it for some days, giving it the appearance of having been amalgamated. This experiment, several times repeated, yielded precisely similar results, from which I think that I am justified in stating that potassa can be reduced by means of the feeble current elicited by a single pair, or as the positive electrode was formed of an oxidizable metal, in the opinion of some, perhaps of two pairs of plates.

16. By submitting in the same apparatus a solution of chloride of sodium to the influence of the battery, analogous results were obtained. An amalgam of sodium being formed, although a much longer time was required, and the result of the experiment, although quite decided, was by no means so distinct as in the case of the reduction of potassium.

17. But of all the saline solutions that I have yet submitted to experiment, none afforded such conclusive and interesting results as those of ammonia. The ammonium being reduced with almost as much ease as copper or tin, when a solution of its chloride (hydrochlorate of ammonia) is submitted to the action of the voltaic current in contact with mercury, in the same manner as chloride of potassium or sodium. The same adhesion and creeping up of the mercury along the wire (15.) is observed, and after a few hours the fluid metal swells to five or six times its former bulk. On removing it quickly and drying it, by allowing it to fall on bibulous paper the amalgam of ammonium is obtained of a buttery consistence, possessing a dull silvery colour, and yielding a peculiar crackling, or (if I may be allowed the expression) an emphysematous sensation to the finger on pressing it: on being immersed in water it very slowly gave off hydrogen, and yielded a solution of ammonia.

18. By far the most satisfactory method of obtaining this amalgam is by using for the negative electrode a piece of platinum wire coiled up at one end, after it has been amalgamated by dipping it into the ammoniacal amalgam obtained by the last described process (17.). A minute quantity of mercury is thus made to adhere to the wire, which being connected with the zinc side of the battery, is dipped into a solution of hydrochlorate of ammonia contained in the smaller tube of the apparatus used in effecting the reduction of silicon (7.). The circuit being completed, a few bubbles

* This peculiar creeping up of the mercury along the wire does not take place if the little tube holding the fluid metal is placed in a vertical position.

of hydrogen are disengaged from the amalgamated wire, which soon cease, and in an hour or two, a leaden grey spongy mass is observed adhering to the wire, which is sometimes sufficiently bulky to fill the tube, and putting on much of the external appearance of a mass of cellular galena. This mass consists of a spongy amalgam of ammonium, containing a very minute proportion of mercury; it is lighter than the solution in which it is immersed, for on adroitly separating a portion of it, it rises to the surface and rapidly decomposes water, hydrogen being evolved and ammonia formed.

19. It is a very curious and interesting fact, that although this spongy ammoniacal amalgam cannot be kept immersed in water even for a few instants without the formation of ammonia, yet as long as it is connected with the negative electrode of the battery, it may be preserved without change for days and weeks. The instant the connexion with the battery is broken, a mass of this amalgam, as large as a walnut, appears to vanish in a few seconds, torrents of minute bubbles being given off, and a scarcely appreciable quantity of mercury being left on the wire. On again closing the connexion with the battery decomposition recommences, and the amalgam is reproduced.

20. From a review of the results of these experiments, we cannot help being struck with the very energetic power of electric currents of weak tension; currents of sufficient energy to reduce to the metallic state oxides on which currents of higher tension from large batteries are comparatively powerless. This fact, although pointed out by Becquerel and other philosophers, has (as far as I am aware) never been before shown to hold good in the reduction of the alkaline metals. Potassium and ammonium not having, I believe, been previously obtained by the weak current emanating from the chemical action of saline solutions on a single pair of plates; and silicon, although obtained by Becquerel combined with iron, has not been procured before in a pure state by electric currents, at least by those of feeble tension*.

In conclusion I may be permitted to observe, that in applying weak electric currents to the reduction of metallic oxides, it is *absolutely necessary that a continuous current be employed, and that its cessation even for an instant is often fatal to the success of the experiment on hand*, which cessation or suspension, as far as my experiments have gone, the modification I have proposed of Professor Daniell's battery appears to be capable of obviating.

Guy's Hospital,
January 20, 1837.

* Some other curious circumstances connected with the decomposing electro-chemical power of currents of low tension have fallen under my observation, but have not yet been sufficiently examined to authorize their publication as facts. Some of these I may perhaps at a future period, with the permission of the Society, have the honour of submitting to its notice.

VI. *Inquiries respecting the Constitution of Salts. Of Oxalates, Nitrates, Phosphates, Sulphates, and Chlorides.* By Thomas Graham, *Esq. F.R.S. Edin., Professor of Chemistry in the Andersonian University of Glasgow, Corr. Member of the Royal Academy of Sciences of Berlin, &c. Communicated by* Richard Phillips, *Esq. F.R.S.*

Received June 23,—Read November 24, 1836.

FROM the results obtained in a former paper upon water as a constituent of sulphates, it seemed likely that a close analogy would generally be found to exist between any hydrated acid and the magnesian salt of that acid. The sulphate of water is constituted like the sulphate of magnesia; and so do I now find the oxalate of water to resemble the oxalate of magnesia, and the nitrate of water to resemble the nitrate of magnesia. Indeed it appears probable that the correspondence between water and the magnesian class of oxides (as we may call the metallic oxides isomorphous with magnesia) extends beyond their character as *bases*,—that in certain subsalts of the magnesian class of oxides we have the metallic oxide replacing the water of crystallization of the neutral salt, or discharging a function which was thought peculiar to water.

In the formation of a double sulphate a certain kind of substitution or displacement was observed, such as the displacement of an atom of water pertaining to the sulphate of magnesia, by an atom of sulphate of potash, to form the double sulphate of magnesia and potash. The same kind of displacement appears to occur likewise in the construction of double oxalates; and the tracing of it enables us to form an idea of the constitution both of the double and of the superoxalates, and to explain their derivation, as in the case of the sulphates.

I. *Of the Oxalates.*

The oxalates promised ample scope for investigation from their number and variety. For we have not only neutral oxalates, double oxalates, and binoxalates, but likewise an unparalleled combination, the quadroxalate of potash, of which the true constitution or proximate composition is a most interesting subject of inquiry.

1. *Oxalate of Water, or Hydrated Oxalic Acid.*

$$\dot{H}\,\dddot{C}\dddot{C}\,\dot{H}^2.$$

The recent and accurate experiments of Berzelius, Gay-Lussac, and Turner, leave no doubt that the crystals of oxalic acid contain three atoms of water. I find

the acid to crystallize with this proportion of water in a variety of circumstances, and believe that it is never deposited from its aqueous solution in any other state. Of these three atoms of water one atom is basic, which is expressed in the formula by placing its symbol *before* that of the acid; while the other two atoms of water are attached to this oxalate of water, and may be termed the *constitutional* water of the oxalate of water. These two atoms of water are found in the oxalate of magnesia, the oxalate of zinc, and the other oxalates of the magnesian class, as well as in the oxalate of water.

It is well known that oxalic acid can likewise exist in combination with no more than one atom of water (its basic water), and is obtained in that state by drying it at a temperature a little above 212° Fahr., or on subliming the hydrated acid by a higher temperature. I have made many experiments in order to discover whether, in the case of the other two atoms of water, one is retained more strongly than the other, or whether an oxalate of water with one additional atom of water, instead of two, could be obtained. The common crystals were dried at various temperatures, both in air and in vacuo, but either none of the water was lost, or the entire two atomic proportions. There is certainly no intermediate hydrate.

2. *Oxalate of Zinc.*

$$\dot{Z}n\,\dddot{C}\dddot{C}\,\dot{H}^2.$$

In the oxalate of water we observe a contracted solubility, and all the oxalates of the magnesian class of oxides are very sparingly soluble in water. They may be obtained by precipitation, on mixing a solution of oxalate of potash with sulphate of zinc, &c. Cold solutions of the salts were always made use of in our experiments; and the precipitates, which were always granular and more or less distinctly crystalline, were washed with cold water, and dried by exposure to the air for a week or two, without the application of artificial heat.

The oxalate of zinc is admitted to possess two atoms of water, and these I find are retained pretty strongly, as in the case of oxalate of water. It was observed that 24·95 grains of the salt lost only 0·44 grain by three days exposure to 212° Fahr.; but by a few hours at 315° Fahr. the salt lost in all 4·87 grains of water, and appears to have become anhydrous.

3. *Oxalate of Magnesia.*

$$\dot{M}g\,\dddot{C}\dddot{C}\,\dot{H}^2.$$

The oxalate of magnesia retains its two atoms of constitutional water very strongly, and it is doubtful whether they can be expelled without decomposing the salt; 13·74 grains of the salt lost only 0·32 grain by an exposure to 212°, protracted for several days; and by two days at 300° Fahr. the whole loss amounted only to 0·47 grain. 22·36 grains of the same salt, ignited, left 5·94 grains of caustic magnesia, or one part

of the salt contains 0·2656 magnesia. A salt constituted with two atoms of water should contain 0·2759 magnesia, of which the specimen analysed falls a little short, probably from containing some hygrometric moisture.

The *oxalate of manganese* lost nothing at 212°, and was found by analysis to contain 0·2416 water, which approaches very closely the quantity equivalent to two atoms, namely, 0·2474 water in one hydrated oxalate of manganese.

In regard to several other oxalates of this class, namely, the oxalate of the protoxide of iron, of oxide of nickel, of oxide of cobalt, and of oxide of copper, I believe it is impossible to obtain them in a state of sufficient purity for analysis. They appear to carry down with them portions of the precipitating salts; and they alter manifestly in appearance and composition during the progress of the washing, to which they must be submitted for the purpose of purification. In the case of oxalate of copper, which was examined most particularly, the results were so anomalous that no inference whatever could be drawn from them.

It will appear, however, that a neutral oxalate of copper with two atoms of water can exist but in combination with oxalate of potash, or with oxalate of ammonia, as a double salt.

None of the oxalates of the magnesian class of oxides is more soluble in oxalic acid than in water, and none of them combines with that acid to form a binoxalate. The crystals, which are obtained on mixing together solutions of binoxalate of potash and sulphate of magnesia, and which have been supposed to be a binoxalate of magnesia, are really a mixture of oxalate of magnesia and of quadroxalate of potash. Hence there is no combination of oxalate of magnesia with oxalate of water; which illustrates the fact that bodies of the same class, such as these two oxalates are, have no disposition to enter into union and form a new compound.

4. *Oxalate of Lime.*

$$\dot{C}a\,\dddot{C}\dddot{C}\,\dot{H}^2.$$

The oxalate of lime contains two atoms of water, like the oxalate of magnesia, but parts with its water more freely than that salt. Thus 12·06 grains of the hydrated oxalate of lime were found to lose 1·6 grain of water at 212° Fahr. in the course of two days, 1·68 grain in three days, 1·84 grain in six days, and nothing more in nine days. The salt originally consisted of 100 oxalate of lime united to 27·85 water, of which last it has lost 19·53 parts, and retained 8·32 at 212°. It is probable therefore that the constitution of hydrated oxalate of lime is the same as that of hydrated oxalate of magnesia, that oxalate of lime forms only one definite hydrate, containing two atoms of water, but that it parts with the whole of its constitutional water at a moderate temperature.

5. *Oxalate of Barytes.*

$$\dot{Ba}\,\dddot{C}\dot{C}\,\dot{H}.$$

This oxalate differs from all the preceding, and contains only one atom of water. It was formed by digesting an excess of oxalic acid upon carbonate of barytes, and afterwards washing the resulting oxalate with cold water. 20·60 grains of the oxalate, calcined by a low red heat, left 16·45 grains carbonate of barytes, equivalent to 12·77 barytes. Hence it follows that the oxalate consisted of

		Composition of $\dot{Ba}\,\dddot{C}\dot{C}\,\dot{H}$.
Barytes	100	100
Volatile matter	61·32	59·08
	161·32	159·08

Before being washed this oxalate had a sour taste, and the volatile portion of it amounted to 67·01 parts instead of 61·32; but it was evidently the neutral oxalate with a little free oxalic acid. It was not a binoxalate; nor did such a salt present itself on digesting the neutral oxalate in oxalic acid, so that I am constrained to deny the existence of a binoxalate of barytes. Indeed it is scarcely a matter of doubt that no supersalt whatever exists of barytes, strontian, lime, or of the magnesian class of oxides.

6. *Oxalate of Potash.*

$$\dot{K}\,\dddot{C}\dot{C}\,\dot{H}.$$

Oxalate of potash is known to crystallize from solution with one atom of water, and with no other proportion. The crystals speedily become white and opake at 212° Fahr., from the loss of water, but cannot, I believe, be made quite anhydrous at that temperature; at least a portion of the salt, which had been exposed to 212° for four days, still retained water, consisting of 100 oxalate of potash and 3·4 water, which is rather less than a third of the water which the salt originally contained (10·8 parts). The oxalate of potash becomes quite anhydrous when dried at 300°. Of salt so dried 100 parts reabsorbed 10·63 water in a damp atmosphere with the greatest avidity. The oxalate of potash has therefore a certain attraction for a single atom of water, and this is an important feature of the salt.

7. *Binoxalate of Potash.*

$$\dot{K}\,\dddot{C}\dot{C} + \dot{H}\,\dddot{C}\dot{C}\,\dot{H}^2.$$

This salt has hitherto been supposed to contain only two, but it certainly contains three atoms of water.

21·37 grains of the salt, calcined by a full red heat, which is necessary for complete

decomposition, left 10·14 grains of carbonate of potash. Allowing the potash an equivalent proportion of oxalic acid, the salt must consist by this experiment of

Potash	32·23
Oxalic acid . . .	49·38
Water	18·39
	100·

The water almost coincides with three atoms, which would amount to 18·42 per cent. of the salt.

In the formation of the binoxalate of potash, the constitutional atom of water of the neutral oxalate of potash appears to be displaced by an atom of hydrated oxalic acid; so that the formula of binoxalate of potash represents anhydrous oxalate of potash, followed by oxalate of water with two atoms of water, as given above. The same principle of derivation applies most happily to that anomalous salt, the quadroxalate of potash.

8. *Quadroxalate of Potash.*

Analytic formula . . . $\dot{K}\,(\dot{C}\dot{C})^4\,\dot{H}^7$.

Rational formula . . . $\dot{K}\,\dot{C}\dot{C} + \dot{H}\,\dot{C}\dot{C} + 2\,(\dot{H}\,\dot{C}\dot{C}\,\dot{H}^2)$.

The formula of the preceding salt is terminated by two atoms of water: let us replace them by two atoms of hydrated oxalic acid, and we have the quadroxalate of potash. We thus derive the quadroxalate from the binoxalate, in the same way that the binoxalate itself is derived from the oxalate.

There can be no doubt, from the accurate analysis of Berzelius, that this salt contains seven atoms of water. He found 100 parts of the quadroxalate of potash to yield by ignition 27·225 carbonate of potash. In an experiment in which 17·3 grains of the salt were ignited by us, there resulted 4·7 carbonate of potash; which is 27·11 carbonate of potash from 100 quadroxalate. Berzelius determined the water directly by igniting the salt with oxide of copper, and found it to amount to 24·8 per cent. of the salt. Calculated from our experiment, the water comes out 25·05 per cent., while the theory of seven atoms of water in the salt requires 24·72 per cent.

10·87 grains of this salt, dried by a nitre-bath, of which the temperature was 240°, lost eventually 1·46 grains; or 100 salt lost 13·43. Four atoms of water amount to 14·12 per cent. of the salt, to which the experimental result approximates sufficiently to prove that this salt parts readily with four of its seven atoms of water. These four atoms of water are evidently the constitutional water of the two atoms of hydrated oxalic acid, which the quadroxalate contains. When the salt is still more strongly heated, oxalic acid itself goes off, partly as a sublimate and partly in a decomposed state.

9. *Oxalates of Ammonia.*

The oxalate and the binoxalate of ammonia agree exactly in composition with the corresponding salts of potash, the hypothetic oxide of ammonium being substituted for potash. It has been supposed that no quadroxalate of ammonia exists; but this is a mistake. Such a salt is formed, on dissolving together equal weights of binoxalate of ammonia and hydrated oxalic acid, and is analogous in form and composition to the quadroxalate of potash.

10. *Oxalate of Soda.*

$$\dot{N}a\,\dddot{C}\dddot{C}.$$

This salt is perhaps the least soluble of the salts of soda, and presents itself as a granular precipitate on saturating carbonate of soda with oxalic acid. Of the oxalate of soda dried in air without the application of heat, 23·44 grains left 18·52 carbonate of soda when strongly ignited, or 100 oxalate yield 79·01 carbonate of soda. Now 100 anhydrous oxalate of soda should yield 79·09 carbonate of soda. Hence the oxalate of soda is correctly stated to be anhydrous. It nevertheless combines with hydrated oxalic acid, and forms a binoxalate. In this compound we have simply the attachment of an atom of the oxalate of water, to the atom of oxalate of soda, without the displacement of an atom of water, as in the formation of the binoxalate of potash. Probably the absence of the atom of water in the oxalate of soda indicates an indifference on the part of this salt to enter into further combination. There is certainly a binoxalate of soda; but this binoxalate cannot support the further attachment to it of two atoms of hydrated oxalic acid, and there is no quadroxalate of soda.

11. *Binoxalate of Soda.*

$$\dot{N}a\,\dddot{C}\dddot{C} + \dot{H}\,\dddot{C}\dddot{C}\,\dot{H}^2.$$

This salt I find to resemble the binoxalate of potash, in containing three atoms of water. 22·11 grains, strongly ignited, left 8·05 grains fused carbonate of soda; or 100 binoxalate leave 40·67 carbonate of soda, equivalent to 23·84 soda; while a binoxalate with three atoms of water should yield 23·95 per cent. soda, or almost exactly the experimental result. The binoxalate of soda lost little more than 1 per cent. of its weight when kept at 212° Fahr. over sulphuric acid in vacuo. But by a heat approaching 300° Fahr. the salt lost 14·64 per cent. of water, which is a little more than two atomic proportions, namely, 13·78 per cent. Hence this salt retains the whole of its constitutional water at 212°, but loses two atoms of it at a higher temperature, retaining strongly the third atom of water, which is basic.

Double Oxalates.

The number of double oxalates is not so great as is generally supposed. On mixing a solution of binoxalate of potash either with the muriate or the sulphate of magnesia,

zinc, &c., the oxalate of magnesia or of zinc precipitates, while the quadroxalate of potash is formed, and remains in solution or crystallizes, being very sparingly soluble, according to circumstances. When binoxalate of potash is digested upon magnesia or upon oxide of zinc, a portion of the oxide is dissolved, but is quickly deposited again as an insoluble oxalate, and no double salt formed. But one member at least of the magnesian class of oxides, namely, oxide of copper, is dissolved by the binoxalates of the alkalies, and forms double salts, which were discovered and carefully examined by M. Vogel of Bayreuth.

12. *Oxalate of Copper and Potash.*

$$\dot{K}\,\dot{C}\dot{C} + \dot{C}u\,\dot{C}\dot{C}\,\dot{H}^2;$$

and also

$$\dot{K}\,\dot{C}\dot{C} + \dot{C}u\,\dot{C}\dot{C}\,\dot{H}^2 + \dot{H}^2.$$

The binoxalate of potash, when considerably diluted, and digested with heat upon the oxide of copper, dissolves it easily, and a double salt of sparing solubility crystallizes, presenting itself generally in two forms, one of which contains two and the other four atoms of water, according to the analyses of M. Vogel, which I have repeated and confirmed so far as the water is concerned. The crystals containing four atoms of water soon become opake by exposure to the air, and lose two atoms of water by efflorescence.

Binoxalate of ammonia likewise dissolves oxide of copper, and does so still more readily than the binoxalate of potash, which may depend upon the circumstance that the resulting double salt of ammonia is considerably more soluble in water than the double salt of potash. The oxalate of copper and ammonia crystallized in plates of a blue colour, and seemed to affect one form only. Of these plates, 9·38 grains were readily decomposed by heat, and left 2·37 grains black oxide of copper, or 25·27 per cent., which is almost exactly the proportion of that of the oxide of copper, which a salt of two atoms water should contain, namely, 25·37 per cent. This salt loses water readily at 212° Fahr.; and of the 11·52 per cent. which it is supposed to have on the theory of its containing two atoms of water, 11·46 per cent. escaped by the exposure of the salt to that temperature. M. Vogel describes two other double oxalates of copper and ammonia; but it is evident that they contain ammonia and not oxide of ammonium; so that they do not come under our consideration at present.

It is to be remarked that the oxalate of copper and potash is represented above by a formula quite analogous to that of binoxalate of potash, oxide of copper being simply substituted for basic water. We have oxalate of potash in both cases, to which there is attached oxalate of copper with two atoms of water in the one formula, and oxalate of water with two atoms of water in the other. It is to be remembered that in the case of the sulphates, the double sulphate of copper and potash was shown to have a similar analogy in constitution to the bisulphate of potash.

Oxalate of Chromium and Potash, of Peroxide of Iron and Potash, of Peroxide of Iron and Soda, &c.

$$\dot{\dot{C}}r\,\dot{\dot{C}}r\,\dot{\dot{C}}\dot{C}^3 + 3\,\dot{K}\,\dot{\dot{C}}\dot{C} + \dot{H}^6.$$

$$\dot{\dot{F}}e\,\dot{\dot{F}}e\,\dot{\dot{C}}\dot{C}^3 + 3\,\dot{K}\,\dot{\dot{C}}\dot{C} + \dot{H}^6.$$

$$\dot{\dot{F}}e\,\dot{\dot{F}}e\,\dot{\dot{C}}\dot{C}^3 + 3\,\dot{N}a\,\dot{\dot{C}}\dot{C} + \dot{H}^{10}.$$

This group of salts has not hitherto been submitted to analysis, although they occupy the same important position among the oxalates which the alums do among the sulphates.

13. *Oxalate of Chromium and Potash.*

This remarkable salt was first described by Dr. Gregory, and its optical properties have been made the subject of a memoir by Sir David Brewster *. It is easily prepared by the following process, which is Dr. Gregory's, with the proportions slightly altered, from a knowledge of the composition of the salt. One part of bichromate of potash, two parts binoxolate of potash, and two parts hydrated oxalic acid, are dissolved together in hot water. There is a copious evolution of carbonic acid gas, arising from the deoxidation of the chromic acid, and nothing fixed remains except the salt in question; of which a pretty concentrated solution crystallizes upon cooling in prismatic crystals, which are black by reflected light, but of a splendid blue colour by transmitted light, when sufficiently thin to be translucent.

This salt, strongly dried without decomposition, was found to lose 11·67 per cent. of water.

The oxide of chromium cannot be precipitated from it completely by means of an alkaline carbonate, and it is very remarkable that only a small portion of the oxalic acid is thrown down from this salt by chloride of calcium.

To determine the proportion of oxalic acid, the salt was heated in strong sulphuric acid, and the gases allowed to escape through a tube containing chloride of calcium. 15·19 grains of the crystals lost 6·71 grains by this treatment, which loss is the weight of the oxalic acid. Hence this salt contains 44·17 per cent. of oxalic acid.

When this double oxalate is ignited, carbonic oxide escapes, and the residuary salt is a mixture of chromate and carbonate of potash, which is entirely soluble in water, and contains no oxide of chromium. In four experiments the fused residuary salt amounted to 0·5458, 0·5411, 0·5454, and 0·5425 of the weight of the crystals operated upon, while it should be 0·5433, provided this residuary salt is a mixture of two atoms chromate and one atom carbonate of potash, and that the composition of the crystals is as follows:

* Philosophical Transactions, 1835.

One atom oxide of chromium, $\dddot{C}r\dddot{C}r$ . . .	1003·6	16·28
Three atoms potash, $3\dot{K}$	1769·7	28·70
Six atoms oxalic acid, $6\dddot{C}\dddot{C}$	2717·4	44·07
Six atoms water, $6\dot{H}$	675·	10·95
	6165·7	100·

The results in regard to the water and oxalic acid narrated above, agree completely with this view, and so does the determination of the oxide of chromium. 26·01 grains of the crystals left, when ignited, 14·08 grains of the mixed chromate and carbonate of potash, which were dissolved in water, and being acidulated with acetic acid, the chromic acid was precipitated by acetate of lead, and gave 17·45 grains chromate of lead, equivalent to 4·28 grains oxide of chromium. Hence by this experiment the crystals contain 16·46 per cent. of oxide of chromium, which approaches very nearly to the theoretical result. The fused residuary chromate and carbonate of potash amounted to 0·5425 of the weight of the crystals, which is so near the theoretical result, namely, 0·5433, that we may safely conclude that the quantity of potash in the salt agrees with our theoretical estimate.

This salt is clearly, therefore, a compound of one atom oxalate of chromium, containing three atoms oxygen in the oxide and nine atoms oxygen in the acid, with three atoms oxalate of potash; and the salt has six atoms of water of crystallization. The oxygen in the oxide of chromium being 1, that in the potash is also 1, that in the water 2, and that in the oxalic acid 6.

I made several attempts to crystallize the oxalate of chromium itself, but without success, so that I had no opportunity of studying its constitution in relation to the constitution of the preceding double salt.

14. *Oxalate of Peroxide of Iron and Potash.*

This salt, which has not hitherto been described, is formed by dissolving the hydrated peroxide of iron to saturation in binoxalate of potash. There is no effervescence, but a sap-green solution results, which, when concentrated, deposits the salt in question in tabular crystals, of which the form has no resemblance to that of the corresponding oxalate of chromium and potash, and which are of a beautiful grass-green colour. These crystals are permanent in the air, unless it is very dry, when they lose water by efflorescence and become brown and opake. The solution of the salt is decomposed by ammonia, and the peroxide of iron completely thrown down. The salt, when ignited, leaves peroxide of iron and carbonate of potash. It loses 10·56 per cent. of water at a temperature not exceeding 230° Fahr., but is partially decomposed at 300°. Below, the theoretical composition of this salt is placed in juxtaposition with the results of an analysis.

	Theory.	Experiment.
One atom peroxide of iron . . .	15·93	16·13
Three atoms potash	28·82	29·07
Six atoms oxalic acid	44·25	43·74
Six atoms water	11·00	10·56
	100·	99·50

Hence its composition is the same as that of the preceding salt, iron being substituted for chromium.

15. *Oxalate of Peroxide of Iron and Soda.*

This salt is formed by dissolving the hydrated peroxide of iron in binoxalate of soda. It crystallizes in solid green crystals. It is composed as follows, the water being calculated from the loss on the analysis:

	Theory.	Experiment.
One atom peroxide of iron . . .	16·32	16·56
Three atoms soda.	19·57	19·66
Six atoms oxalic acid	45·34	45·51
Ten atoms water	18·77	18·27
	100·	100·

Of the ten atoms water which this salt contains it readily loses six at 212° Fahr., and retains four atoms water at that temperature. It differs, therefore, in composition, and it does so also in form, from either of the preceding double oxalates.

A corresponding oxalate of chromium and soda was produced by a similar process, and crystallized with some difficulty in solid dark crystals, which appeared to have the same form as the preceding soda-salt, and were found, like it, to contain ten atoms of water.

There is also a double oxalate of alumina and potash, which may be made by dissolving hydrated alumina in binoxalate of potash, and crystallizes in white tables of a pearly lustre, which have the same form as the oxalate of iron and potash.

II. *Of Nitrates.*

1. *Hydrated Nitric Acid, the Nitrate of Water.*

$$\dot{H}\dot{N}H^3.$$

Nitric acid combines with one atom of water as base, and with three atoms more by a less powerful affinity. The well-defined character of the combination containing four atoms water, which is the acid of specific gravity 1·42, is evinced in its high boiling point and in an appearance of saturation which it exhibits. The true and complete nitrate of water has therefore three atoms of constitutional water attached to it. And in the case of the nitrates of those metallic oxides which correspond with

water in their basic character, we find the water of crystallization likewise to be three atoms, or a multiple of three, and no other number.

2. *Nitrate of Copper.*

$$\dot{\mathrm{Cu}}\,\dddot{\mathrm{N}}\,\dot{\mathrm{H}}^3;$$

and also

$$\dot{\mathrm{Cu}}\,\dddot{\mathrm{N}}\,\dot{\mathrm{H}}^3 + 3\,\dot{\mathrm{H}}.$$

There are two nitrates of copper, one of which crystallizes in prisms, and the other in rhomboidal plates of a lighter blue colour than the prisms; the first of which I find to contain three and the other six atoms of water. Both are deliquescent to a certain degree, the salt which contains the large proportion of water being more so than the other.

(1.) Of the dark blue prisms, 28·12 grains carefully calcined left 8·98 grains black oxide, or 31·94 per cent. In a second experiment, 22·9 grains left 7·34 grains oxide, or 32·05 per cent. The salt being neutral in composition, the quantity of nitric acid may be inferred from the oxide of copper, and the difference between their sum and the weight of the salt operated upon is the water. By the first analysis the water amounts to 3·38, and by the second to 3·28 atomic proportions. The excess above three atoms is probably hygrometric moisture, to remove which from this salt we cannot employ the ordinary means. In a third experiment upon a portion of the same salt, which had been dried over sulphuric acid till it began to effloresce, 33·19 grains of nitrate left 11·04 grains oxide, which gives 2·83 atomic proportions of water to the salt, or the result is a little below the three atoms. Hence this nitrate may safely be supposed to possess three atoms of water.

(2.) Of the lighter coloured crystals in plates, 10·60 grains left 2·78 grains oxide of copper when ignited, or 27·36 per cent. Hence the salt is composed of

			With six atoms water.
Nitrate of copper . . .	6·57	100·	100·
Water	4·03	61·33	57·54
	10·6	161·33	157·54

The experimental determination is a little above the theoretical estimate, as might be expected from the deliquescent nature of the salt.

The crystals speedily became opake over sulphuric acid in vacuo, and 10·6 grains lost 2·18 grains water in a night, retaining 1·85 water; which is 28·16 water retained to 100 anhydrous salt, or almost exactly three atoms of water. Hence this salt parts easily with half its water. The other three atoms of water are retained more strongly; for by a second day's exposure over sulphuric acid there was an additional loss of only 0·15 grain water; or the water retained was reduced to 25·87 parts united to 100 anhydrous salt.

3. *Subnitrate of Copper.*

$$\dot{H}\dot{N}Cu^3.$$

It is well known that when the nitrate of copper is heated to the temperature of 400° or 500° Fahr., it is decomposed, nitric acid and water being expelled, and a subnitrate remaining, which consists of one atom of nitric acid, one atom of water, and three atoms of oxide of copper. This decomposition I find to take place and be completed at a very moderate temperature, not exceeding 150° Fahr.; and it appears, besides, that none of the three constitutional atoms of water of the nitrate of copper can be expelled without a certain corresponding loss of acid: that on heating the salt in question, nitric acid and water go off together, in the form of nitrate of water with its three atoms water. Thus, three atoms of crystallized nitrate of copper, containing three atoms acid, three atoms oxide, and nine atoms of water, are resolved into two atoms nitrate of water, each containing one atom acid and four water; and one atom of subnitrate of copper, which contains one atom acid, one water, and three oxide of copper.

Experiment.—In a stove of which the temperature never exceeded 150° Fahr., 27·54 grains crystallized nitrate of copper, containing three atoms of water, exposed on a capsule, suffered the following gradual reduction of weight: a loss of 2·59 grains in one day, of 9·62 in six days, of 11·1 in seven days, of 13·35 in eleven days, of 13·47 in twelve days, of 13·58 in sixteen days, of 13·60 in eighteen days, and nothing more afterwards by a heat of 300° Fahr., continued for several hours. Of the crystallized nitrate, 27·54 grains have left 13·94 grains subnitrate; or we have 0·5062 subnitrate from 1 nitrate. By calculation the residuary subnitrate should be 0·5026, with which the experimental result closely corresponds.

Another portion of the same nitrate of copper, dried exactly in the same way, lost 1 per cent. of its weight when afterwards heated to 400° Fahr.; and thereafter, being ignited, was found to consist of

	Experiment.	Theory.
Oxide of copper	100·	100·
Volatile matter	53·19	53·1
	153·19	153·1

I am satisfied that no other subnitrate except the preceding, which contains three atoms of oxide of copper, can be obtained by the decomposition of the neutral nitrate by means of heat. For a quantity of the subnitrate of copper of the first experiment narrated above being gradually exposed in a platinum crucible to a heat above the melting point of lead, by means of a sand-bath, so as actually to reduce a portion of the subsalt in contact with the bottom of the crucible to the state of black oxide, yet the major portion of the subsalt, which still retained its green colour, was found to be little altered in composition. After this extreme heating the subsalt consisted of

Oxide of copper . .	100·
Volatile matter . .	50·6
	150·6

Or the proportion of volatile matter in the subsalt has suffered only a small reduction, namely, from 53·1 to 50·6 parts. This last subnitrate afforded drops of nitric acid with fumes of nitrous acid when heated in a tube, so that the subnitrate of copper retains water even at a temperature above the melting point of lead.

The subnitrate of copper merits a careful consideration; for the subsalts of the magnesian class of oxides, which can be had of a definite composition, are really much fewer in number than is generally supposed. What constitution ought to be assigned to this salt? It will be observed that I have represented it by the singular formula

$$\dot{H}\,\dddot{N}\,\dot{Cu}^3;$$

implying that the single atom of water which it contains is really the base of the salt, and that the three atoms of oxide of copper are in the place of the constitutional water of this nitrate of water. This opens a new view of the constitution of subsalts. The excess of metallic oxide which they contain may not be basic at all in certain cases like the present, but discharge a function in the constitution of the salt which has hitherto been recognised only as executed by water. For if we find water and oxide of copper strongly resembling each other as bases, why may not the analogy between them extend further, and oxide of copper be capable of discharging the function of constitutional water or water of crystallization in the composition of a salt? Indeed the speculation that all salts whatever are neutral in composition is highly probable. Where the metallic oxide is in excess, as in what are called subsalts, we can attribute another function than that of base to the whole or a portion of the metallic oxide, and thus preserve the salt neutral in composition, or according to its formula. To this subject I shall again recur.

The following observation is particularly favourable to the view which we are taking of the constitution of subnitrate of copper. When the black oxide of copper is drenched with the strongest nitric acid, it is a subnitrate of copper which is formed, although the nitric acid may be in great excess. The black oxide is converted into a green powder, from which the excess of nitric acid should be drained off as well as possible, and the powder will be found to be in great part insoluble in water. The explanation seems to be, that the concentrated nitric acid employed does not contain the constitutional water which the neutral nitrate of copper requires, and accordingly that salt is not formed; but the nitrate of water supplies itself with oxide of copper in the place of its deficient constitutional water; so that the result is a nitrate of water with three atoms of oxide of copper attached. But when nitric acid of a specific gravity not exceeding 1·42 is digested upon the same black oxide of copper, the neutral nitrate of copper only is formed, and no subnitrate.

This view seems likewise to be necessary to account for the great force with which the single atom of water is retained by the subnitrate of copper. The water cannot be expelled without decomposing the salt, notwithstanding the great excess of oxide of copper present.

4. *Nitrate and Subnitrate of Bismuth.*

$$\dot{\dot{B}}i\,\dot{\dot{N}}\,H^3$$

$$H\,\dot{\dot{N}}\,Bi^3.$$

The neutral nitrate is admitted to contain three atoms of water, like the nitrate of copper, and its constitution appears to be similar.

No portion of the constitutional water of this salt can be expelled without decomposing the salt. Indeed this salt loses acid by exposure to dry air at a temperature not exceeding 80° Fahr. The crystals of the salt are resolved by a heat of 212° into a solid and fluid portion, the first of which is probably the subnitrate, while the last is hydrated nitric acid containing much nitrate of bismuth in solution, and not a supernitrate of bismuth. But the fluid portion fixes so readily upon cooling that the solid product cannot be obtained in a definite state.

Experiment. 28·61 grains of nitrate of bismuth in good crystals, being exposed to a gradual ignition, left 14·16 grains of fused oxide of bismuth. This result accords with the view which is taken above of the composition of this salt:

		Experiment.	Theory.
Oxide of bismuth	14·16	100·	100·
Nitric acid and water . . .	14·45	102·04	102·72
	28·61	202·04	202·72

It appears likewise that three atoms of the hydrated nitrate of bismuth are resolved, when dried at a high temperature, into two atoms hydrated nitrate of water and one atom subnitrate of bismuth, which last is of the same constitution as the subnitrate of copper.

Experiment 1. Dried on the sand-bath at a temperature above the melting point of tin, 28·61 grains of nitrate of bismuth lost 9·29 grains, and retained 5·16 grains of volatile matter, or consisted of

		Experiment.	Subnitrate by theory.
Oxide of bismuth.	14·16	100·	100·
Volatile matter	5·16	36·44	34·24
	19·32	136·44	134·24

Experiment 2. A portion of nitrate dried in a stove at a temperature not exceeding 180° Fahr., till it ceased to lose weight, was thereafter found to consist of

Oxide of bismuth	21·24	100·
Volatile matter	7·57	35·64
	28·81	135·64

It appears from the second of these experiments that the subnitrate of bismuth may be produced at a temperature so low as 180° FAHR., and from the first experiment that the subnitrate may be exposed to a temperature of 500° FAHR. without decomposition.

Several experiments were made to produce another definite subnitrate, containing a greater proportion of oxide of bismuth, by the action of heat upon this subnitrate, but without success. The salt was partially decomposed at various temperatures under redness, but no definite compound resulted. Hence the subnitrate described is probably the only definite subnitrate of bismuth that can exist. The small pearly crystals obtained on throwing the neutral nitrate of bismuth into a moderate quantity of water, are of the same composition as the subnitrate obtained by heat.

5. *Nitrate of Zinc.*

$$\dot{Z}n\dddot{N}\dot{H}^3 + 3\,\dot{H}.$$

This salt is easily obtained by dissolving zinc in nitric acid. It is very soluble in water, and moderately deliquescent.

Experiment. 29·17 grains of the crystals ignited, left 7·86 grains oxide of zinc. In this experiment we have 0·2694 oxide from one salt, which is very near 0·2713 oxide, the proportion which should be left, supposing the salt to contain six atoms of water. By efflorescence at 212° one part of this salt loses 0·18 water, which is one half of the whole water which the salt is assumed to contain, namely, 0·3639 water. It loses no acid at 212°. Hence this salt is of the same constitution as the nitrate of copper, but is not decomposed at so low a temperature. The proportion of water, however, cannot be reduced below three atoms without a loss of acid, and there appears to be a subnitrate of zinc resembling the subnitrate of copper.

6. *Nitrate of Magnesia.*

$$\dot{M}g\dddot{N}\dot{H}^3 + 3\,\dot{H}.$$

Experiment. 27·12 grains of crystals of nitrate of magnesia, when calcined, left 4·3 grains caustic magnesia; a result which indicates 6·17 atomic proportions of water in the salt, or the salt contains six atoms of water.

The nitrate of magnesia stands exposure to a heat which would melt lead without losing acid. At that high temperature the proportion of water is reduced to one atom, which cannot be expelled without loss of acid. The salt remains in a fused state and transparent, and dissolves afterwards completely in water.

Experiment. 18·40 grains of the crystals, containing 7·71 grains water, lost 6·60 grains by a strong sand-bath heat continued till the salt ceased to lose weight. This is a loss of exactly five sixths of the water contained in the salt.

Experiment. 19·76 grains, containing 8·28 water, by similar treatment lost 6·77,

which approaches very closely to 6·90 grains, the number representing five atomic proportions of water.

This single atom of water retained by the nitrate of magnesia, is not displaced and expelled upon heating the salt, together with an atomic proportion of nitrate of potash to 600° or 700° Fahr., so that the retention of an atom of water does not indicate a disposition, upon the part of nitrate of magnesia, to form a double salt. It is probable that this peculiar and intimate combination of nitrate of magnesia with one atom water does not exist in the crystals or ordinary hydrate of nitrate of magnesia, but is the result of a new arrangement of the constituents of the salt at a high temperature. There are indications of the existence of a similar nitrate of water.

There does not appear to be a subnitrate of magnesia like the subnitrate of copper.

Supposed Double Nitrates and Supernitrates.

As double nitrates are said to exist, I have repeatedly attempted to form them; but when nitrate of magnesia, nitrate of zinc, or nitrate of copper was mixed with nitrate of potash or with nitrate of ammonia, the salts uniformly separated again in crystallizing. There is no proof of the existence of a single supernitrate.

Most of the nitrates of oxides not belonging to the magnesian class are anhydrous salts, such as the nitrates of potash, soda, barytes, strontian, lead, &c., and do not suggest any new subject matter of inquiry.

III. *Of Phosphates.*

In the present state of our knowledge phosphoric acid is quite peculiar in being capable of combining with bases in three different proportions, forming, besides the usual class of salts containing one atom of acid to one atom of protoxide as base, two other anormal classes of salts, in which two and three atoms of base are united to one atom of acid, namely, the pyrophosphates and the common phosphates. Arsenic acid forms only one class of salts, but that class is anormal, every member of it containing three atoms of base to one atom of acid, like the common phosphates. These anormal classes of phosphates and arseniates, with perhaps the phosphites, are, I believe, the only known salts to which the ordinary idea of a subsalt is truly applicable; or in the formulæ of these salts only, ought more than one atom of any protoxide to appear in a basic relation to one atom of acid. All other reputed subsalts are probably neutral in composition, as I have endeavoured to show in the case of the subnitrate of copper; for to this salt they all bear an analogy in their small solubility and other properties, while they exhibit little resemblance to those classes of phosphates and arseniates which really possess more than one atom of base. The following Table contains the formulæ of the most important phosphates, with a new nomenclature of these salts, which I offer for consideration.

First Class.

Monobasic phosphate of water (metaphosphate of water) $\dot{H}\dddot{P}$.

Monobasic phosphate of soda (metaphosphate of soda) $\dot{Na}\dddot{P}$.

Second Class.

Bibasic phosphate of water (pyrophosphate of water) . $\dot{H}^2\dddot{P}$.

Bibasic phosphate of soda and water (bipyrophosphate of soda) . . . . . $\dot{Na}\dot{H}\dddot{P}$.

Bibasic phosphate of soda (pyrophosphate of soda) . . $\dot{Na}^2\dddot{P} + 10\dot{H}$.

Third Class.

Tribasic phosphate of water (common phosphate of water) $\dot{H}^3\dddot{P}$.

Tribasic phosphate of water and soda (biphosphate of soda) . . . . . $\dot{Na}\dot{H}^2\dddot{P} + 2\dot{H}$.

Tribasic phosphate of soda and water (phosphate of soda) $\dot{Na}^2\dot{H}\dddot{P} + 24\dot{H}$.

Tribasic phosphate of soda (subphosphate of soda) . . $\dot{Na}^3\dddot{P} + 24\dot{H}$.

Tribasic phosphate of soda, ammonia, and water (microcosmic salt) . . . . . $\dot{Na}\dot{N}\dot{H}^4\dot{H}\dddot{P} + 8\dot{H}$.

Tribasic phosphate of magnesia and water (phosphate of magnesia) . . . . . $\dot{Mg}^2\dot{H}\dddot{P} + 2\dot{H} + 12\dot{H}$.

Tribasic phosphate of magnesia and ammonia (ammoniaco-magnesian phosphate) . . . . . $\dot{Mg}^2\dot{N}\dot{H}^4\dddot{P} + 2\dot{H} + 10\dot{H}$.

It is my object to get rid of the trivial names pyrophosphates, metaphosphates, and common phosphates, which have tended to keep up an erroneous impression that the phosphoric acid is of a different nature in these classes of salts, or is modified in some way unknown. This notion has arisen from the pertinacity with which phosphoric acid continues combined with a constant number of atoms of base, whether it be one, two, or three, although the base itself be repeatedly changed by decomposing the original combination. But this is an occurrence quite analogous to the formation of different sets of sulphurets or of chlorides, when we decompose two or more different oxides of the same metal, such as the oxide and suboxide of mercury, by sulphuretted hydrogen or by muriatic acid. The metal continues in the same relative state of saturation throughout a series of such decompositions; and so does the phosphoric acid, because in both cases the decomposition is effected by an equivalent substitution.

A difficulty occurs in naming two members of the tribasic class, so as to distinguish them from each other, namely, the biphosphate of soda and phosphate of soda, both of which contain soda and water as base. But this difficulty is obviated by placing

first in the name that base of which two atoms are present. Thus the biphosphate of soda is "the phosphate of water and soda," and the phosphate of soda is the phosphate of soda and water, both being at the same time characterized as "tribasic."

What I have to add at present in regard to the phosphates relates chiefly to the last three salts, of which formulæ are given in the preceding Table, which belong to classes of tribasic phosphates that were not examined in my former paper upon the phosphates*. But I may premise a few observations, which are more strictly supplementary to the results of that paper.

1. The bibasic phosphate of water (pyrophosphate of water) is possessed of very considerable stability. Both weak and concentrated solutions of this salt have been kept for five or six months without any sensible change or production of the tribasic phosphate of water.

2. It appears to be impossible to crystallize any bibasic phosphate (pyrophosphate) of potash. Such salts can exist in solution, but not in the dry state. The same observation applies to the bibasic phosphates of ammonia, or we have no pyrophosphates of ammonia except in solution. Indeed, the solution of the bibasic phosphate of water and ammonia assumes another atom of basic water when the evaporation is carried far, and crystallizes as the tribasic phosphate of water and ammonia (biphosphate of ammonia).

3. In the case of tribasic phosphates containing potash, I have succeeded in crystallizing the tribasic phosphate of potash, and the tribasic phosphate of water and potash, but not the tribasic phosphate of potash and water, or what would be considered on the old view as the neutral phosphate of potash.

4. Both the bibasic and tribasic phosphates of water may be treated with an excess of caustic potash in solution without the formation of any precipitate or sparingly soluble combination. It is only in the monobasic phosphate of water that a sparingly soluble combination is formed by potash, such as that which is described by Dr. Thomson under the name of diphosphate of potash.

I. *Tribasic Phosphate of Soda, Ammonia, and Water.* (*Phosphate of Soda and Ammonia: Microcosmic Salt.*)

$$\dot{N}a\,\dot{N}H^4\,\dot{H}\,\dddot{P} + 8\,\dot{H}.$$

I have repeated more than once the analysis of this salt, and obtained the same result as M. Mitscherlich. It appeared to contain 0·5094 of volatile matter: and there may be derived from an atom of this salt one atom of phosphoric acid, of soda and of ammonia respectively, and ten atoms of water. It has hitherto been viewed as a double phosphate or combination of phosphate of soda with phosphate of ammonia; but no reason can be assigned why these particular salts should combine together, and combinations of salts of soda and ammonia are exceedingly unusual.

* Philosophical Transactions, 1833.

The view expressed above in the formula is much more likely to be true, namely, that this salt is simply a tribasic phosphate, of which the three atoms of base are all different: they are soda, oxide of ammonium, and water; and the salt possesses eight atoms of water of crystallization. By a graduated heat it is possible to expel the water of crystallization of this salt, and likewise the ammonia of its oxide of ammonium; and the water of the last remaining as base, the salt $\dot{Na}\,\dot{H}^2\,\dddot{P}$ is produced.

M. Mitscherlich now admits that there is no tribasic phosphate corresponding with this, but containing potash instead of oxide of ammonium, a conclusion of which I have ascertained the accuracy.

I endeavoured to form a tribasic phosphate to contain two atoms soda and one atom of oxide of ammonium, but such a salt appears to have no existence. For when ammoniacal gas was passed into a strong and hot solution of the common phosphate of soda, a slight deposition of the tribasic phosphate of soda took place, followed by the rhomboidal crystals of the common phosphate unchanged.

It likewise appears that when the bibasic phosphate of soda and the bibasic phosphate of potash (pyrophosphates) are mixed together, no new salt is produced; but the former may be crystallized out, and the latter remains uncrystallizable.

II. *Tribasic Phosphates containing Oxides of the Magnesian Class.*

1. *Tribasic Phosphate of Zinc and Water.* (*Phosphate of Zinc.*)

$$\dot{Zn}^2\,\dot{H}\,\dddot{P} + 2\,\dot{H}.$$

This salt, which is nearly insoluble, is obtained in minute silvery plates, by mixing three ounces of sulphate of magnesia with four ounces of phosphate of soda, each dissolved in two pounds of cold water. These crystalline plates consist of

		Theory of $\dot{Zn}^2\,\dot{H}\,\dddot{P} + 2\,\dot{H}$.
Anhydrous salt . . .	100·	100·
Water	19·63	17·77
	119·63	117·77

Dried above the melting point of tin the crystals still retained a glistening appearance, but had lost two thirds of their water; for they now consisted of

		Theory of $\dot{Zn}\,\dot{H}\,\dddot{P}$.
Anhydrous salt . . .	100·	100·
Water	6·08	5·92
	106·08	105·92

The two atoms of water which are expelled in the above experiment are, notwithstanding, pretty strongly attached to the salt, being retained at the boiling point of water. Indeed these two atoms of water are highly constitutional, and are found in all the phosphates of this class.

This phosphate fuses at a red heat, after it becomes anhydrous, but it continues soluble in dilute acids.

2. *Tribasic Arseniate of Magnesia and Water.* (*Arseniate of Magnesia.*)

$$\dot{Mg}^2\dot{H}\dddot{As} + 2\dot{H} + 12\dot{H}.$$

This salt precipitated on mixing dilute solutions of 500 grains of arseniate of soda and 300 grains of sulphate of magnesia. It consisted of

		Theory of $\dot{Mg}^2\dot{H}\dddot{As} + 14\dot{H}$.
Anhydrous salt . . .	100·	100·
Water	86·58	86·25
	186·58	186·25

This salt contains in all fifteen atoms of water, of which three are retained and twelve expelled at the boiling point of water. Dried at 212° it consisted of

		Theory of $\dot{Mg}^2\dot{H}\dddot{As} + 2\dot{H}$.
Anhydrous salt . . .	100·	100·
Water	17·17	17·25
	117·17	117·25

It therefore retains pretty strongly two atoms of water besides its basic atom, resembling the preceding salt in this respect.

This arseniate and the corresponding phosphate are rendered insoluble in dilute acids by the effect of a strong red heat.

3. *Tribasic Phosphate of Magnesia and Water.* (*Phosphate of Magnesia.*)

$$\dot{Mg}^2\dot{H}\dddot{P} + 2\dot{H} + 12\dot{H}.$$

This salt appears in distinct prismatic crystals in the course of twenty-four hours, upon mixing two ounces of sulphate of magnesia with three ounces phosphate of soda, each dissolved in two pounds of water. Cold water is capable of dissolving about one thousandth part of its weight of these crystals. They have been stated erroneously to be much more soluble. The proportion of water which they contain has hitherto been stated at fourteen atoms instead of fifteen, which is the truth. By analysis the crystals were found to consist of

		Theory of $\dot{Mg}^2\dot{H}\dddot{P} + 14\dot{H}$.
Anhydrous salt . . .	100·	100·
Water	121·7	119·76
	221·7	219·76

I find that the proportion of water retained by this salt is readily reduced at 212°, from fifteen atoms to seven, by the escape of eight atoms of water. Of the seven atoms retained one is basic, and therefore expelled with difficulty; but from a variety

of experiments which I have performed it appears probable (although I have never attained very precise results) that the other six atoms go off in pairs at different temperatures between 212° and 350° FAHR. But even at 410° the quantity of water retained by this salt was sensibly above one atomic proportion. We may with considerable probability represent the consecutive combinations of this salt with water by such a formula as the following:

$$\dot{Mg}^2\dot{H}\dddot{P} + 2\dot{H} + 2\dot{H} + 2\dot{H} + 8\dot{H}.$$

Besides the preceding salt there is a tribasic phosphate of magnesia, which is obtained as an insoluble precipitate on mixing tribasic phosphate of soda with sulphate of magnesia. Of this salt the whole three atoms of base are magnesia, as its name implies. Dried at 212° it retains five atoms of water. At a red heat it glows, but it continues soluble in acids even after exposure to a white heat. But I did not succeed in forming the other tribasic salt, containing two atoms of water and one atom of magnesia as bases, which is wanted to complete the series. Such a salt does not appear to exist.

It may be mentioned here in reference to the monobasic phosphate of magnesia (metaphosphate of magnesia), that although this salt does not present itself on mixing the monobasic phosphate of soda with the sulphate of magnesia, yet it is readily precipitated in the form of a soft viscid body, on using the acetate of magnesia instead of the sulphate.

4. *Tribasic Phosphate of Magnesia and Ammonia.* (*Ammoniaco-magnesian Phosphate.*)

$$\dot{Mg}^2\dot{N}H^4\dddot{P} + 2\dot{H} + 10\dot{H}.$$

This salt is the well-known granular precipitate formed on adding a salt of magnesia to any soluble tribasic phosphate with which ammonia or a salt of ammonia has been mixed. I was much interested in ascertaining the true constitution of this salt, and have carefully analysed seven or eight different specimens of it, prepared with and without free ammonia in the liquors. The result is that only one tribasic salt of these constituents exists, although two have often been admitted; while in this compound there exists only one atom of ammonia instead of two, as M. RIFFAULT has supposed. I subjoin the preparation and analysis of one specimen of this salt. 350 grains of crystallized phosphate of soda, 100 grains of chloride of ammonium, and 200 grains of *aqua ammoniæ* were dissolved together in four pounds of cold water, and 200 grains of crystallized sulphate of magnesia were added to that mixture. The precipitation was gradual, and the liquor remained alkaline. The precipitate was slightly washed with cold water, and afterwards dried in the air for ten days, the thermometer being 65° FAHR., without artificial heat. The true proportions of water, which this and many other precipitates affect, have often been mistaken, and definite hydrates not obtained, from using hot solutions in their preparation. Of this preci-

pitate 26·8 grains lost by ignition 14·5 grains, or one part of the precipitate contains 0·541 volatile matter. For the ammonia, the volatile matter from 9·65 grains of the precipitate was sent over quicklime contained in a tube, so as to arrest the water. The loss, or the ammonia, amounted to 0·67 grain, or to 0·0695 of the precipitate. Hence this precipitate consists of

		Theory of $Mg^3 \dddot{N} H^4 \dddot{P} + 2\dot{H} + 10\dot{H}$.
Anhydrous salt . . .	45·90	45·85
Ammonia	6·95	6·98
Water	47·15	47·17
	100·	100·

From the manner in which this specimen of the salt was prepared, it should contain the maximum proportion of ammonia of which the salt admits, and yet that proportion is one atom only, and not two, as it was estimated by Riffault. A salt of the same composition was obtained from the same materials, omitting the caustic ammonia. In that case the product was not so abundant, and the mother liquor remained acid from the production of tribasic phosphate of water and soda, which has an acid reaction. When this salt, contained in a little retort, is heated in a very gradual manner to 212° by means of a water-bath, it is possible to distil over ten atomic proportions of the water without any ammonia whatever. Of the three atoms of water which remain, (the whole quantity originally present in the salt being thirteen atoms,) one appears to be combined with the ammonia in the formation of oxide of ammonium, while the other two are the constitutional water of the tribasic phosphate of magnesia and water.

It appears, then, that this salt is not a double phosphate, or combination of two phosphates, but that it is formed from the tribasic phosphate of magnesia and water, by the substitution of oxide of ammonium for the basic water of that salt; and it is a tribasic phosphate of magnesia and oxide of ammonium. The oxygen in the magnesia is double that in the oxide of ammonium.

This salt is the type of a class of tribasic phosphates, in which the magnesia is replaced by the other oxides, which are isomorphous with that base. Two of these salts were discovered and carefully examined by Dr. Otto of Brunswick *.

Dr. Otto's analysis of what we may call the tribasic arseniate of manganese and ammonia corresponds exactly with the analysis given above of the magnesian salt, except that he derives only twelve instead of thirteen atoms of water from his salt. The deficiency in the proportion of water found by him, I attribute to the use which he made of hot water in washing his salt.

His analysis of the tribasic phosphate of the protoxide of iron and ammonia is particularly interesting, as it proves that this salt is precipitated, containing no more than three atoms of water, or exactly of the composition of the magnesian salt dried

* Journal für Praktische Chemie von Erdman und Schweigger-Seidel, 1834, p. 409.

at 212°, as we have described. The constitution of this salt of iron I would therefore represent by the formula

$$\dot{Fe}^2\dot{N}H^4\dddot{P} + 2\dot{H}.$$

In the same paper Dr. Otto describes another extraordinary phosphate, under the name of paraphosphate of soda, ammonia, and oxide of manganese, which does not belong to any class of phosphates that I have examined, but may possibly be a combination of two bibasic phosphates. Its constituents are $2\dddot{P}$, $2\dot{M}n$, $\dot{N}H^4$ and $6\dot{H}$. It is prepared from bibasic phosphates, and would be said in the old language to contain pyrophosphoric acid.

IV. *Of Sulphates.*

In a former paper upon water as a constituent of sulphates *, I examined particularly the constitution of hydrated sulphuric acid and of the sulphates of the magnesian class of oxides. All these salts contain one atom of constitutional water, which is displaced in the formation of the double sulphates by an atom of an alkaline sulphate. This view is illustrated by the following formulæ:

Sulphate of water (acid of sp. gr. 1·78)	$\dot{H}\ddot{S}\dot{H}$
Sulphate of magnesia	$\dot{M}g\ddot{S}\dot{H} + 6\dot{H}$
Sulphate of magnesia and potash	$\dot{M}g\ddot{S}(\dot{K}\ddot{S}) + 6\dot{H}$
Sulphate of water and potash (bisulphate of potash) .	$\dot{H}\ddot{S}(\dot{K}\ddot{S})$.

It will be found upon experiment that the salts sulphate of magnesia and sulphate of zinc become anhydrous at much lower temperatures when mixed with sulphate of potash than by themselves, the sulphate of potash displacing the constitutional water of the other salt at a very moderate heat, although the salts are mixed in the state of dry powders.

In that paper the opinion was supported, originally suggested I believe by M. Mitscherlich, that the bisulphate of potash is a double sulphate of water and potash, and therefore really neutral in composition. The only difficulty which stood in the way of generalizing this result, and maintaining that *all* the salts usually considered as bisalts are really neutral in composition, was the composition of the bichromate or red chromate of potash, a salt which unquestionably is anhydrous. Here, it might be said, is a true bisalt. But M. H. Rose has lately published some observations in regard to anhydrous sulphuric acid, which, I think, afford a clue to the discovery of the true constitution of the red chromate of potash. It appears that the vapour of anhydrous sulphuric acid is absorbed by sulphate of potash and by chloride of potassium, without decomposition, and definite compounds formed; which, however, are destroyed by solution in water. Here we appear to have a class of

* Edinburgh Transactions, vol. xiii. p. 297; or London and Edinburgh Philosophical Magazine, 3rd series, vol. vi. pp. 327, 417.

combinations of sulphuric acid *with salts*. Chromic acid, which is isomorphous with sulphuric, forms combinations which I consider as analogous to these. With the neutral or yellow chromate of potash it forms the red chromate of potash, and with chloride of potassium it forms M. Peligot's salt; which differ only from M. Rose's corresponding combinations of sulphuric acid, in being more permanent. The superior stability of these chromic acid combinations unquestionably depends upon the little affinity for water which their acid possesses, while the affinity of sulphuric acid for water is very great. Hence we may suppose that the red chromate of potash is not a direct combination of two atoms of chromic acid with one atom of potash, but a combination of one atom of chromic acid with one atom of yellow chromate of potash; and it may be represented as follows:

$$\dot{\mathrm{K}}\,\dddot{\mathrm{C}}\mathrm{r})\,\dddot{\mathrm{C}}\mathrm{r}.$$

The red chromate of potash will thus belong to a new order of combinations, differing essentially from proper salts, which contain an oxide as base. This salt, therefore, cannot be adduced as militating against the law that "all salts are neutral in composition"; the only known exceptions to which law are, I believe, afforded by the anormal classes of phosphates, phosphites, and arseniates.

I have devoted much time to the examination of *subsulphates* of the magnesian class of oxides, particularly of the subsulphate of zinc and the subsulphate of copper. These salts were generally formed by the partial precipitation of sulphate of zinc or sulphate of copper by means of caustic potash. They have both a disposition to carry down sulphate of potash, which is never entirely removed from them by washing; while one of them, the subsulphate of zinc, is itself decomposed by washing. When most successfully prepared, they were found to contain four atoms of metallic oxide to one atom of acid, (instead of three atoms oxide, as M. Berzelius supposed,) together with four atoms of water. I have not hitherto been able to form a distinct idea of their constitution, or to decide between different views which may be taken of it. But the force with which water is retained in these subsalts is very remarkable. The subsulphate of copper loses no portion of its four atoms of water at 212°, and I have not been able to reduce the quantity of water retained by this salt so low as one atomic proportion, even at the melting point of lead.

The constitution of the subsulphate of copper appears to be changed when it is made anhydrous by heat. In the progress of the desiccation of the salt, its colour passes from a dull blue to an olive green, and it finally becomes of a chocolate brown, and is then anhydrous. Water poured upon the brown matter comes off of a blue colour, dissolving out a considerable portion of the soluble sulphate of copper. It appears, therefore, that the water originally present in the subsulphate must discharge some important function in its constitution, the subsalt being obviously decomposed when made anhydrous.

The *Alums* form a most important class of the sulphates, but I have never had it in my power to compare their constitution with that of the sulphate of alumina itself,

which is not easily obtained in a crystallized state. This salt, however, is described as containing eighteen atoms of water, while the alums have twenty-four. At present I would merely throw out the conjecture, that in the alums we may have simply an alkaline sulphate with the sulphate of alumina attached, that salt carrying along with it its whole water of crystallization, and acquiring six atoms more. The quantity of water in potash alum may be reduced by efflorescence to six atoms in a stove of the temperature of 150° FAHR. Hence potash alum may perhaps be represented as follows:

$$\dot{K}\dddot{S} + (\dot{A}\dot{l}\dot{A}\dot{l}\dddot{S}^3 + 6\,\dot{H} + 18\,\dot{H}).$$

I have shown by an analysis conducted in very favourable circumstances, that soda-alum contains, like potash-alum, twenty-four, and not twenty-six atoms of water.

V. *Of Chlorides.*

The affinity which the hydracids exhibit for water is weak. Of the lower hydrates of muriatic acid we know nothing, the volatility of the acid putting it out of our power to form and examine such hydrates; but it is likely that they will correspond with the hydrates of the chloride of magnesium, &c., which can be examined.

The law in the case of the chlorides of the magnesian class of metals appears to be, that they have two atoms of water pretty strongly attached to them, and which we may consider as constitutional. Thus chloride of copper crystallizes with two atoms of water, and with no lower proportion; but several chlorides of this class have two or four atoms more, the proportion of water advancing by a multiple of two atoms.

1. *Chloride of Copper.*

$$\mathrm{CuCl}\,\dot{H}^2.$$

The blue prismatic crystals of chloride of copper become brown and lose the greater proportion of their water at a temperature not exceeding the boiling point of water. Fifteen grains of the crystals, exposed to a much higher temperature, lost 3·23 grains of water, leaving 11·77 grains of chloride of copper; and when this quantity of chloride of copper was exposed to the atmosphere, it quickly recovered 3·16 grains of water, and resumed the blue colour of the crystallized salt. I believe this method of reabsorption, in the case of constitutional water, often to give hydrates of which the composition is even more exact than if they had been obtained from solution, owing to the absence of that water, which is often mechanically interposed between the plates of crystals. The hydrated chloride of copper obtained in this way consisted of

			Theory of $\mathrm{CuCl}\,\dot{H}^2$.
Chloride of copper . . .	11·77	100.	100.
Water	3·16	26·85	26·84
	14·93	126·85	126·84.

2. *Chloride of Manganese.*

$$MnCl\dot{H}^2 + 2\dot{H}.$$

Experiment.—Of the flesh-coloured crystals, 15·53 grains, precipitated by nitrate of silver, gave 22·57 grains of chloride of silver, equivalent to 5·56 chlorine, or to 9·92 chloride of manganese, which leaves 5·61 grains water in the salt, or 36·12 per cent. of water. Now a chloride of manganese with four atoms of water would contain 36·33 per cent. of water.

This salt readily lost half its water when dried at 212° in open air, or when dried over sulphuric acid in the vacuum of an air-pump at the ordinary temperature. But when the exposure of the salt in such circumstances was long protracted, a little of the constitutional water also was lost.

3. *Protochloride of Iron.*

$$FeCl\dot{H}^2 + 2\dot{H}.$$

In three experiments made upon different specimens of crystallized protochloride of iron, all newly and very carefully prepared, 13·69 grains chloride of silver were precipitated from 9·72 salt, 17·20 chloride of silver from 12·44 salt, and 15·75 chloride of silver from 11·21 salt. These experiments almost coincide in their results, which are, that 1 part of the salt contains 0·3466, 0·3463, and 0·3461 of chlorine. But such proportions of chlorine are decidedly under the proportion which a neutral salt with four atoms of water should contain, namely, 0·3593 chlorine. Indeed, the quantity of water in the salt is indicated by these experiments to be four and a half atomic proportions almost exactly. By crystallizing from an acid solution Bonsdorff has lately obtained this salt in a state of purity, and containing four atoms of water.

4. *Chloride of Magnesium.*

$$MgCl\dot{H}^2 + 4\dot{H}.$$

Of the crystals of this salt, which are decidedly deliquescent, 12·65 grains were found to contain 4·29 chlorine; or the salt contains 33·91 per cent. of chlorine, which approaches sufficiently near to the theoretical proportion 34·69 per cent., supposing the salt to contain six atoms of water.

5. *Chloride of Calcium.*

$$CaCl\dot{H}^2 + 4\dot{H}.$$

The crystals of this deliquescent salt, dried in vacuo till they began to effloresce, were found to contain six atoms of water, the proportion usually allotted to them; but it is remarkable, that, continued in vacuo over sulphuric acid for ten days during

the heat of summer, the crystals became opake and of a talky lustre, without being disintegrated, and their proportion of water was reduced to two atoms.

6. *Double Chloride of Copper and Ammonium.*

$$NH^4Cl + CuClH^2.$$

Hydrated chloride of copper dissolved with chloride of ammonium, in the proportion of eleven of the first to seven of the last, readily affords a double salt, in which we appear to have an atom of chloride of ammonium with an atom of the hydrated chloride of copper attached. This double salt is less soluble than the chloride of copper itself, and retains more strongly the two constitutional atoms of water of that salt; illustrating in both of these points what appear to be two very general occurrences: namely, 1st, the reduced solubility of double salts; and, 2nd, the closer attachment which constitutional water exhibits for a salt when that salt itself enters into combination.

	Analysis.	Theory of $NH^4Cl + CuClH^2$.
Chlorine	51·03	51·08
Copper	23·35	22·83
Ammonium (NH^4) .	13·20	13·10
Water	12·09	12·99
	100·67	100·

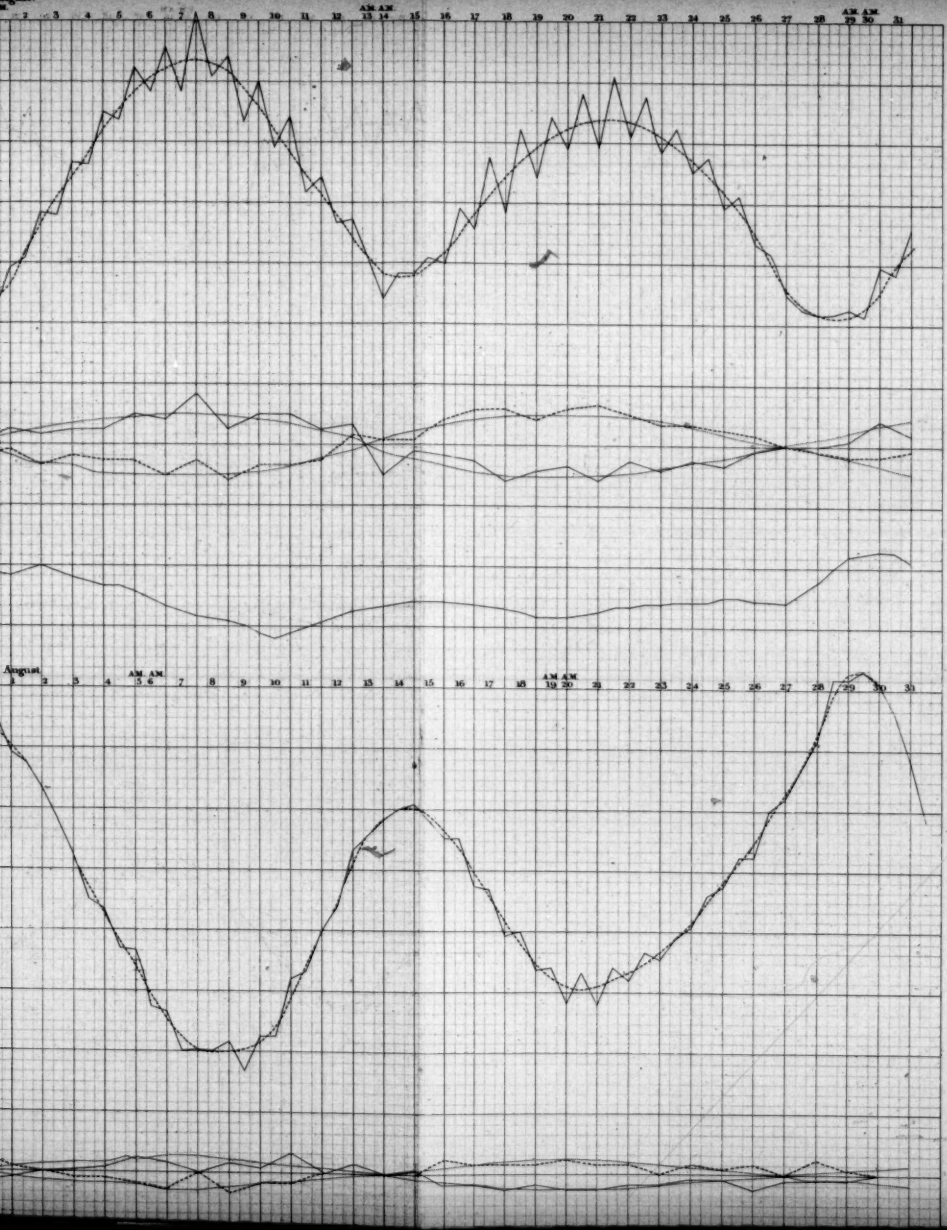
The water cannot be entirely expelled without risking the sublimation of chloride of ammonium, and hence the quantity of water obtained is under the truth. The copper is above the truth, from having been precipitated by caustic potash in the state of oxide, which last when so obtained always retains a little potash.

There is a corresponding chloride of copper and potassium, but I did not succeed in forming analogous double salts with chloride of magnesium or with any other chloride of the class in the place of the chloride of copper.

The chlorides have probably their analogues in the cyanides, but with the single cyanides of iron, copper, &c., we are less acquainted. It is worthy of remark, however, that the disposition of the protocyanide of iron and of the cyanide of copper to combine with two atoms of cyanide of potassium may depend upon the cyanides of iron and of copper possessing two atoms of constitutional water, (like the corresponding chlorides,) which are displaced by two atoms of the alkaline cyanide in the formation of the double cyanides. In "ferrocyanic acid" we have the protocyanide of iron combined with two atoms of hydrocyanic acid, in the place of the same two atoms of water.

PLYMOUTH 1834. RANGE OF TIDES. RANGE OF

Phil. Trans. MDCCCXXVII. Plate II. p. 10.



VII. *Researches on the Tides.—Seventh Series. On the Diurnal Inequality of the Height of the Tide, especially at Plymouth and at Singapore; and on the Mean Level of the Sea. By the Rev.* W. Whewell, *M.A. F.R.S., Fellow of Trinity College, Cambridge.*

Received March 7,—Read March 9, 1837.

THE Inequality of the Tides which is the subject of the present paper, though theoretically very curious, and practically very important, has hitherto been hardly noticed, and its laws have never been generally stated. By means of the materials which I have had in my hands, I have not only been able to obtain a rule agreeing with the observations to an extraordinary degree of precision, but I have found and analysed a case in which this inequality assumes a very remarkable form, so as materially to disguise the general circumstances of the tides, and to explain other cases in which the usual features are entirely obliterated.

The inequality of which I speak is the Diurnal Inequality, by which the tide of the morning and evening of the same day differ. The difference is often very considerable, especially in the height of the water; and naval officers have often found the preservation or destruction of a ship to be caused by this difference, without being aware that it was subject to steady rules, and was capable of being predicted. The small number of places for which I have been able to procure the proper observations will not permit me at present to state the circumstances of the inequality as they occur all over the surface of the ocean; but I am, by fortunate circumstances, able to trace its laws in some very remarkable instances, situated in very widely separate regions of the globe.

Sect. I. *Diurnal Inequality at Plymouth.*

I will first treat of the diurnal inequality as it appears at Plymouth, at which port good tide observations are regularly made under the direction of Mr. Alexander Lumsdale and Mr. William Walker, the Master Attendant and Assistant Master Attendant of the Dock-yard.

It has long been known that both at Plymouth and at other places there is commonly a difference in the morning and evening tide of the same day. It is stated by Colepress in 1668*, that at that port "the diurnal tides from about the latter end of March till the latter end of September are about a foot higher in the evening than in the morning; that is, every tide which happens after twelve in the day before twelve at

* Philosophical Transactions, vol. iii. p. 633.

night, and *vice versâ* the rest of the year." But we shall soon see that this way of expressing the fact, by speaking of morning and evening tides, is quite inaccurate.

It is easily seen that the theory of the tides, which supposes them to be produced by the ocean assuming its form of equilibrium under the influence of the moon's attraction, would give a diurnal difference of the tides: for if the moon have 20° north declination, the tide spheroid will have one pole in latitude 20° north, and the other in 20° south latitude; and as the earth revolves, a place in 50° north latitude will have the tide which belongs to these two poles alternately: and as it is 30° from one pole and 70° from the other, the two tides will be very unequal.

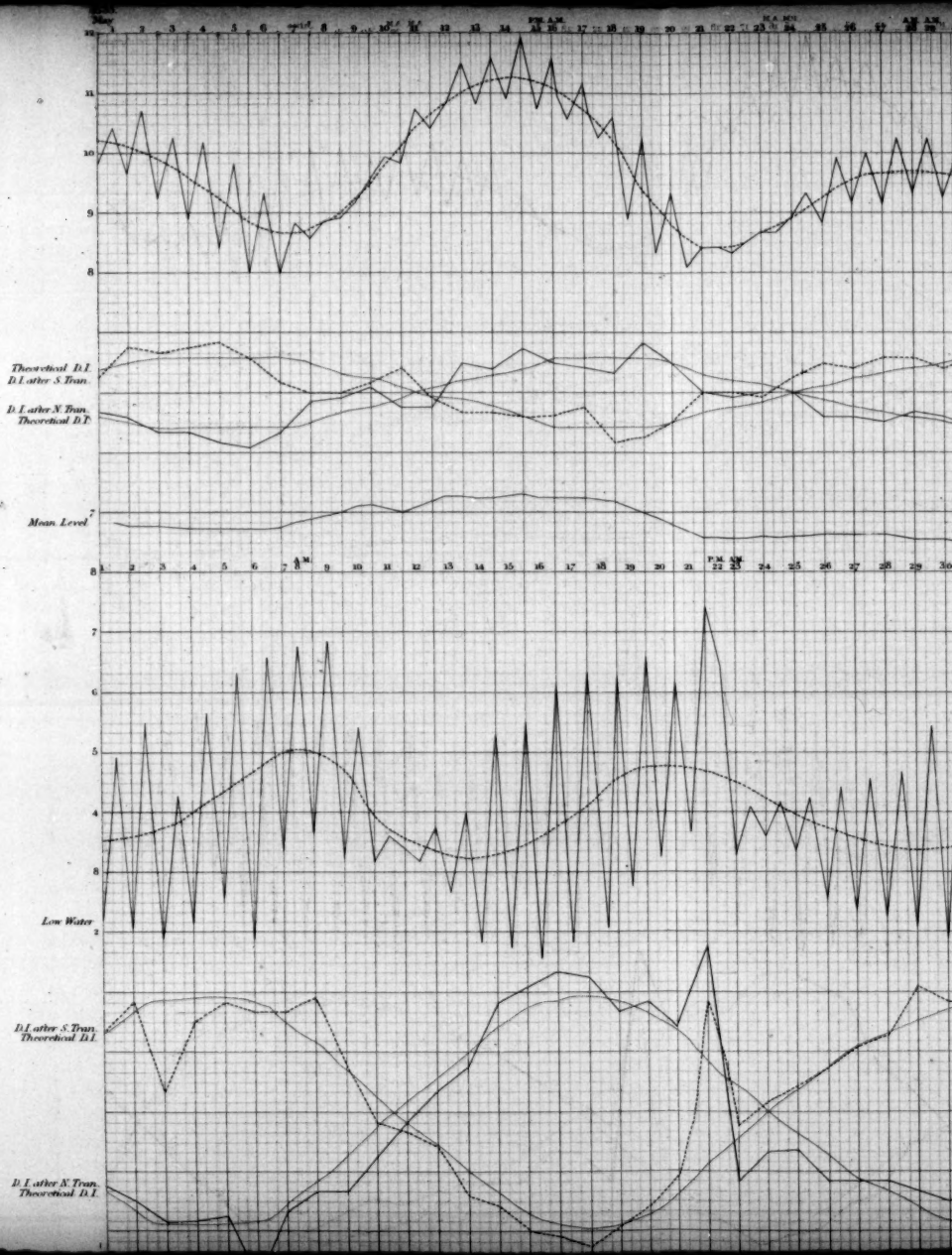
Now it has been found, with regard to all the other inequalities of the tides, that they follow the *laws* of the tides of the equilibrium-theory, although the *constant elements* (the *magnitudes* and *epochs*) can be determined only by observation. Finding that the diurnal inequality was very clearly marked in the Plymouth observations, I did not hesitate to attempt to trace its laws, by assuming this kind of correspondence with the equilibrium-theory. The result confirmed the assumption in the most striking manner, as I shall show.

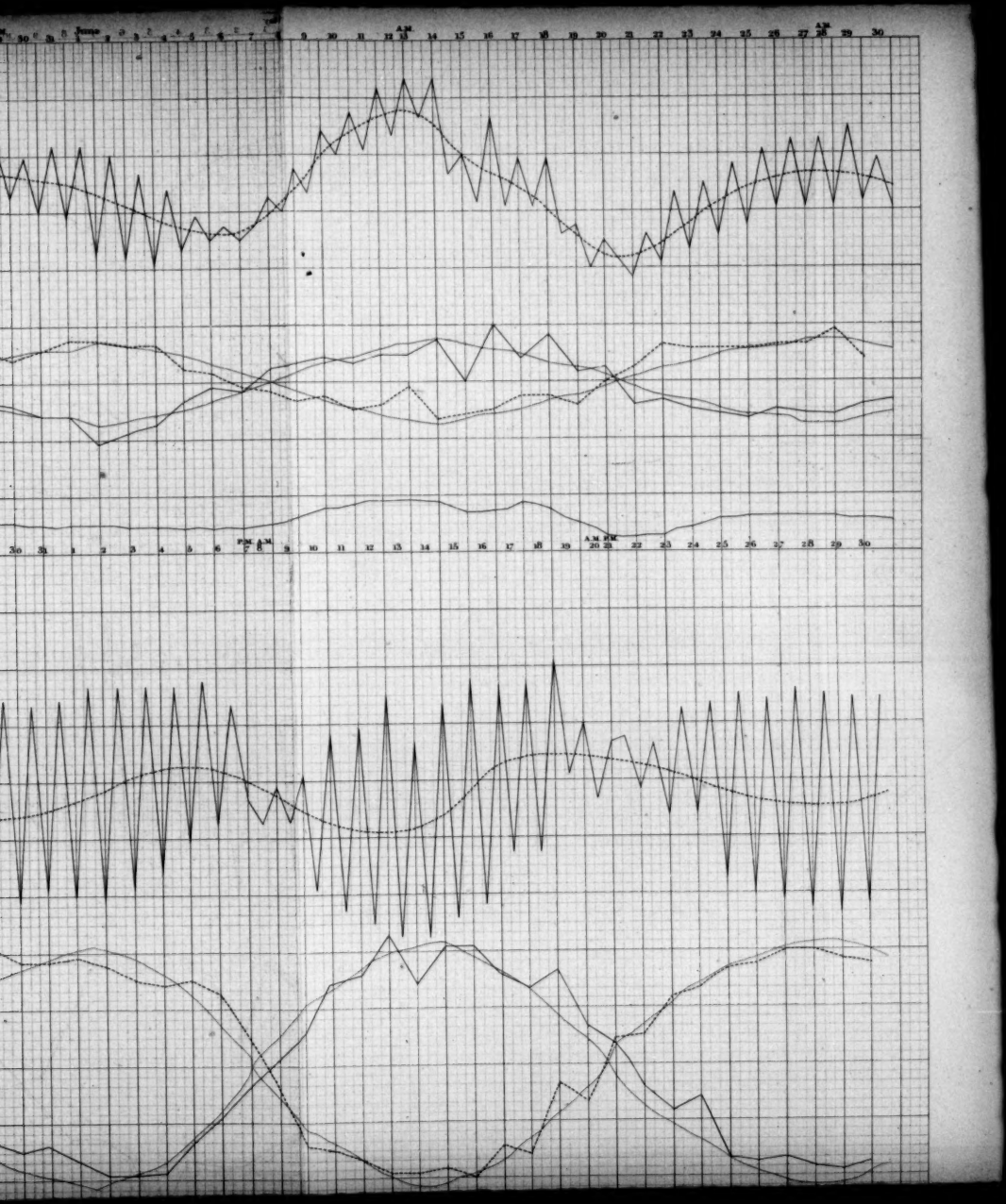
According to the equilibrium-theory, the tide which belongs to a *south* transit of the moon should be the *greater* (of the two on the same day) when the moon's declination is *north*; when the moon crosses the equator, the difference of the two tides vanishes; when she has *south* declination, the tide which belongs to her *south* transit is the *smaller*. The contrary (as to greater and smaller) will be true of the tide which belongs to the *north* or *inferior* transit.

We cannot know, except by observation, to what transit of the moon any tide *belongs*; but it is manifest that if we begin with any tide, the tides must belong alternately to south and north transits, and therefore the above alternation of greater and smaller tides, as the moon has north or south declination, must come into view.

Accordingly I set off the observed heights of high water at Plymouth as ordinates of a curve, as seen in Plates II. and III. The zigzag form of the lines, appearing, vanishing, and reappearing, about once a fortnight, with great steadiness, showed that the diurnal inequality really existed in the observations. A line was drawn *by the eye*, cutting off from these zigzags equal portions above and below, and this was taken as the mean high water cleared of the diurnal inequality. The *excesses* or *defects* of the observed height and this mean height were then set off from another axis; those which belonged to the high water next following a south transit forming one curve, and the alternate tides, which follow a north transit, forming another curve. These curves were both of them found to have their ordinates alternately positive and negative at intervals, corresponding to the change of the moon's declination from north to south, and the contrary.

But though the order and cycles of these changes were the same for the observed diurnal inequality and for the lunar declination, the epochs of the changes were not the same. According to theory, as we have said, the diurnal inequality ought to


Theoretical D.I.
D.I. after S. Tran.
D.I. after N. Tran.
Theoretical D.I.
Mean Level
Low Water
D.I. after S. Tran.
Theoretical D.I.
D.I. after N. Tran.
Theoretical D.I.



vanish when the moon is in the equator. But it appeared that in fact the diurnal inequality did not vanish till about four days after that period.

By taking the moon's declination four days anterior to the day of observation, and reducing it to a proper scale, it was found that the amount of the diurnal inequality could be represented with great accuracy, as may be seen in the Plates, which are specimens of a comparison of this kind made for the whole of the years 1833 and 1834.

It is to be observed, however, that the calculation of the diurnal inequality from the declination was made by means of a coefficient which was somewhat different in different months. Thus the usual multiplier of the declination for the diurnal inequality of high water at Plymouth is $\frac{1}{4}$; that is, 4° of lunar declination produce a difference of height of 1 inch: but in some cases the coefficient is $\frac{1}{3}$ or more; in others it is $\frac{1}{5}$. These differences appear to arise in part from the height of the tide itself; for the inequality is theoretically proportional to the whole lunar tide; partly to the effect of the sun, according to different seasons of the year. Yet there appears to be still some other unexplained cause of the variation of this multiplier; for there are differences in its value which cannot be referred to the causes just mentioned, and which operate, during a lunation or a semilunation, too uniformly to be accidental. We may take the coefficient at Plymouth to be $\frac{1}{4}$, which is the value on which the curves represented in Plate II. are constructed. I am persuaded that no one accustomed to the comparison of theoretical formulæ with observation can look at those curves without being persuaded that the formula exhibits the true law of nature.

As has been said, the declination of an anterior period has been taken. The period employed was the *fifth* lunar transit preceding the tide. Thus the diurnal inequality of January 6, 1834, is determined by the declination on January 2, at $5^h\ 44^m$ A.M., the time of the moon's transit. The assumption of this period is confirmed by the general agreement of the results.

From what has been said, the inaccuracy of the statements of this inequality, as an excess of the evening tide at particular seasons, and of the morning tide at other seasons, will readily appear; for the high water at Plymouth is, on the average, five hours after the moon's transit. Suppose the moon to move in the ecliptic, which is her average path: when the sun's right ascension is five hours, (that is, about June 7,) the tide which follows the moon's transit will follow the sun's transit also, as soon as the moon is north of the equator; that is, if the diurnal inequality were regulated by the moon's place on the same day, the afternoon tide would be greatest; and so it would continue till the moon was seven hours after the sun, at which period the tide would be twelve hours after the sun, and the tide following the moon would become the morning tide. But at the same time the moon would pass to the south of the equator; and therefore the tide following the moon would be the smaller. Therefore in this situation the evening tide would be the greater during the whole lunation.

But suppose the sun's right ascension to be eight hours, (July 21,) then, when the

moon begins to have north declination, the tide which follows her, and which is therefore the greatest, is three hours before the sun, and is a morning tide. When the moon's right ascension becomes three hours, (that is, after about one eighth of a lunation,) the tide following the moon, which is still the greater, (because the declination is still north,) becomes the evening tide. The evening tide continues the greater till the moon's right ascension becomes twelve hours, when she passes to the south of the equator, and the tide following the moon, which is nine hours after the sun, and still the evening tide, becomes the smaller; and this continues till the moon is seven hours from the sun, or in fifteen hours right ascension, at which period the tide which follows her becomes the morning tide, and the evening tide is again the greater. Thus in this position the morning tide is greater during six hours of the moon's motion in right ascension (from the sun), and the evening tide is greater during the remaining eighteen hours; that is, the evening tide is the greater during three fourths of the lunation.

We might in the same manner trace the changes which take place in other positions of the sun; but this is unnecessary. The effect of the inequality may be calculated by the tables which are added at the end of this paper.

The height of low water at Plymouth is also affected by a diurnal inequality. It follows the same law as the inequality of high water; its epoch is the same; and its multiplier for May, June, July, August, 1834, is $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{9}$, respectively.

Sect. II. *Diurnal Inequality at Singapore.*

By the Hydrographical Office of the Admiralty I was furnished with about a year's observations of the tides of Singapore, from August 1834 to August 1835, made by Mr. W. Scott, the Master Attendant at that port, in pursuance of directions given by the Directors of the East India Company.

These observations, from the very curious nature of the results to which they lead, I consider as more remarkable and valuable than any series of equal extent which has fallen under my notice.

On laying down the heights of high water, it appeared that the early part of the series was very irregular, obviously from the imperfection of the observations; but beginning with January 1835, the curve was tolerably regular; and during the greater part of the subsequent time, the inequalities (which the observers could not know) were so clearly marked, and so steady in their course, that it was impossible to doubt the goodness of the observations.

I proceeded to examine these in the manner already described for the Plymouth observations, and found a diurnal inequality nearly agreeing in law and in amount with that at Plymouth; the only difference being, that instead of four days it was here found necessary to take the lunar declination *a day and a half* preceding the tide, or, more exactly, at the *interpolated* or north lunar transit which intervened between the second and third south transit preceding the tide.

The amount of the inequality is nearly the same as at Plymouth, or rather greater, being, in the most regular parts of the series, one inch of height for every three degrees of declination.

In these parts of the series (May, June, July, 1835,) the coincidence of the formula with observation is as close as at Plymouth. In other months (March, April, and August,) there are discrepancies; but we cannot consider these as throwing any doubt on the general correctness of the formulæ, when we see how well it represents the observed diurnal inequality of low water, which is much more marked than that of high water.

The diurnal inequality of low water at Singapore is of a magnitude which it would have been impossible to anticipate. It makes a difference in many cases of not less than six feet between the height of the morning and evening tide; the whole rise of the mean tide being only seven feet at spring tides, and the difference of mean spring and neap tides not more than two feet.

This enormous diurnal inequality conforms, with deviations which are slight considering its magnitude, to the same formula which we have already stated, the epoch being the same as that for high water; that is, thirty-six lunar hours anterior to the last transit. The multiplier is different in different months, varying from $\frac{3}{4}$ to 1; so that each degree of the moon's declination produces an effect of nearly an inch in the height of low water, or two inches in the difference of two successive low waters.

Sect. III. *On the Diurnal Inequality at some other places, and on the General Laws of its Progress.*

I have not found any register of tide observations which exhibits the diurnal inequality so clearly and regularly as Plymouth and Singapore, although I have tried many series observed in different parts of the world. It may however be detected in many, perhaps in most, places. The comparison of the circumstances of this inequality in different places is curious and interesting, and especially the change which the *epoch* undergoes; that is, the anterior period at which the moon's declination corresponds to the amount and direction of the inequality.

Bristol.—Mr. Bunt, who has bestowed very great labour upon the analysis of tide observations made at Bristol, has, among other inquiries, endeavoured to determine the diurnal inequality at that port. The results are not very regular, but they lead him to the conclusion that the inequality vanishes at nearly the distance of five days' motion of the moon from her nodes; that is, the epoch is *five days*. The amount of the inequality is five or six inches each way, at the greatest.

Liverpool.—The diurnal inequality of the heights at Liverpool has been detected by Mr. Bywater from the observations, and introduced by him into his tide tables. I have already remarked in these Researches *, that the epoch of the diurnal inequality at this port is about *six days and a quarter*; but I do not conceive the determina-

* Fifth Series. Philosophical Transactions, 1836, p. 133.

tion to be very exact, since the inequality has been tabulated by means of the calendar months, and thus has been referred to the moon's mean motion in the ecliptic instead of being referred to her actual motion in her own orbit. The greatest effect is about *half a foot* in excess and in defect.

Leith.—Tide observations have been made at Leith Harbour for several years. I have examined these for the diurnal inequality, but it does not appear with any great steadiness and regularity. Still its existence is very obvious; and as the determination of its epoch is a curious point, I attempted it in the following manner:

Leith Tides, 1835.

Periods of Max. Diurn. Ineq.	Tides after S. Transit.	Middle of Max.	Inequality vanishes.	Moon's dec. vanishes.	Difference. Days.
Feb. 15 to 25.	less	Feb. 20.	Feb. 27.	N. Feb. 15.	12
March 1 to 13.	greater	March 7.	March 13.	S. March 1.	12
March 15 to 25.	less	March 20.	March 27.	N. March 15.	12
March 28 to April 8.	greater	April 3.	April 11.	S. March 29.	13
April 15 to 22.	less	April 19.	April 26.	N. April 11.	13
April 29 to May 9.	greater (*small*)	May 4.	May 9.	S. April 24.	15
May 11 to 18.	less	May 15.	May 22.	N. May 9.	13
May 22 to June 7.	greater (*small*)	May 30.	June 6.	S. May 21.	[16]
June 9 to 14.	less	June 12.	June 17.	N. June 5.	12
June 15 to 30.	irregular	June 23.	June 29.	S. June 18.	[11]
July 1 to 8.	less (*small*)	July 5.	July 10.	N. July 2.	[8]
July 12 to 19.	greater (*irreg.*)	July 16.	July 20.	S. July 15.	[5]
July 21 to 27.	less (*small*)	July 24.	August 3.	N. July 30.	[5]
August 10 to 17.	greater	August 14.	August 22.	S. August 11.	11
Aug. 26 to Sept. 4.	less	August 31.	Sept. 6.	N. August 26.	10
September 8 to 18.	greater	Sept. 13.	Sept. 19.	S. Sept. 8.	11
September 20 to 30.	irregular	Sept. 25.	Sept. 30.	N. Sept. 22.	[8]
October 1 to 11.	greater	October 6.		S. October 5.	

Among all the irregularities of the Leith tides, it is easily seen from the curves, when they are laid down, that there is a diurnal inequality, in consequence of which the *tide following the south transit* of the moon becomes alternately the greater and the smaller, as the moon's declination changes from south to north, and the reverse. The times when this inequality is large can be picked out more decidedly than the times when it vanishes, and I therefore determined the epoch by means of the greatest inequality, supposing the times when it vanishes to be midway between two successive maxima, as may be seen in the preceding Table.

Rejecting those cases in which the inequality is very small or altogether irregular, it appears that the inequality vanishes *twelve days* after the moon's inclination vanishes. This is certainly a very extraordinary result; for it is difficult to conceive how the effect of the moon's action can require so much time to manifest itself. Yet there can hardly be any doubt of the fact; for it is verified in 11 semilunations out of 17, and is inconsistent with none; the variations in the interval being not greater than might be expected, supposing the law to be true. It may be observed, that by these variations the inequality is in some cases thrown back more than a whole semilunation. Thus the inequality which prevails before April 26, and vanishes about that day, is not produced by the series of declinations which vanish on April 24, but by the series which vanish on April 11. To suppose the reverse would be impossible; for that would make it necessary to suppose that the inequality vanishes on Feb. 27, in consequence of the declination vanishing two days *later*, or March 1; that is, that the effect precedes the cause.

In the system of tide observations made on the coasts of Europe and America in June 1835, of the results of which an account was given in the Sixth Series of these Researches*, it appeared that the diurnal inequality on the east coast of Scotland was, during that semilunation, irregular, passing over a tide in the middle of the series. This and other anomalies in the diurnal inequality, as it appears on the coasts of the German Ocean, appear to show that the waters in that region are affected by the mixture of more than one tide. In the most material point, however, the observations of June 1835 confirm the results of our present inquiry; namely, in showing that the diurnal inequality travels more slowly than the other inequalities. On the east coast of America, the changes of this inequality appear to be contemporaneous with the corresponding changes of the moon's declination, and the epoch is *zero*. On the coasts of Spain, Portugal, and France, it is successively *two* and *three days*. And this is quite consistent with the fact that this epoch is *four* days on the coast of Cornwall and Devonshire, *five* days at Bristol, *six* at Liverpool, and *twelve* at Leith. That the diurnal inequality should thus creep from place to place on successive days is difficult to explain; but the laws of fluid motion are so little known, that we cannot collect from hydraulical views any good reason for doubting this curious fact. The fact is certainly not easily reconciled with our conception of the

* Philosophical Transactions, 1836, Part II. p. 304.

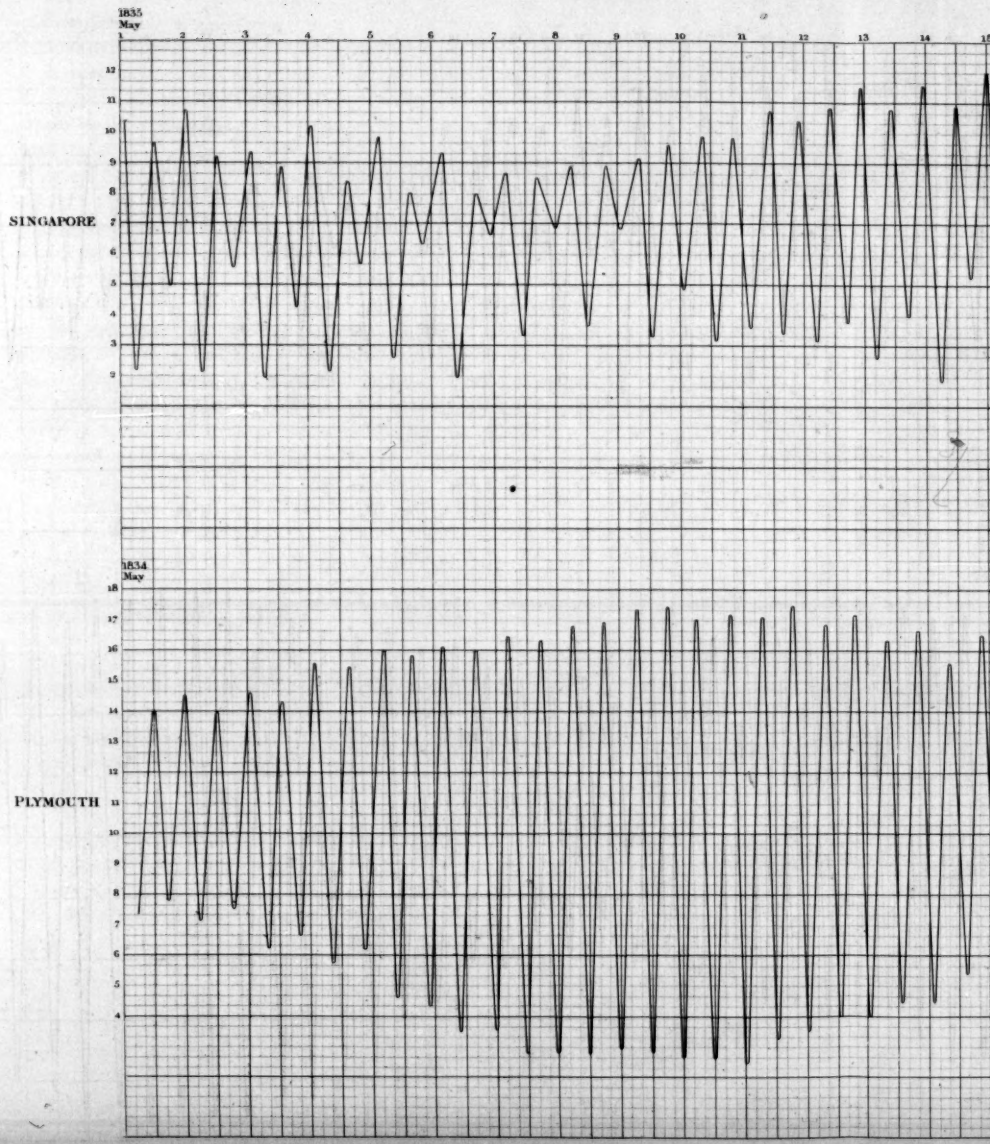
tides of remote places, as produced successively by the motion of the same "tide-wave;" but it is already established beyond doubt, by the observations made on the two sides of the Atlantic in 1835, that tides which were supposed to be brought by the same tide-wave differ materially in their circumstances. As I have already stated*, "On the 9th, 10th, and 11th of June 1835, when the diurnal inequality was great in America it was nothing in the West of Europe; and on the 18th and 19th, when this inequality had vanished in America, it was great in Europe." Are we to doubt whether the tide-wave which brings high water to America and to Europe at the same moment be the same wave? A sound hydrodynamical view of all the circumstances must enable us to decide; but for this purpose more observations are needed, especially observations on the coast of America, where the diurnal inequality is great, and where, on several accounts, a knowledge of its laws would be interesting to us.

Another remarkable circumstance in the progress of the diurnal inequality is, that it appears much more distinctly and steadily at some places than at others which are near them: nor does it seem easy to assign any rule which it follows in this respect. It is very marked and almost universal on the coast of the United States, and was conspicuous in the observations of June 1835 on the coasts of Spain and Portugal, the west coast of France, and parts of the west coast of Ireland. Yet at places interjacent among those at which it was thus displayed it could not be detected; nor did the circumstances easily allow of my ascribing this to any defect of exactness in the observations. In like manner it is large on the east coast of New Holland, as we know from Cook's account of his getting his ship off a reef by means of it; and the north and south coasts of Australia appear to exhibit the extreme case of it, as we shall see. We might therefore suppose that it affects the whole of the Indian Ocean: yet at Keeling Island, in the centre of that ocean, it does not decidedly show itself. Such, at least, is the result of observations made by Captain Fitz Roy, from April 2 to April 8, 1836, with which I have been furnished by his kindness.

Sect. IV. *On extreme Cases of great Diurnal Inequality.*

If we consider the motion of the surface of the water in cases where, as at Singapore, the diurnal inequality is very great, we shall see that this motion is very different from the alternate equal ascent and descent which would occur if there were no such inequality. In order to exhibit this peculiarity, I have represented this motion in Plate IV. for Plymouth and for Singapore, as observed in the months of May and June. It will be seen that at Plymouth the curve of the motion of the surface oscillating upwards and downwards by nearly equal distances; the main feature of inequality is the difference of spring and neap tides, although the diurnal inequality is very clearly visible. But at Singapore the alternate oscillations make no approach to equality; at some parts of the series the alternate tides seem to be on the point

* Philosophical Transactions, 1836, Part II. p. 302.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SINGAPORE
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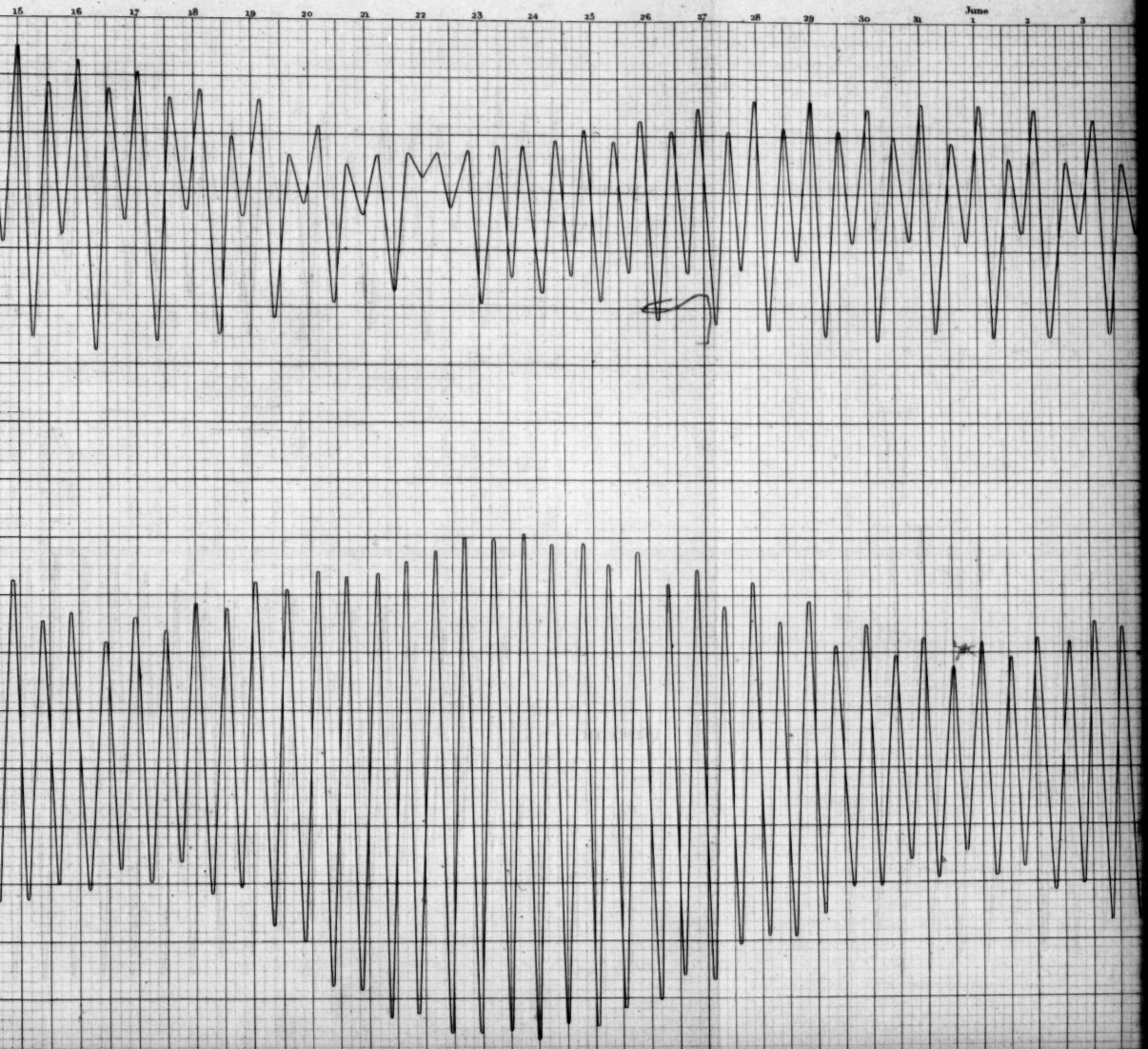
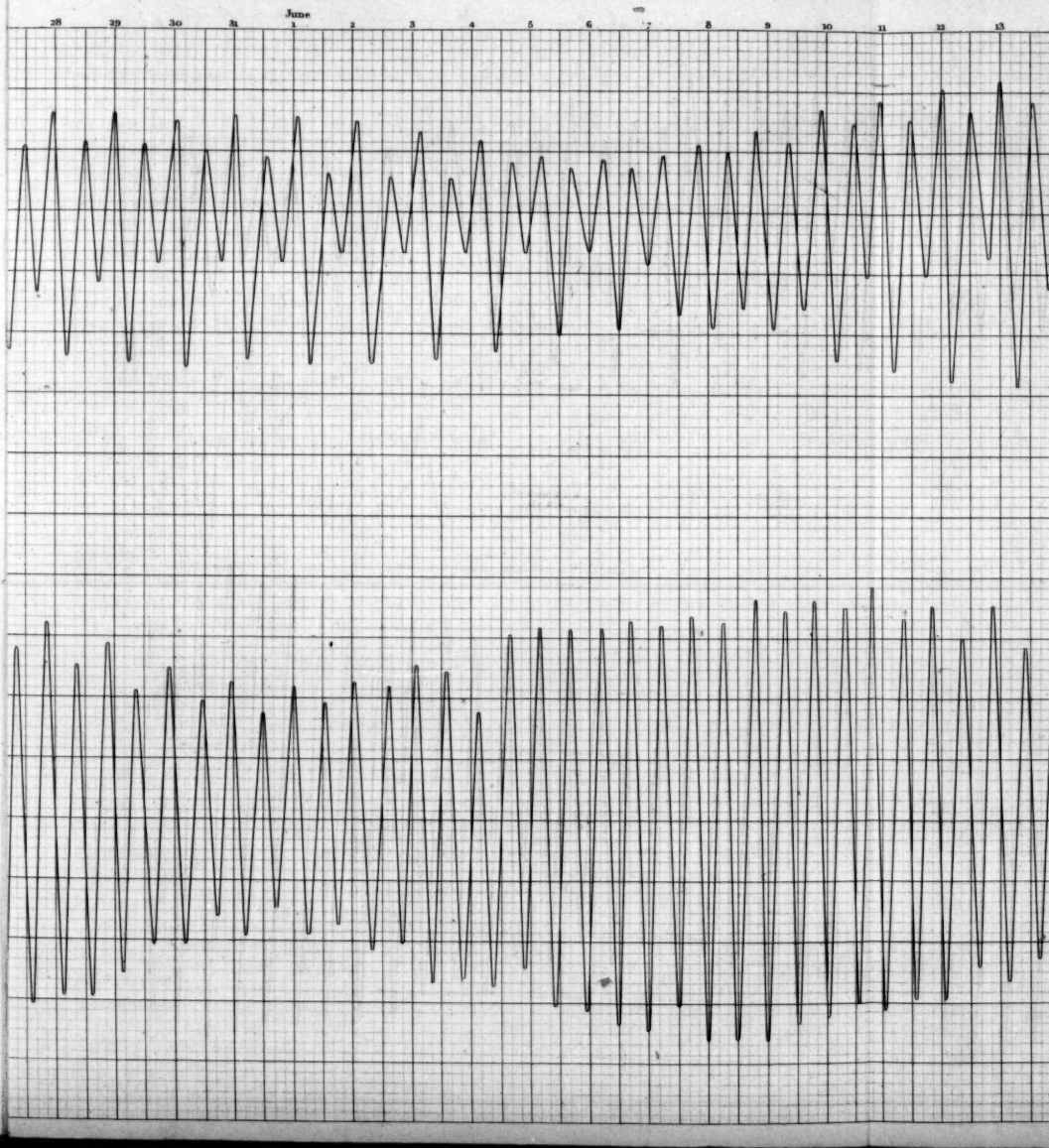
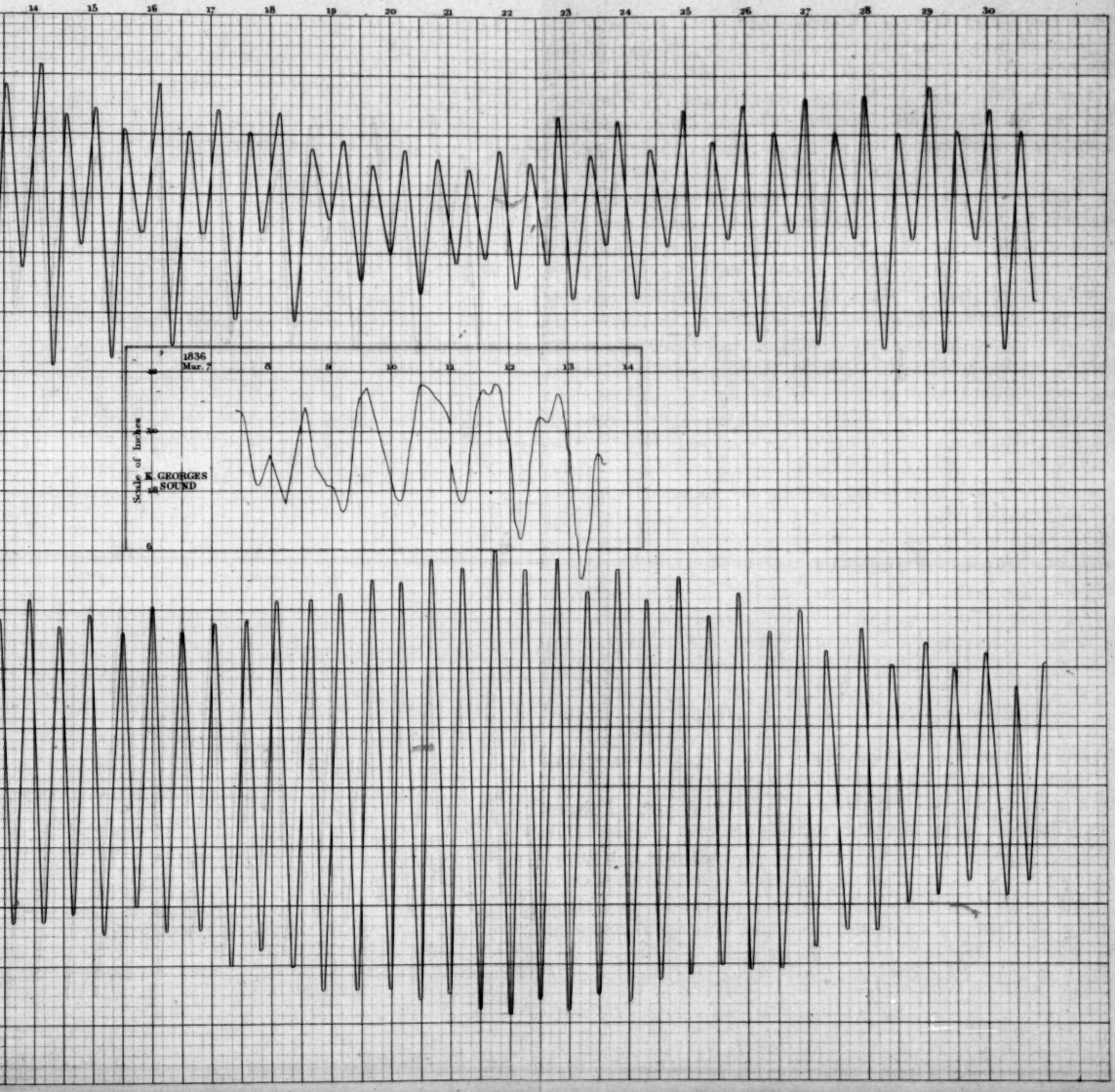
PLYMOUTH

RANGE OF TIDES.



RANGE OF TIDES.

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Scale of Inches
K. GEORGES SOUND

of disappearing; and the progress of this alternation affects the tides as much as the independent alternation of springs and neaps.

It is easy to conceive the diurnal inequality carried a little further than it is at Singapore; so that at a certain stage of it the alternate tides would vanish. This is equivalent to supposing the highest low water and the lowest high water to have the same height.

There are statements of navigators respecting various places at which there is "only one tide in twenty-four hours." From what has been said it appears that this may happen during a part of each semilunation by the effect of the inequality now under consideration, but that it cannot in this way be constantly the case.

I am fortunately enabled to throw some light on this subject by the kindness of Captain Fitz Roy. King George's Sound on the south coast of New Holland is one of the places to which these *Single-day Tides* have been ascribed*. In March 1836 Captain Fitz Roy, aware of the interest of this position in respect to tide phenomena, caused observations to be made every half hour for some days, and for a portion of the time, every quarter of an hour. The result was that on March the 7th and 8th there were two very unequal tides, and that on the 9th and 10th there was only one tide; but a recession and return in the high water, which had been barely perceptible on the 11th, became more and more marked on the 12th, 13th, and 14th, so as again to give two tides each day. Thus at this place it appears to be only at one particular period of the semilunation that we have a single-day tide, agreeably to our general view. I insert the curve of the motion of the surface at King George's Sound in Plate IV.

The single-day tides of Tonquin† were referred by Newton to the interference of two tides, which arrive by different channels. The great diurnal inequality of Singapore, which is in the same seas, appears to be clearly due to the effect of the moon's declination; and the establishment of this point, and the circumstances ascertained to occur in the reputed single-day tide of King George's Sound, throw some doubt on the explanation just referred to, which cannot be removed till the tides of those seas have been more fully observed.

Sect. V. *On the Mean Height of the Sea.*

The question of the fluctuations of the mean height of the sea is not especially connected with the diurnal inequality. But as the curves which I had to draw in the course of this investigation give me the means of exhibiting very clearly these fluctuations, I will here say a word on the subject.

In Plate II. a line is drawn representing the *mean height*; that is, the height midway between low water and high water each day. It is obtained by taking the mean of the two curves of high and low water by which the diurnal inequality is cut off. The same is done for Singapore in Plate III.

* Flinders, vol. i. p. 71. King, vol. ii. p. 380. † See Philosophical Transactions, vol. xiv. p. 162.

It appears that in all these cases the mean height of the sea is very nearly constant. This is most remarkable at Singapore, where, though the successive low waters often differ by six feet, the mean level only oscillates through a few inches. At Plymouth the mean level is not quite so steady. The fact is, that at that port the low water varies more by the difference of springs and neaps than the high water does; and hence the mean level slightly follows the low water, and is lowest at spring tides, and highest at neap tides, or perhaps more exactly a day or two later.

"The level of the sea at low water," a phrase sometimes used by surveyors, is altogether erroneous, and may lead to material error. From the instances just quoted (and indeed from the nature of the case) it is certain that the mean height of the sea is far more nearly constant than low or high water, under whatever assumed conditions. A *level surface* drawn from any point (that is a surface of *stagnant* water) would probably be nearly parallel to the points of mean water at different places. This becomes more manifest when we consider that at places near each other the tide often differs greatly in amount. At St. David's Head in Pembrokeshire the range of the tides is near thirty feet; on the opposite coast of Ireland it is only two or three: if the sea were level at low water the difference of the mean heights on the two sides of the Channel (which is only about fifty miles) would be fourteen feet. Such an average elevation of one side of a narrow sea above the other is quite inconsistent with the laws of fluids.

I cannot conclude this paper without again pointing out that a great number of curious facts in fluid motion are established by these Tide Researches, of which it may be hoped the theory of hydrodynamics will one day be able to render a reason. Why is it that at places near each other the range of the oscillations of the sea from low to high water is so different? Why is it that the sun affects the low water at Plymouth more than the high water, and that the moon's declination at Singapore affects the low water four times as much as the high water, while at Plymouth it affects it less? Above all, why is it that while the effect of the sun, and of the moon's declination and parallax, in the monthly course of the tides, produces the effect due to the equilibrium of the forces in one or two days, the moon's declination does not produce its effect upon the diurnal oscillation till after three, four, five, and six days; and in some cases probably not till the moon is exerting forces which tend absolutely to reverse the effect?

TABLE of the DIURNAL INEQUALITY of the Height of High Water AT PLYMOUTH.

To be used with the moon's declination *four days anterior*.

For *N.* decl., ***add*** to the tide following moon's S. transit, ***subtract*** from the tide following moon's N. transit.
For *S.* decl., ***subtract*** from the tide following moon's S. transit, ***add*** to the tide following moon's N. transit.

Moon's Declination	0° to 4°	5° to 9°	10° to 14°	15° to 18°	19° to 21°	22° to 24°	25° to 26°	27° to 28°	29°	30°
Diurnal Inequality	0in	1in	2in	3in	4in	5in	6in	7in	8in	9in

Postscript.

I will take the liberty of mentioning the only way in which it appears to me mechanically possible to conceive the slow propagation of the diurnal inequality which I have described in Sect. III.

If we suppose equal semidiurnal tides to be propagated along the length of a wide canal; and if we suppose, in addition to these, a *transverse* oscillation of the water to take place in the direction of the width of the canal, the time of this oscillation (from maximum to maximum) being a whole tide day; we shall have successive tides alternately greater and less by a diurnal inequality. And we may easily suppose this transverse oscillation to be propagated *gradually and slowly along* the canal, by the contact of the particles of the water. In this manner we may represent phenomena following laws like those above described.

But it may be further observed, that we may conceive the semidiurnal tide, as well as the diurnal inequality, to be propagated along the canal by means of transverse oscillations, the time of this oscillation being half a lunar day; and the rate of propagation of this undulation may easily be supposed to be different from that of the diurnal oscillation. In this way we may conceive the possibility of the different inequalities of the tides being propagated from place to place at different rates, and thus having different epochs, as from the recent researches on the subject contained in the Philosophical Transactions they appear to have.

Moreover, it is by no means necessary, in order to make this explanation applicable, that the transverse undulations should be perpendicular to the direction in which the tide is propagated: they may be oblique to it at any angle, and the result will still be the same.

It appears possible, also, that such a supposition may be modified, so as to explain other phenomena of the tides; for instance, the smallness of the tides in the central parts of wide seas.

But the application of such a supposition to the actual phenomena of the ocean, and the determination of those tracts of sea which must, on this view of the case, be looked upon as tide-canals, would be a matter of no small difficulty, even if our materials were sufficient for the purpose, and would probably be impossible without more knowledge of the tides on the shores of the great oceans than has yet been published.

Trinity College, Cambridge,
May 5, 1837.

VIII. *On the Structure of the Brain in Marsupial Animals.* By Richard Owen, *Esq. F.R.S. Hunterian Professor of Anatomy to the Royal College of Surgeons.*

Received October 31, 1836,—Read January 26, 1837.

THE brain in Mammalia is essentially characterized by the complexity and magnitude of the apparatus by which its different masses are brought into communication with one another. With respect to size, the cerebral hemispheres are in many species proportionally inferior to those of Birds; and in most Insectivorous and Rodent Mammalia they present an equally smooth and uniform external surface; but notwithstanding the absence of convolutions and diminished size of the cerebral hemispheres in such Mammalia, a large apparatus of medullary fibres is present, which connect together either the opposite hemispheres, or the distant parts of the same hemisphere; and this apparatus, or great commissure, is superadded to the anterior, posterior, and soft commissures, which, with the exception of a very slight rudiment of the fornix, are alone developed in birds for the purpose of uniting the opposite hemispheres. In the higher Mammalia, in which the cerebral hemispheres acquire superior size and increased extent of surface by means of convolutions, the superadded commissural apparatus presents a corresponding development and a highly complicated structure; its several parts being distinguished as the corpus callosum, fornix, and their intercommunicating laminæ, termed the septum lucidum. The fornix, by means of its posterior crura and the intermediate medullary tract termed the lyra, brings the hippocampi majores into communication with each other, and with the posterior folds of the corpus callosum*; by means of its anterior crura it establishes a communication between the hippocampi and the optic thalami; and by means of the septum lucidum its connexion with the corpus callosum is continued to the anterior fold of that body†.

In the Human brain the fornix, though of complex structure and developed as a very distinct part, is of small size as compared with the corpus callosum; while the delicate

* "The fasciculi from the fornix form in part the covering of the hippocampus, and in part its loose fold, the tænia hippocampi."—Reil in Mayo's Anatomical Commentaries, p. 116.

"L'envelope medullaire de la corne d'Ammon se continue avec la partie posterieure du corps calleux, et en partie aussi avec le pilier posterieur de la voute: c'est dans ce dernier que va se jeter le corps frangé tout entier."—Meckel, *Anatomie Descript.* tom. ii. p. 679.

† "Ainsi la voute represente une chaine très complexe qui unit les deux hémisphères l'un avec l'autre sur plusieurs points, et qui, de plus, établit une communication entre la partie anterieure et la partie posterieure de chaque hémisphère."—Meckel, *Anatomie Descript.* tom. ii. p. 658.

laminæ of the septum lucidum by which the fornix is connected with the corpus callosum, present an extent of surface corresponding to the degree to which the corpus callosum and fornix recede vertically from one another as they advance from behind forwards. In tracing the modifications of these different parts through the mammiferous series, the disproportion of the fornix to the corpus callosum is found to decrease as the parts, to the connexion of which they are subservient, alter in their relative size. For as the superincumbent masses of the cerebral hemispheres diminish in the placental Mammalia, the corpus callosum is proportionally restricted in its development; while the hippocampi and their free processes, called the tæniæ hippocampi, maintaining a remarkable uniformity in their absolute size, the fornix also continues large, and undergoes modifications of form which more distinctly manifest its relation as a commissure to the hippocampi than its structure in the human brain would indicate. Thus in the brain of the Sheep the tæniæ hippocampi, instead of being lost in the posterior crura of the fornix, are continued along its lateral margins, augmenting its breadth: they converge and unite above the anterior crura of the fornix, which here appear as small subordinate appendages sent off into the optic thalami below, from the union of the tæniæ above; the tæniæ are then again separated and are continued downwards and forwards into the anterior lobes of the hemispheres, bringing these parts into communication with the hippocampi behind, whilst the point of union of the opposite tæniæ becomes continuous with the anterior fold of the corpus callosum above.

As the corpus callosum and fornix recede vertically from one another in a less degree in most Mammalia than in Man, the two laminæ of the septum lucidum are consequently of less extent, but are proportionally stronger, as they are formed not merely by the epithelium of the lateral ventricles, but by fibrous laminæ extending from the anterior and upper surface of the fornix to the opposite surface of the corpus callosum. In the simple and depressed forms of brain, such as the Rodentia present, the fornix, or hippocampal commissure, and the corpus callosum, or hemispheric commissure, are in contact, so that their uniting medium cannot with propriety be termed the septum lucidum.

The corpus callosum is the principal bond of union between the opposite hemispheres, extending horizontally above the ventricles, its middle fibres passing transversely, while those of its extremities, which are more or less bent beneath its body, radiate, and all intermix, in apposition with the ascending and diverging fibres of the peduncles of the cerebral hemispheres. It has hitherto been considered as the great characteristic of the brain in the Mammalia, and, taking the human brain as the term of comparison, to be developed in the ratio of the magnitude of the cerebral hemispheres.

In the placental Mammalia this is a pretty accurate expression of the relations of the corpus callosum; and as the posterior lobes of the hemispheres are the first to disappear in the descending comparison, so the corpus callosum diminishes in longi-

tudinal extent from behind forwards, and thus the corpora quadrigemina, pineal gland, and posterior part of the optic thalami are successively brought into view on divaricating the cerebral hemispheres in the different Mammalia which exhibit this progressive degradation of the great commissure.

The researches of TIEDEMANN, as is well known, have shown that the anterior part, which is the most constant in the mammiferous series, is that from which the development of the corpus callosum commences in the human brain.

The aim of the present paper is not, however, to trace step by step the various modifications of the commissural apparatus of the hemispheres through the mammiferous class, but is limited to the description of a remarkable modification in that apparatus in the brains of the marsupial animals, to the detection of which I was led by observing that the commissural system presented the essential difference between the brains of the oviparous and mammiferous Vertebrata, and by associating the greater perfection of the brain, resulting from the development of the great commissure with the placental mode of development in the true Mammalia.

The connexion subsisting between placentation and high cerebral organization may be one of simple coincidence, yet it is certain that of all the great organic systems, the cerebral or sentient organ is that which alone offers a marked improvement of gradational structure in the animals developed by a placenta.

An attentive study of the manners of different Marsupiata in confinement, and an inspection of the exterior forms of the brain in some of the species, induced me to allude in a former paper to an inferiority of intelligence and a low development of the cerebral organ as being the circumstances in the habits and structure of these singular animals which were most constantly associated with the peculiarities of their generative economy*. I have since derived the most satisfactory confirmation of this coincidence from repeated dissections of the brains of Marsupiata belonging to different genera; and although unable to explain how a brief intra-uterine existence and the absence of a placental connexion between the mother and fœtus can operate (if it be really effective) in arresting the development of the brain, yet it is a coincidence which has been so little suspected, and is so interesting in various points of view, that I believe the evidence of it will be acceptable both to the physiologist and the naturalist.

In order to obtain satisfactory proof of the difference in the structure of the brain in the marsupial and placental quadruped, I dissected and compared together, step by step, the brains of a Wombat and Beaver. These animals, as is well known, are of nearly similar bulk, and manifest so many mutual affinities in their structure, that they have been, and still are, by some naturalists, classed in the same order of Mammalia. The Wombat is, in fact, in all its exterior characters, save the marsupial pouch, a Rodent; and in its internal anatomy, especially its digestive organs, more nearly resembles the Beaver than do many of the true rodent animals. The

* Philosophical Transactions, 1834, p. 358.

brain of the Beaver was also preferred for this comparison of internal organization, because on an outward inspection it would be pronounced to be the less highly organized of the two; the hemispheres in the Wombat presenting a few convolutions (Plate V. fig. 3.), whilst in the Beaver they are perfectly smooth (Plate V. fig. 1.).

In the Beaver, however, the cerebrum is extended further backward, though still leaving the cerebellum quite uncovered; while in the Wombat a portion of the optic lobes (corpora quadrigemina) is also exposed.

On divaricating the hemispheres of the brain in the Beaver, we bring into view, about three lines below the surface, the corpus callosum; and on removing the cerebral substance to a level with this body, its fibres are observed to diverge into the substance of each hemisphere, in the usual manner, some bending upwards, but a greater proportion arching downwards, and embracing the cerebral nuclei; the anterior fibres radiating into the anterior, the posterior fibres into the posterior extremities of the hemispheres. (Plate VI. fig. 3.)

The portions of the brain which are removed in thus tracing the extent of the corpus callosum, bring into view the corpora bigemina and the pineal gland; but the optic thalami are concealed by the great commissure above described.

On separating the hemispheres of the brain of the Wombat, not only the bigeminal bodies and pineal gland, but the optic thalami are immediately brought into view, and instead of a broad corpus callosum, we perceive, situated deeply at the bottom of the hemispheric fissure, a small commissural medullary band, *m*, (Plate VII. fig. 4.) passing in an arched form over the anterior part of the thalami, and extending beneath the overlapping internal or mesial surfaces of the hemispheres, which thus appear, as in the Bird, to be wholly disunited.

On gently raising the hemispheres from above the commissure, and pressing them outwards with the handle of a scalpel, the instrument passes into the fissure upon which the hippocampus is folded; and on continuing the pressure the hippocampus is torn through, and the lateral ventricle is exposed. The mesial wall of the hemisphere is continued from the superior and internal border of the hippocampus, and is composed in the Wombat, as in the Bird, of a thin lamina of medullary substance analogous to the septum lucidum. In the Kangaroo, the mesial parietes of the lateral ventricles are stronger, being about two lines in thickness.

The posterior transverse fibres of the commissure are continued outwards and backwards, beneath the more longitudinal fibres, which overlap them as they pass from the tæniæ hippocampi forwards to the anterior cerebral lobes. All the fibres of the commissure pass along the floor of the lateral ventricles into the substance of the hippocampi majores, which are of proportionally very large size. (See Plate VI. and VII. fig. 4, *n*.)

Thus the commissure which is brought into view on divaricating the cerebral hemispheres in the Wombat is seen to be partly the bond of union of the two hippocampi majores in the transverse direction, and partly of the hippocampus and anterior

lobe of the same hemisphere in the longitudinal direction. It also fulfils the other function of the fornix by sending down from the inferior surface two small nerve-like processes, which extend vertically, behind the anterior commissure, through the substance of the optic thalami, near their mesial surfaces, to the corpus albicans, at the base of the brain.

The superior view of the connexions of the hippocampal commissure of the Wombat is given at Plate VI. fig. 4.

Returning to the Beaver's brain, we raise the posterior thickened margin of the corpus callosum, and at the middle of its inferior surface we find it closely connected with the centre of a commissural band of fibres, arching over the anterior part of the optic thalami, and passing outwards and backwards along the floor of the lateral ventricles into the substance of the hippocampi, which are as largely developed as in the Wombat. The anterior part of the corpus callosum is bent downwards, and is attached along the middle line of its inferior surface by a uniting medium of medullary substance, representing the septum lucidum, to the hippocampal commissure or fornix. The tæniæ hippocampi, which form the lateral parts of this commissure, extend forwards, as in the Wombat, into the anterior lobes.

The corpus callosum being removed, and the commissural fibres of the hippocampi being left behind (as shown on the left side at Plate VI. fig. 5.), the view of the Beaver's brain now corresponds with that obtained in the previous dissection of the brain of the Wombat; which we regard, therefore, as wanting the corpus callosum, septum lucidum, and consequently the fifth ventricle. The artery of the plexus choroides, in both the Beaver and Wombat, enters the lateral ventricle, where the hippocampus commences at the base of the hemisphere, and the plexus is continued along the under surface of the tænia hippocampi, and passes beneath the fornix, through the usual foramen, to communicate with its fellow in the third ventricle, immediately behind the anterior crura of the fornix, which are sent down in the Beaver, as in the Wombat, from the centre of the inferior surface of the hippocampal commissure.

If we expose the lateral ventricle by removing its outer parietes in a marsupial and placental quadruped, as shown in Plate VII. figg. 4 and 5, in the Kangaroo and Ass, the hippocampus major (*n*), the tænia hippocampi (*o*), the plexus choroides (*p*), and the foramen Monroianum (*γ*) are brought into view. If a style be thrust transversely through the internal wall of the ventricle, immediately above the hippocampus, in the placental quadruped, it perforates the septum lucidum (*q*), and enters the opposite ventricle below the corpus callosum. If the same be done in the marsupial brain, the style passes into the opposite ventricle, but is immediately brought into view from above by divaricating the hemispheres, and is seen lying above the commissure of the hippocampi.

This commissure may nevertheless be regarded as representing, besides the fornix, he rudimental commencement of the corpus callosum; but this determination does not invalidate the fact that the great commissure which unites the supraventricular

masses of the hemispheres in the Beaver and all other placentally developed Mammalia, and which exists in addition to the hippocampal commissure, is wanting in the brain of the Wombat: and as the same deficiency exists in the brain of the Great and Bush Kangaroos, the Vulpine Phalanger, the Ursine and Mauge's Dasyures, and the Virginian Opossum, it is most probably the characteristic of the marsupial division of Mammalia.

In the modification of the commissural apparatus above described, the Marsupialia present a structure of brain which is intermediate to that of the placental Mammalia and Birds, in which class the great commissure is wholly wanting, and the hemispheres, though comparatively larger than in many of the Mammalia, are brought into communication only by means of the anterior, posterior, and soft commissures, and a slight trace of the fornix or hippocampal commissure.

Of the other peculiarities of the marsupial brain, the relatively large size of the anterior commissure is most worthy of notice; its development is correspondent with the large size of the cerebral ganglion, which forms the chief origin of the olfactory nerve, and some of the anterior fibres arch forwards, and are directly continued into those nerves.

In the position, superficial transverse fissure, and solidity of the bigeminal bodies, the marsupial brain adheres to the mammiferous type, as also in the exterior transverse fibres of the commissure of the cerebellum, forming the pons Varolii, the presence of which is in relation with the development of the lateral lobes of the cerebellum.

Other minor points of difference between the brains of the Marsupiata themselves will be explained in the description of the figures.

Meanwhile their agreement in so important a modification of the cerebral organ as the absence of a corpus callosum and septum lucidum, affords additional and strong grounds for regarding the Marsupiata as a distinct and peculiar group of Mammalia; and when to this modification of cerebral structure are added the traces of the oviparous type of structure presented in the circulating and absorbent systems, together with the peculiarities of the osseous and generative apparatus, we may with reason suspect that distribution of the Marsupiata to be artificial and founded on an imperfect knowledge of their mutual affinities which, from a modification of the teeth and extremities alone, would separate and disperse the species amongst corresponding groups of the placental Mammalia.

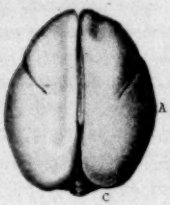
Cuvier has observed that the marsupial group of quadrupeds embraces forms which typify different orders of the ordinary Mammalia*; and M. De Blainville regards them as forming, with the Monotremata, a division apart from the placental Mammalia. The metropolis of this subclass is the continent of Australasia, where the different carnivorous, insectivorous, omnivorous, and herbivorous genera act

* "Les Marsupiaux—nous paraissent devoir former un ordre à part, tant ils offrent de singularités dans leur économie, et surtout parceque l'on observe en quelque sorte la representation de trois ordres bien différents."—Règne Anim. i. p. 172.

Fig. 1.

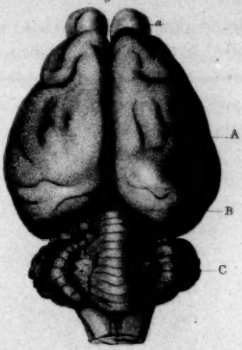
Beaver

2



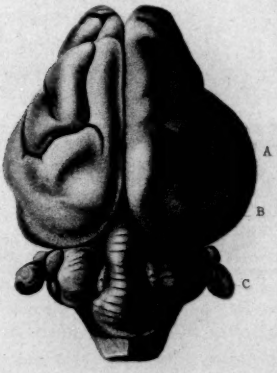
Midas rufimanus

3



Wombat

4



Kangaroo

6

Opossum

9

Opossum

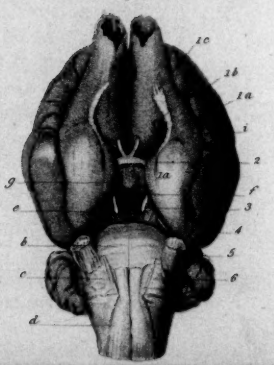
7

Beaver

5

Ursine Dasyure

8



Wombat

Richard Owen. del.

E. Baxter. sc.

corresponding parts to those performed by the placental Mammalia on a larger theatre, in which the avoidance of more numerous and powerful enemies, or the capture of more varied and subtle prey, demands the manifestation of more courage, the practice of more address, and the possession of more resources than appear to be called for by the exigencies of the Marsupiata in their more limited sphere.

Description of the PLATES.

PLATE V.

External form.

Fig 1. The upper surface of the brain of a Beaver (*Castor Fiber*, L.).
2. The upper surface of the brain of a Monkey (*Midas rufimanus*, GEOFF.).
3. The upper surface of the brain of a Wombat (*Phascolomys Wombatus*, BL.).
4. The upper surface of the brain of a Kangaroo (*Macropus major*, SHAW.).
5. The upper surface of the brain of an Ursine Dasyure (*Dasyurus ursinus*, GEOFF.).
6. The upper surface of the brain of a Virginian Opossum (*Didelphys Virginiana*, SHAW).
7. The base of the brain of a Beaver.
8. The base of the brain of a Wombat.
9. The base of the brain of a Virginian Opossum.

From these figures it will be seen that the convolution of the surface of the hemispheres of the brain does not take place in proportion as the hemispheres themselves are developed in superficial extent. They are fewer, for example, in the *Midas*, in which the hemispheres extend, as in most of the Quadrumana, over the greater part of the cerebellum, than in the Kangaroo or Wombat, where the cerebellum is left quite exposed. The brains of two species of herbivorous and two of carnivorous Marsupials are figured in this plate, to show indications of superior development which distinguish the brain of the herbivora, in the greater proportional development of the cerebrum, its convoluted surface, and the smaller proportional size of the olfactory tubercles. In all the species, but especially the carnivorous Marsupials, the greater relative size of the vermiform process is deserving of notice, as indicating the approach to the oviparous type of cerebral structure: it is associated with a corresponding diminution of the pons Varolii, as is strikingly shown in fig. 9.

PLATE VI.

Fig. 1. Side view of the brain of the Kangaroo.
2. Side view of the brain of the Virginian Opossum.

Structure.

Fig. 3. Brain of the Beaver, with the substance of the hemispheres removed to the level of the corpus callosum.

4. Brain of the Wombat, with the substance of the hemispheres removed to the level of the hippocampal commissure, except on the right side, where part of the thin internal wall of the lateral ventricle is left.

5. Brain of the Beaver, with the left cerebral hemisphere cut down to a level with the commissure of the hippocampus, and the lateral ventricle exposed. The corpus callosum has been vertically divided, and the left half removed, together with the hemisphere: the right hemisphere is entire.

6. A similar dissection of the brain of the Kangaroo, with the right hemisphere entire, and turned aside, showing the absence of the hemispheric commissure, corresponding to the corpus callosum of the Beaver.

The small size of the corpus striatum, *r*, as compared with the Wombat and Beaver, is shown in this view. The posterior bigeminal bodies are the broadest, the anterior the longest, in this animal as in the Beaver and Wombat.

PLATE VII.

Fig. 1. A vertical bisection of the brain of the Opossum (*Didelphys Virginiana*, Shaw), showing the large proportional size of the anterior commissure, *y*.

2. A vertical bisection of the brain of a Goose.

3. A lateral section of the left hemisphere, showing the lateral ventricle and hippocampus major in the Opossum.

The roof of the lateral ventricle is raised, showing it to be formed by fibres arching over the hippocampus, and continued from the inner margin of that part into those which radiate from the corpus striatum externally.

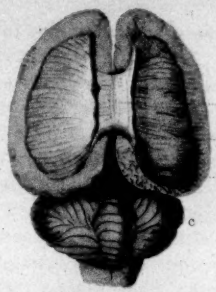
4. A similar dissection of the brain of the Kangaroo.

In this species the roof of the ventricle is proportionally thicker than in the carnivorous Opossum. Besides the diverging fibres of the crus cerebri, and those which pass in an arched form from the inner border of the hippocampus, over that body to the corpus striatum, there are others which form a thin layer, and pass into the tænia hippocampi, closely embracing the hippocampus: some of these are shown at *x x*.

The general disposition of the hemispheric fibres is such, that, supposing them contractile, they would draw the superficies of the hemisphere towards the crus cerebri, as to a fixed point, and compress the bodies projecting into the ventricles.

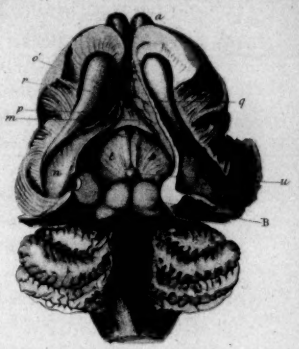
5. A similar dissection of the brain of the Ass. The dotted line shows the extent of the corpus callosum.

Fig. 3.



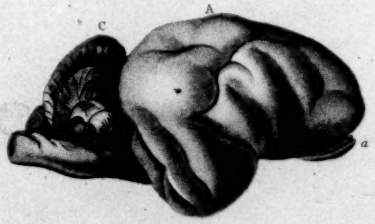
Beaver. N°II.

Fig. 4.



Wombat. N°12.

Fig. 1.

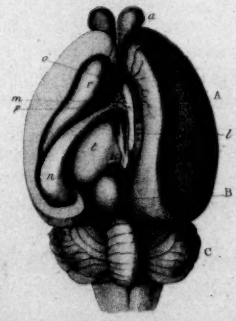


Kangaroo

Fig. 2.

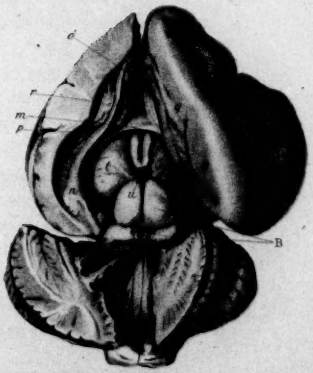
Opossum.

Fig. 5.



Beaver.

Fig. 6.



Kangaroo.

Richard Owen. del.

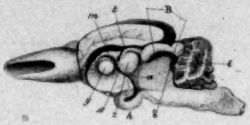
J. Basire sc.

The same letters indicate the same parts in each figure.

A. Cerebral hemispheres.
B. Optic lobes, or corpora quadrigemina.
C. Cerebellum.
* Place between the vermiform process and lateral lobes, where the medullary matter of the cerebellum is superficial.
a. Olfactory lobes or ganglions.
b. Pons Varolii, or cerebellic commissure.
c. Corpus trapezoideum.
d. Corpora pyramidalia.
e. Crura cerebri.
f. Corpus albicans.
g. Infundibulum.
h. Pituitary gland.
1 *a*. Natiform protuberance, giving off the external root of the olfactory nerve.
1 *b*. Pyriform protuberance, forming the origin of the internal root of the olfactory nerve.
1 *c*. Medullary root of the olfactory nerve emerging from a longitudinal fissure in the natiform protuberance.
i. Fissure dividing the external root of olfactory nerve from the superincumbent hemisphere.
2. Chiasma of the optic nerves.
3. Third pair of nerves.
4. Fourth pair of nerves.
5. Fifth pair of nerves.
6. Sixth pair of nerves.
k. Corpus trapezoideum.
l. Corpus callosum, or commissure of the hemispheres.
m. Fornix, or commissure of the hippocampi.
n. Hippocampus major.
o. Tænia hippocampi.
o'. Anterior fibres of the tænia hippocampi continued into the anterior lobes of the hemispheres.
p. Plexus choroides.
q. Septum lucidum, or internal wall of the lateral ventricle (Plate VII. fig. 5.).
r. Corpus striatum.
s. Continuation of the lateral ventricle into the olfactory nerve.
t. Optic thalami.
u. Pineal gland.
x. Part of a thin stratum of medullary fibres arching over the hippocampus major, and continued into the internal wall of the ventricle.

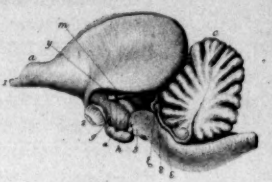
y. The anterior commissure.
z. Soft commissure.
m. Hippocampal commissure.
α. The third ventricle.
β. The iter ad infundibulum.
γ. The foramen Monroianum.
δ. The iter ad quartum ventriculum.
ε. The valvula Vieussenii.
ζ. The fold of the valve corresponding to the posterior commissure.

Fig. 1.



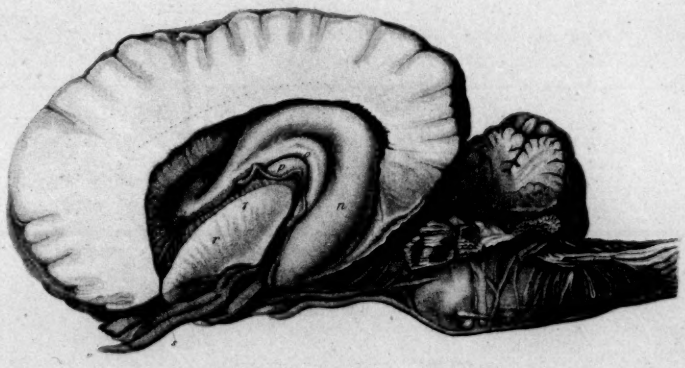
Opossum.

Fig. 2.



Goose.

Fig. 5.



Ass.

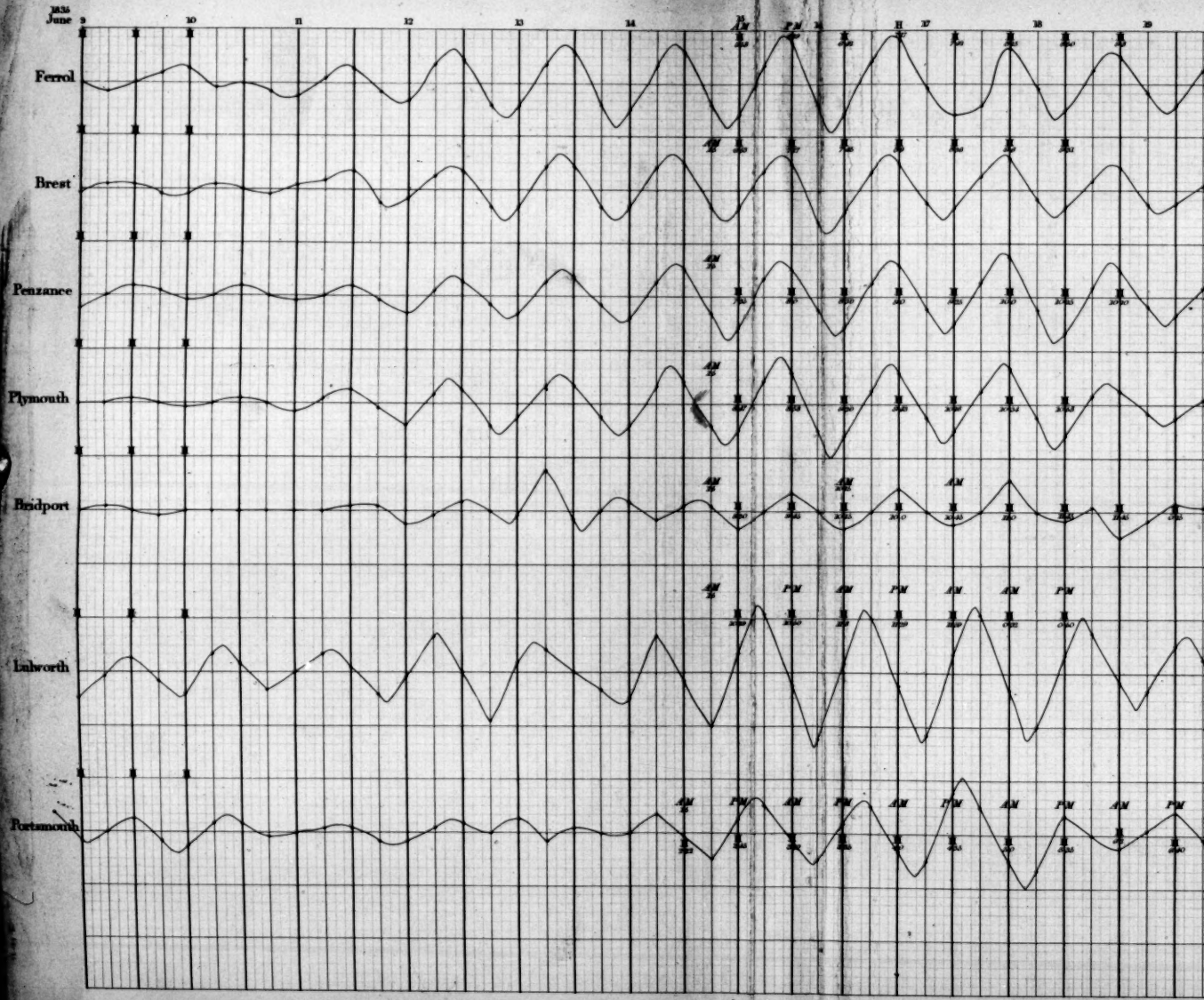
Fig. 3.

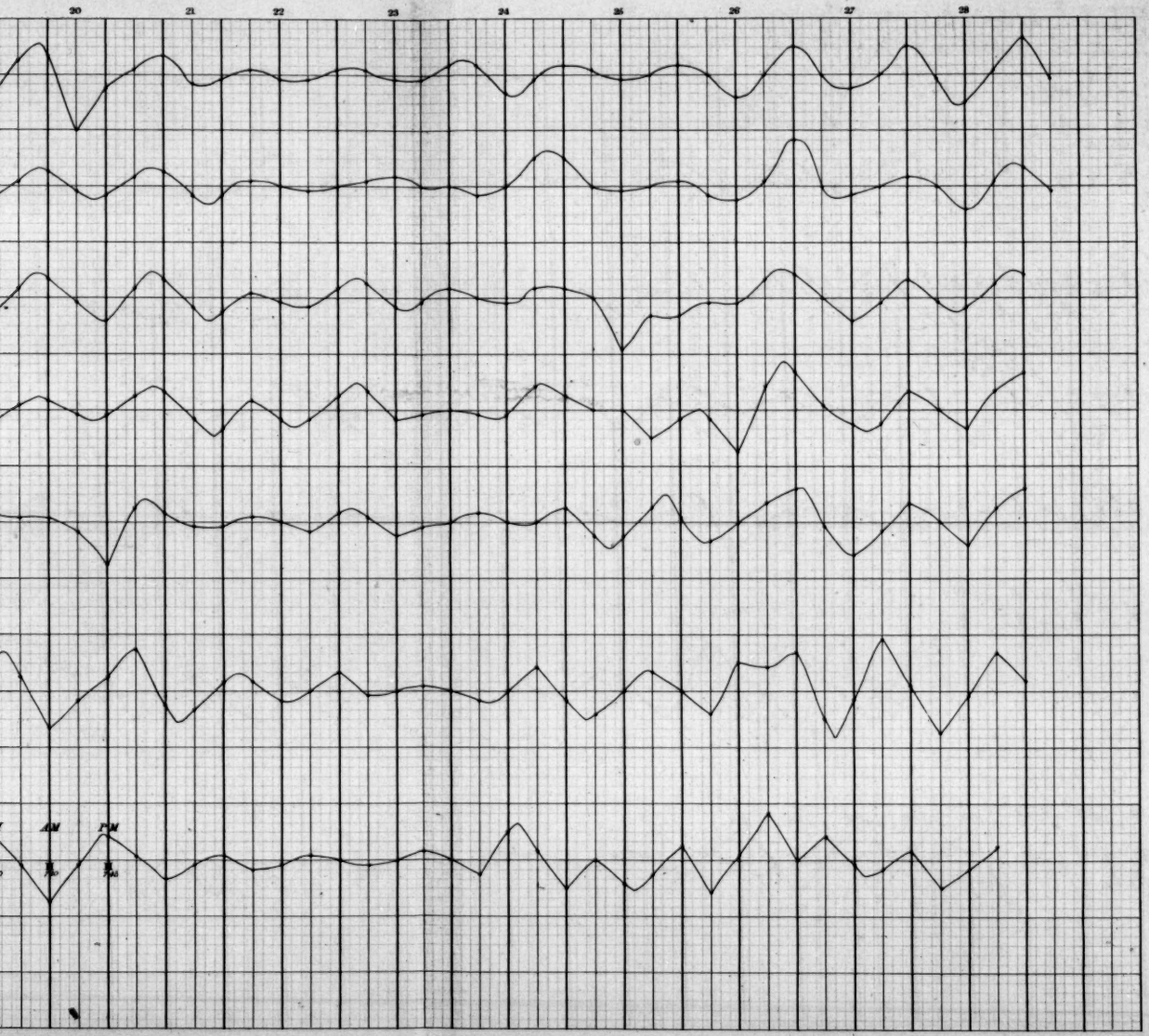
Opossum.

Fig. 4.

Kangaroo.

DIURNAL WAVE




20
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AM
PM

IX. *On the Tides.* By John William Lubbock, *Esq.*, *V.P.R.S.*

Received March 15,—Read March 16, 1837.

In my last paper on the Tides I endeavoured to point out the remarkable agreement which obtains in some respects between Bernoulli's theory and results obtained from observations at the London Docks. Since that time my attention has been directed to the following points:

1. To ascertain, from a discussion of the Liverpool observations with reference to a previous transit *, whether they present the same kind of agreement with theory as those of London. (See Tables I. to XII.)

2. To ascertain whether, by taking into account a greater number of observations, the results given in my last paper remain sensibly unaltered. (See Tables XV. to XXVIII.)

3. To ascertain whether the *establishment* of the port varies sensibly in different years, and whether the removal of the old Bridge has occasioned any difference at London. (See Tables XIV. and XXX.)

Numerous tables have been computed for me, in order to elucidate these points, by Mr. Jones and Mr. Russell, having been enabled to procure their valuable assistance in these laborious investigations by means of a further sum of money placed at my disposal for the purpose by the British Association for the Advancement of Science, to which distinguished body I take this opportunity of offering my grateful acknowledgements.

The succeeding transits of the moon being denoted by the letters A, B, C, D, E, F; and F being the time of the moon's transit which immediately precedes the time of high water at London, my last discussion of the London Dock observations was instituted with reference to transit B: the present discussion of the observations made at Liverpool between the 1st of January 1774 and the 31st of December 1792, by Mr. Hutchinson, has been instituted with reference to transit A†, or that which precedes the time of high water at Liverpool by about $2^d\ 0^h\ 20^m{\cdot}9$. This paper contains, in fact, two sets of tables precisely similar; the one set deduced from 13,391 observations of high water made at Liverpool by Mr. Hutchinson, and the other set deduced from 24,592 observations of high water at the Wapping entrance of the

* The former discussion by Mr. Dessiou, given in the Philosophical Transactions for 1835, was made with reference to the transit immediately preceding.

† I had intended the transit B to be used as the argument: the mistake was not perceived until the work was too far advanced to make it worth while to have recommenced.

London Docks made under the direction of the late Mr. Peirce; the latter tables differing from those given in the Philosophical Transactions for 1836, Part I., chiefly by being founded upon nearly double the number of observations. The high waters at Liverpool considered in this paper occur about 48 hours after the transit of the moon to which they are referred; the tides at London which are considered occur about 50 hours after the transit to which they are referred in this discussion, so that in fact all the *intervals* of the Liverpool Tables given in this paper ought to be increased by 36 hours, and all the *intervals* in the London Tables by 48 hours. The tide which makes high water at Liverpool arrives at the same instant somewhere on the north-east coast of Scotland, and at London about fifteen hours later. This is proved by the epoch of the semimenstrual inequality.

I find by interpolation from Table II. the interval for the moon's transit A

	d	h	m
At 3 o'clock . . .	1	23	40·5
and at 9 o'clock . . .	2	1	2·0

The difference is $1^h\,21^m{\cdot}5$, which converted into space

$$= 20^\circ\,22' \qquad \log\tan 20^\circ\,22' = 9{\cdot}56965 = \log(A).$$ (See p. 117.)

If we take the difference between the greatest and least heights = 5·52 from Table III.,

$$(E) = \frac{5{\cdot}52}{2(A)} = 7{\cdot}4353 \text{ for Liverpool,} \quad \log(E) = 0{\cdot}87130.$$

If we take the greatest height = 17·66 from Table III.,

$$17{\cdot}66 = D + \{1 + (A)\}(E) = D + \{1{\cdot}3712\}(E);$$

and hence definitively for Liverpool in the year 1786,

$$\log(A) = 9{\cdot}56965, \quad \log(E) = 0{\cdot}87130, \quad D = 7{\cdot}46,$$

D being reckoned from the datum in the East Wall of the Canning Dock.

And I find in the same manner for London in the year 1820

$$\log(A) = 9{\cdot}58418, \quad \log(E) = 0{\cdot}64690, \quad D = 16{\cdot}69,$$

D being reckoned from the sill of the London Dock gates at the Wapping entrance.

I conceive that the best if not the only method of investigating alterations in the height of the land above the water in any given locality where the water is influenced by the tides, will be to examine carefully whether any alteration has taken place in the values of the constants D and (E) for that place, the height of high water being of course always reckoned from some fixed mark in the land.

The semimenstrual* inequality of the interval at Liverpool presents the same remarkable agreement with observation which has been noticed before, while the form or law of the semimenstrual inequality of the height is also the same as that indi-

* The semimenstrual inequality is an inequality of high water or of the semidiurnal wave.

cated by the observations; but in order to render the agreement complete, it would be necessary to change the epoch by half an hour*. This remarkable circumstance also obtains in the London correction, as may be seen by reference to the plate which accompanies my last paper.

The results contained in the Tables here given are laid down in diagrams, without which they could not be so readily understood; but as they are similar in nature to those contained in my last paper, they do not require an extended description.

The calendar month inequality at Liverpool, considered as resulting implicitly from the corrections due to changes in the declinations of the luminaries and in the sun's parallax, agrees generally with the equilibrium theory, and with the results deduced from the London observations given in my last paper. The diagrams in Plate I. show that the spring equinoctial tides are greater than the neap equinoctial tides, and that the neap solstitial tides are greater than the spring solstitial tides, confirming what is stated by LAPLACE in the *Exposition du Système du Monde*, 5e ed., p. 83, and by NEWTON: "In quadraturis autem solstitialibus majores ciebunt æstus quam in quadraturis æquinoctialibus, eo quod Lunæ jam in æquatore constitutæ effectus maxime superat effectum Solis. Incidunt igitur æstus maximi in syzygias et minimi in quadraturas luminarium, circa tempora æquinoctii utriusque. Et æstum maximum in syzygiis comitatur semper minimus in quadraturis, ut experientiâ compertum est." LAPLACE says, "Elles [les marées] augmentent et diminuent avec le diamétre et le parallaxe lunaire, *mais dans un plus grand rapport*;" but the diagrams in Plate II. appear to confirm the truth of this passage only at neap tides.

It is desirable to establish the laws which regulate the diurnal inequality in the height of high water in different parts of the globe; at present the data are very insufficient. Mr. WHEWELL remarks, "that it would be easy to enumerate many actual cases in which the safety or loss of a ship has been determined by this inequality." Mr. WHEWELL was the first specially to notice, in his examination of the results of the tide observations made on the coasts of Europe and America in June 1835, contained in the Philosophical Transactions for 1836, the changes which this inequality presents in passing from one place to another.

This inequality depends chiefly upon the sign and amount of the moon's declination. The observations at London and Liverpool indicate no difference between tides corresponding to upper and lower transits, or between those corresponding to A.M. transits and transits P.M. six months afterwards; hence in endeavouring to determine the *diurnal inequality* at London and Liverpool, I have confounded in Tables XII. and XIII. the results corresponding to upper and lower transits, and those corresponding to A.M. transits and transits P.M. six months afterwards. I have also added to these those which ought, according to the preceding remarks, to differ only

* Or, adopting BERNOULLI's views in other respects, the epoch of the correction for the height is not the same as that for the interval.

in sign, and I have taken the mean of the whole for the result, as in the following example.

	h	m		ft.		ft.
Moon's transit A.—Liverpool, Jan.	0	30	A.M.	−·63	substituting	−·56
			P.M.	+·57	mean of all	+·56
July			A.M.	+·60	with proper	+·56
			P.M.	−·42	sign	−·56
				4)2·22		
				·56		

In the comparison of the heights in Plates I. II. and III. the London corrections have been multiplied by 1·7, that being the ratio of the quantities (E) for London and Liverpool, agreeably to the remark made in my last paper, p. 223. As the London discussion contained in my last paper was instituted with reference to transit B, and this discussion of the Liverpool observations has been made with reference to transit A, and as the tides which correspond to P.M. transits B correspond to A.M. transits A about twenty-five minutes less, in comparing our London and Liverpool results in all the Plates it was necessary to change the epoch, or to place the London corrections more to the left by half an hour, and to substitute in Plate III. for the London results corresponding to transits P.M. those corresponding to transits A.M. The diurnal inequality therefore, as it is laid down in Plate III. for London and Liverpool, has reference to the same tide or semidiurnal wave, making high water at London about fifteen hours later than at Liverpool.

I have already remarked that the laws to which the wave producing the semidiurnal inequality is subject, agree remarkably with Bernoulli's theory. The equilibrium theory also implies the existence of another wave producing a *diurnal inequality*. $2\psi - 2\varphi$ and 2ψ are the arguments of the *semidiurnal* inequality, $\psi - \varphi$ and ψ of the *diurnal inequality*. If we suppose the diurnal inequality-wave to move with a different velocity from the other, the diurnal inequality in the height may still be represented by the expression

$$\mathrm{d}h = B\{A \sin 2\delta \cos(\psi - \varphi) + \sin 2\delta' \cos\psi\},$$

and may be calculated by means of Tables X. and XI., δ being the sun's declination, and δ' that of the moon, but the constants which accompany ψ and φ will be different from those which accompany 2ψ and 2φ; and if we consider the constants to be included in the quantities ψ and φ, at high water, $\cos\psi$ may no longer be nearly equal to ± 1 in the last expression, but it will nearly equal $\pm$ some other constant, supposing the angle ψ still to increase by 180° in passing from one high water to the next; and the diurnal inequality, if the smaller term due to the sun's declination be neglected, may be represented approximately by

$$\mathrm{d}\psi = \frac{G \tan\delta'}{1 + A\cos 2\varphi} \qquad \mathrm{d}h = C \sin 2\delta' \text{ (for a given transit A.M. or P.M.)},$$

C and G being constant for any given place. Probably the amount depends also upon the moon's parallax, and then the expression for d h will be

$$C \times \frac{P^s}{(57')^s} \sin 2\,\delta' \text{ (for a given transit A.M. or P.M.).}$$

But this expression will not afford results agreeing with those which I have obtained from the observations at Liverpool if the declination of the moon be employed belonging to the time of the transit A; and it is necessary to employ the moon's declination at some time previous; that is, several days before the high water under consideration. This is not at variance with what is stated in the *Exposition*, except that, although LAPLACE considers the two waves separately*, he has not, I think, referred distinctly to the change in the epoch for different places, or to the difference between the epoch of the original diurnal and semidiurnal waves, which produce the derived tides observed on our coasts. If, however, the diurnal inequality-wave travels more slowly than the semidiurnal inequality-wave, the epoch also will be different, and thus it may depend upon the moon's declination several days earlier.

If this view be correct, the diurnal inequality of high water has a maximum (geographically) at those places on the coast at which the diurnal inequality-wave and the semidiurnal inequality-wave arrive simultaneously, and there will be places intermediate at which the diurnal inequality of high water is imperceptible, but where the diurnal inequality of low water is a maximum. This theory agrees with observation in giving no difference in the diurnal inequality for upper or lower transits.

The diurnal inequality in the interval at Liverpool is inappreciable; the diurnal inequality in the height has been laid down in Plate III. from the approximate expression

$$\mathrm{d}\,h = B\,\{A \sin 2\,\delta \cos(\psi - \phi) + \sin 2\,\delta' \cos \psi\} \quad \text{(for transits A.)}$$

The moon's declination δ' was taken from Table X., where it is given for the time of the moon's transit A, but the curve evidently requires to be shifted more to the right; it is difficult to decide exactly how much. In the observations of the height at Liverpool in May 1836 (see Plate IV.) the diurnal inequality vanishes on the 15th, the moon having crossed the equator on the 11th. If we consider that the theory curve requires to be shifted to the right about two hours, this would amount to referring the diurnal inequality at Liverpool to the moon's declination about four days previously.

Mr. RUSSELL has extended the discussion of the London observations given in my last paper by employing those made between 1802 and 1807, and those between 1827 and 1835, omitted before, so that we have now obtained tables similar to those contained in my last paper from the concurrence of no less than 24,592 observations.

* J'ai determiné la grandeur de ce flux et l'heure de son *maximum* dans le port de Brest. J'ai trouvé un cinquième de mètre [7·4 inches] à fort peu près pour sa grandeur; et un dixième de jour environ, pour le temps dont il precède à Brest, l'heure du *maximum* de la marée semidiurne. (Exposition du Système du Monde, 5e ed., p. 286.)

In consequence of this additional number of observations some of the jumps or irregularities which the former tables presented have been removed*, but the differences are in general less than I anticipated.

It is evident from the diagrams in Plate III. that a diurnal inequality in the interval at London is distinct although small. The value of the constant C is different from that which obtains for Liverpool. It is evident from Plate III. that the diurnal inequality in passing from Liverpool to London becomes reversed, that is to say, if a and b denote two successive heights of high water at Liverpool, and a', b' successive heights at London caused by the same tides,

$$\text{if } a > b \text{ then generally } \quad a' < b'.$$

The character of the diurnal inequality is generally manifest in the observations of a single month, as may be seen by those which are laid down in Plate IV. When the change is remarked which takes place in the diurnal inequality in passing from Plymouth to Portsmouth, it will not excite surprise that this inequality should be so different for places more distant from each other, as for London and Liverpool.

The calculations or predictions of the time of high water at any given place have long been made to depend upon what is called the *establishment of the port*, or a certain quantity presumed to be constant and independent of the distances and declinations of the luminaries, but which may be influenced by local circumstances. It seemed to me desirable to ascertain carefully how much this quantity has fluctuated during the time the observations were made at Liverpool by Mr. Hutchinson, which we have employed, and since the observations at the London Docks were instituted. Tables XIV. and XXX., which give these fluctuations, have been computed by Mr. Russell. The changes of the Liverpool establishment, and the fluctuations of the average height of high water at Liverpool are given in Table XIV., and are exhibited in fig. 1. Plate V.: which shows the time and height of high water from 1802 to 1835 at the London Docks on the full and change of the moon; the moon's parallax being 57′, and the declinations of the luminaries 15°, i. e. *the establishment* and the fluctuations in the average height of high water during the same interval. All the intervals and heights have been carefully reduced to horizontal parallax 57′ and declination 15°.

The changes of the London establishment, and the fluctuations of the average height, are given in Table XXX., and are exhibited in fig. 2. Plate V. These fluctuations in the *interval* and in the *height* present an insuperable obstacle to extreme accuracy in tide predictions, unless they can be explained.

"In 1832 none of the lower portions of old London Bridge, (with the exception of two piers,) which prevented the natural flow of the tidal waters, were removed; and in the second year (1833) almost the whole of that structure was cleared away as

* See, for instance, the calendar month correction for the interval in January, and the correction for the height corresponding to H. P. 56′.

regarded the masonry and starlings, although the section of the river was far from completed, many portions still remaining one or two feet above low-water mark, and which were finally removed in the year 1834*."

The time of high water appears now to be nearly as late as in 1804; in 1821 it was about ten minutes sooner.

I am much indebted to Mr. YATES for notice of a very ancient tide table which exists in a MS. in the British Museum. It is in the Codex Cottonianus, Julius DVII., which appears to have been written in the 13th century, and to have belonged to St. Albans Abbey. It contains calendar and other astronomical or geographical matters, some of which are the productions of JOHN WALLINGFORD, who died Abbot of St. Albans A.D. 1213. At p. 45 b. is a table on one leaf, showing the time of high water at London Bridge, "flod at london brigge", thus:

Ætas Lunæ.	h	m
1	3	48
2	4	36
3	5	24
4	6	12
....	....	....
....	....	....
28	1	24
29	2	12
30	3	0

N.B. The numbers increase by a constant difference of forty-eight minutes. The first column gives the moon's age in days.

Hence it would appear that high water at London on full and change was at that epoch $3^h\ 48^m$, or more than an hour later than at present. The time of high water at London on full and change is given in Mr. RIDDLE's *Navigation* and in other works $2^h\ 45^m$: FLAMSTEED made it 3^h†.

Note.—*On the Fluctuations of the Height of High Water due to changes in the Atmospheric Pressure.*

Read June 15.

M. DAUSSY having ascertained that at Brest the ocean rises when the barometer is depressed, I verified the existence of the same fact at Liverpool and London, and I found that at Liverpool when the barometer falls ·91 inch the tide rises 10·1 inches. As the *range* of the barometer is 3 inches‡, the correction which arises from change in the atmospheric pressure is by no means inconsiderable, its *range* being at Liverpool about 33 inches. At London I have found that when the barometer falls ·9 inch

* RENNIE, Report on Hydraulics, p. 512.

† See Philosophical Transactions, vol. xii. p. 12.

‡ Between the tropics the fluctuations of the barometer do not much exceed one fourth of an inch, while beyond this space they reach to 3 inches. DANIELL's Meteorological Essays, p. 108.

the tide rises 6·3 inches, and hence the *range* of the correction here is about 21 inches. Hence it is evident that in many inquiries relative to the tides, and particularly when observations are employed throughout only a limited period, the *correction* due to the atmospheric pressure may require to be attended to. Here, however, a question arises of some interest; does the surface of the ocean rise in narrow seas *simultaneously* with the depression of the barometer, or otherwise? In order to acquire some information upon this point, I requested Mr. Russell to calculate carefully from our Tables the height of high water at Liverpool and London for May and June 1836, and to compare the calculations with the observations, which is done in the accompanying Table, and the *errors*, together with the height of the barometer at Liverpool and London, are exhibited in fig. 3. Plate V.

Table showing the difference between the Height of High Water as calculated, and the Heights derived from observations at the London and Liverpool Docks.

1836. May.	Liverpool. O − C.	London. O − C.	1836. May.	Liverpool. O − C.	London. O − C.	1836. June.	Liverpool. O − C.	London. O − C.	1836. June.	Liverpool. O − C.	London. O − C.
	inches.	inches.		inches.	inches.		inches.	inches.		inches.	inches.
1	− 1	− 1	17		+ 1	1.		+ 6	17.	+ 11	+ 9
	− 7	+ 5		− 8	0		− 3	+ 4		+ 12	+ 9
2	− 11	+ 13	18	− 6	− 1	2.	0	+ 6	18.	+ 14	+ 5
		+ 18		− 4	+ 1		+ 6	+ 6		+ 14	+ 5
3	− 15	+ 11	19	− 3	+ 1	3.	+ 11	+ 1	19.	+ 14	+ 7
	− 11	+ 8		− 1	− 1		+ 14	+ 1		+ 13	+ 10
4	− 6	+ 6	20	+ 1	− 1	4.	+ 17	+ 4	20.	+ 12	+ 8
	− 2	+ 5		+ 2	+ 1		+ 18	+ 4		+ 12	+ 5
5	− 1	+ 2	21	+ 3	+ 4	5.	+ 14	+ 8	21.	+ 11	+ 5
	− 4	+ 5		+ 3	+ 5		+ 12			+ 10	+ 4
6	− 5	+ 7	22	+ 4	+ 2	6.	+ 12	+ 17	22.	+ 12	0
	− 3	+ 5		+ 1	+ 1		+ 5	+ 6		+ 14	0
7	− 4	+ 6	23	0	+ 5	7.	+ 12	+ 6	23.	+ 16	+ 1
	− 2	+ 4		− 3	+ 7		+ 9	+ 8		+ 15	+ 2
8	− 2	+ 4	24	− 1	+ 6	8.	+ 6	+ 7	24.	+ 11	+ 2
	0	+ 3		+ 1	0		+ 9	+ 6		+ 10	+ 1
9	+ 1	+ 4	25	+ 1	0	9.	+ 9	+ 6	25.	+ 9	0
	0	+ 2		− 3	+ 1		+ 9	+ 4		+ 4	+ 1
10	0	+ 1	26	− 2	+ 5	10.	+ 9	+ 3	26.	0	+ 3
	− 1	− 3		− 2	+ 1		− 5	+ 5		− 2	+ 2
11	0	− 5	27	− 2	− 1	11.	+ 3		27.	− 1	
	0	− 5		− 2	− 1		+ 5	+ 6		+ 2	+ 1
12	0		28	− 0		12.	+ 5	+ 4	28.	+ 3	0
	− 3	+ 1		− 2	0		+ 3	+ 2		+ 2	+ 2
13	− 5	0	29	− 3	0	13.	+ 1	+ 3	29.	0	+ 3
	− 8	− 2		− 3	+ 1		+ 1	+ 4		0	+ 3
14	− 9	− 2	30	− 2	0	14.	+ 4	+ 5	30.		+ 3
	− 9	− 1		− 3	+ 1		+ 5	+ 7		− 1	+ 2
15	− 9	− 1	31	− 0	+ 3	15.	+ 7	+ 9			
	− 7	− 1		− 3	+ 2			+ 11			
16	− 7	0				16.	+ 11	+ 10			
	− 7	+ 1					+ 11	+ 9			

The above differences, O − C, are not the differences between Calculation and *actual* observation, but between Calculation and what it is presumed observation would be if freed of diurnal inequality by drawing an intermediate curve between those given in Plate IV.

Results deduced from Observations made at

LIVERPOOL.

Table I. (*a*.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A* in each month of the year, from 13,391 observations made at the Liverpool Docks, between the 1st of January 1774 and the 31st of December 1792.

January.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
91	0 30·1	12 10·1	17 10·1	18·6	57·6
94	1 30·4	11 57·1	17 3·7	14·9	57·4
96	2 29·7	11 45·5	16 4·1	9·9	57·4
104	3 29·7	11 38·8	15 0·8	5·6	57·0
100	4 29·9	11 42·7	13 4·8	4·9	56·7
104	5 30·0	12 3·5	12 6·8	8·5	56·8
96	6 30·1	12 36·2	12 6·8	13·5	56·7
92	7 37·6	13 2·6	13 9·1	17·9	56·9
92	8 29·2	13 7·3	15 3·8	20·6	57·1
86	9 30·2	12 59·4	16 2·2	22·7	57·5
89	10 30·2	12 44·7	17 3·5	22·6	57·6
86	11 30·7	12 28·7	17 9·0	21·9	57·7
	Sun's Declination S. 21°.				

February.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	° ′	′
88	0 31·4	12 12·9	18 3·6	9·9	57·3
91	1 30·7	11 57·7	17 7·1	5·2	57·4
90	2 29·9	11 47·1	16 4·9	4·8	57·2
91	3 29·1	11 38·2	14 8·7	8·3	57·1
91	4 29·3	11 42·3	12 11·5	13·4	56·9
86	5 29·5	12 3·6	11 9·5	18·4	56·9
85	6 29·0	12 40·6	11 11·2	20·9	56·7
83	7 29·5	13 5·2	13 7·0	22·3	57·0
81	8 29·0	13 8·7	15 0·5	22·9	57·0
83	9 29·8	12 59·4	16 6·3	21·7	57·2
81	10 30·4	12 43·9	17 7·9	18·9	57·5
87	11 30·3	12 28·2	18 2·8	14·9	57·3
	Sun's Declination S. 13°.				

March.

102	0 28·8	12 13·1	18 4·3	4·5	57·7
101	1 29·1	11 58·9	17 5·0	8·0	57·7
100	2 29·9	11 45·5	15 11·9	12·8	57·3
92	3 30·0	11 35·9	14 1·1	17·5	56·9
91	4 29·3	11 37·7	12 3·5	20·7	56·8
89	5 29·7	12 0·2	11 2·5	22·3	56·7
89	6 31·2	12 42·2	11 8·8	22·8	56·2
84	7 30·1	13 6·2	13 3·6	21·7	56·8
94	8 29·7	13 8·8	15 0·7	19·7	56·8
98	9 30·2	12 58·4	16 8·2	15·2	57·2
98	10 30·9	12 44·1	17 10·3	10·6	57·4
92	11 30·4	12 28·4	18 6·4	6·0	57·7
	Sun's Declination S. 2°.				

April.

94	0 29·0	12 13·1	17 6·8	12·4	57·5
93	1 30·0	11 57·7	16 9·6	16·8	57·6
87	2 30·7	11 43·1	15 4·8	20·6	57·5
86	3 30·6	11 34·1	13 10·1	22·2	57·2
88	4 30·7	11 37·7	12 3·3	23·0	57·0
85	5 30·7	12 2·2	11 5·8	22·3	57·0
91	6 30·3	12 39·6	11 10·9	19·8	56·7
91	7 30·5	13 1·6	13 8·3	15·8	57·1
94	8 29·4	13 4·5	15 5·1	11·1	57·1
96	9 28·4	12 56·9	16 10·1	6·2	57·2
103	10 29·0	12 43·5	17 8·3	4·4	57·2
94	11 29·4	12 29·1	18 0·5	7·4	57·3
	Sun's Declination N. 10°.				

May.

87	0 28·6	12 12·8	16 9·0	20·3	57·3
89	1 29·8	11 57·1	16 2·3	22·1	57·4
83	2 30·6	11 43·1	15 3·3	22·8	57·5
87	3 29·3	11 35·2	14 0·7	22·1	57·3
95	4 29·4	11 38·9	12 11·7	20·0	57·1
96	5 29·7	12 3·2	12 4·1	15·1	57·1
101	6 28·9	12 35·7	12 10·8	11·9	57·1
106	7 29·5	12 57·1	14 1·6	6·6	57·2
103	8 30·0	13 1·3	15 4·4	4·5	57·0
101	9 30·5	12 55·5	16 5·7	7·2	57·3
98	10 31·3	12 43·0	16 11·6	11·9	57·2
89	11 29·9	12 29·3	17 3·5	16·9	57·4
	Sun's Declination N. 19°.				

June.

82	0 28·7	12 11·1	16 9·1	23·1	57·3
85	1 28·9	11 56·8	16 3·3	22·1	57·3
87	2 29·4	11 44·2	15 5·9	19·5	57·2
92	3 28·6	11 38·7	14 5·5	15·9	57·1
99	4 30·0	11 44·5	13 8·0	11·5	57·1
96	5 29·7	12 4·6	12 11·2	6·4	57·0
108	6 30·2	12 35·4	13 3·0	4·8	57·0
97	7 30·9	12 57·2	14 0·5	7·5	57·2
96	8 30·1	13 2·6	15 2·5	13·4	57·3
91	9 30·4	12 57·2	16 2·0	17·0	57·4
85	10 29·8	12 45·1	16 8·0	20·4	57·6
84	11 29·5	12 29·4	16 9·4	22·2	57·3
	Sun's Declination N. 23°.				

* The succeeding transits of the moon are denoted by the letters A, B, C, D, E, F; F being the transit immediately preceding the time of high water at London.

TABLE I. (*a*.) (Continued.)

July.

Number of Obser-vations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declina-tion.	Mean Horizontal Parallax.
	h m	h m	ft. in.	° ′	′ ″
93	0 30·3	12 13·3	17 2·5	19·4	57·4
90	1 30·1	11 58·5	16 10·1	16·1	57·3
101	2 29·8	11 45·4	16 1·1	11·0	57·1
101	3 30·8	11 41·8	14 9·0	6·4	56·9
100	4 29·7	11 45·3	13 8·2	4·7	56·8
102	5 29·0	12 5·2	12 8·5	7·5	56·8
99	6 29·1	12 37·0	12 7·9	12·7	56·9
97	7 29·7	13 0·8	13 10·8	17·2	57·1
92	8 30·6	13 6·2	14 11·1	20·7	57·1
85	9 29·5	13 0·2	16 1·6	22·2	57·6
88	10 29·1	12 44·3	16 10·1	22·9	57·6
89	11 30·0	12 29·2	17 3·3	21·9	57·6

Sun's Declination N. 21°.

August.

Number of Obser-vations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declina-tion.	Mean Horizontal Parallax.
	h m	h m	ft. in.	° ′	′ ″
97	0 29·6	12 14·3	18 0·5	10·7	57·7
103	1 28·3	12 0·0	17 4·1	6·1	57·6
105	2 28·0	11 47·0	16 5·0	4·5	57·3
100	3 28·4	11 40·1	14 9·5	7·0	56·9
100	4 28·7	11 41·3	13 1·1	12·8	56·8
93	5 28·8	12 2·4	11 9·4	17·5	56·7
93	6 29·0	12 39·7	12 0·4	20·6	56·7
84	7 29·1	13 4·9	13 4·7	22·2	56·9
90	8 29·3	13 9·7	14 9·0	22·9	56·9
88	9 30·4	13 0·2	16 3·3	21·6	57·1
92	10 30·6	12 44·8	17 2·8	19·4	57·4
92	11 30·3	12 28·5	18 0·6	15·5	57·7

Sun's Declination N. 13°.

September.

98	0 30·5	12 13·5	18 4·9	4·5	57·6
98	1 30·9	11 58·7	17 6·5	7·0	57·5
96	2 30·4	11 45·6	16 2·5	12·4	57·5
93	3 30·0	11 36·6	14 4·9	16·8	57·1
88	4 29·6	11 37·9	12 6·7	19·9	56·9
82	5 28·4	11 59·3	11 6·0	22·1	56·8
92	6 29·2	12 39·5	12 1·6	23·3	56·7
83	7 30·4	13 4·9	13 6·9	22·1	57·0
88	8 29·7	13 8·2	15 2·4	20·0	56·9
91	9 29·9	12 58·7	16 9·2	16·0	57·2
94	10 29·7	12 44·0	17 11·2	11·2	57·4
98	11 30·2	12 28·5	18 3·8	6·5	57·5

Sun's Declination N. 3°.

October.

95	0 28·7	12 12·9	18 0·0	11·3	57·3
93	1 28·3	11 57·0	17 3·1	16·3	57·5
92	2 28·7	11 41·3	15 8·0	20·2	57·3
91	3 28·2	11 35·0	14 3·2	22·0	57·4
97	4 29·8	11 34·9	12 7·4	23·0	57·1
89	5 31·6	12 0·4	11 8·1	21·9	57·2
88	6 29·7	12 37·7	12 1·9	20·2	56·9
100	7 29·6	13 3·3	13 10·6	17·0	57·0
96	8 30·7	13 5·7	15 6·8	12·4	56·9
100	9 30·0	12 57·3	17 0·6	6·9	57·1
98	10 29·3	12 43·5	17 8·9	4·5	57·3
100	11 28·7	12 28·8	18 6·8	6·6	57·3

Sun's Declination S. 9°.

November.

86	0 31·1	12 11·9	17 5·2	19·7	57·4
82	1 30·6	11 56·0	16 6·8	22·2	57·2
83	2 29·9	11 41·7	15 6·4	22·9	57·2
85	3 29·6	11 34·1	14 2·8	22·2	57·1
94	4 31·3	11 37·2	12 10·7	20·2	57·0
89	5 31·0	12 2·4	12 5·5	16·4	57·1
100	6 30·2	12 37·5	12 11·5	12·3	56·9
98	7 30·0	12 57·7	14 2·9	7·3	57·0
98	8 28·7	13 2·4	15 9·6	4·5	57·1
99	9 28·4	12 54·4	16 11·7	6·2	57·3
94	10 28·3	12 43·9	17 8·0	11·6	57·4
94	11 29·9	12 28·8	17 10·1	16·5	57·4

Sun's Declination S. 19°.

December.

85	0 29·6	12 12·0	17 1·2	23·1	57·3
87	1 29·8	11 55·2	16 5·8	22·1	57·3
86	2 29·1	11 42·7	15 7·6	19·3	57·1
97	3 28·7	11 38·3	14 9·2	15·8	57·1
101	4 30·3	11 41·6	13 6·0	11·0	56·9
104	5 30·1	12 3·7	12 10·3	6·6	56·9
106	6 30·2	12 35·8	13 2·0	4·9	56·9
103	7 30·0	12 57·8	14 3·4	7·1	57·1
101	8 29·5	13 2·3	15 7·7	12·3	57·3
90	9 29·1	12 57·0	16 7·7	16·3	57·5
93	10 29·1	12 45·1	17 3·3	20·1	57·5
85	11 30·3	12 28·2	17 6·3	22·2	57·6

Sun's Declination S. 23°.

TABLE II. (*b*.) (Interpolated from Table I.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit A, and the Time of High Water at the Liverpool Docks for each month in the year.

Apparent Solar Time of Moon's Transit A.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 30	12 9·9	12 13·2	12 12·6	12 13·6	12 12·3	12 10·6	12 13·3	12 14·0	12 12·4	12 12·4	12 12·1	12 11·9	12 12·3
1 30	11 58·8	11 57·5	11 58·1	11 57·2	11 56·1	11 56·2	11 58·2	11 59·1	11 58·4	11 59·1	11 55·9	11 54·9	11 57·0
2 30	11 44·8	11 46·8	11 45·0	11 42·4	11 43·4	11 43·8	11 45·2	11 46·1	11 44·9	11 40·6	11 41·4	11 42·3	11 43·9
3 30	11 38·8	11 38·0	11 36·1	11 33·7	11 34·5	11 38·4	11 42·0	11 40·2	11 36·4	11 34·0	11 33·9	11 38·0	11 37·0
4 30	11 43·5	11 42·7	11 38·3	11 37·7	11 38·8	11 44·3	11 45·9	11 42·1	11 38·3	11 34·7	11 36·9	11 41·8	11 40·4
5 30	12 3·7	12 4·0	12 0·8	12 1·9	12 3·2	12 4·8	12 5·9	12 3·4	12 0·3	11 59·4	12 1·8	12 3·8	12 2·8
6 30	12 25·8	12 40·7	12 40·6	12 39·0	12 36·4	12 35·3	12 37·4	12 39·8	12 39·5	12 37·7	12 37·8	12 35·6	12 37·9
7 30	13 2·8	13 5·3	13 5·7	13 1·7	12 57·7	12 57·5	13 1·1	13 4·8	13 4·8	13 3·4	12 57·7	12 58·0	13 1·7
8 30	13 7·5	13 8·7	13 8·3	13 4·7	13 1·3	13 3·3	13 6·4	13 9·5	13 8·0	13 5·5	13 2·7	13 3·0	13 5·7
9 30	13 0·2	12 59·7	12 58·7	12 56·9	12 56·1	12 57·9	13 1·1	13 0·5	12 59·0	12 57·5	12 53·6	12 57·6	12 58·2
10 30	12 45·2	12 44·5	12 44·7	12 43·5	12 43·5	12 45·6	12 44·7	12 45·3	12 44·3	12 43·6	12 43·9	12 45·4	12 44·5
11 30	12 29·1	12 28·4	12 28·7	12 29·0	12 29·4	12 29·4	12 29·4	12 28·8	12 28·8	12 28·5	12 28·9	12 28·5	12 29·0

Moon's Hor. Par. 57′.

TABLE III. (*c.*) (Interpolated from Table I.)

Showing the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, in each month of the year.

Apparent Solar Time of Moon's Transit A.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0 30	17·59	18·18	18·06	17·35	16·61	16·62	17·04	17·75	18·16	17·86	17·27	16·97	17·46
1 30	17·15	17·43	17·12	16·55	16·02	16·13	16·71	17·07	17·33	17·02	16·50	16·35	16·78
2 30	16·17	16·33	15·87	15·21	15·33	15·40	16·05	16·26	16·02	15·53	15·45	15·57	15·77
3 30	15·06	14·66	14·13	13·76	13·93	14·38	14·81	14·79	14·37	14·06	14·18	14·70	14·40
4 30	13·51	12·98	12·33	12·30	12·93	13·63	13·75	13·14	12·59	12·58	12·92	13·55	13·02
5 30	12·65	11·82	11·33	11·49	12·30	12·93	12·78	11·89	11·56	11·62	12·43	12·90	12·14
6 30	12·69	12·06	12·03	12·03	12·87	13·25	12·71	12·16	12·26	12·20	13·00	13·21	12·54
7 30	13·85	13·58	13·38	13·64	14·06	13·94	13·87	13·45	13·58	13·88	14·24	14·24	13·81
8 30	15·30	15·07	15·15	15·40	15·37	15·09	14·87	14·81	15·26	15·59	15·79	15·53	15·27
9 30	15·98	16·45	16·60	16·79	16·35	16·01	15·90	16·23	16·69	17·01	16·89	16·46	16·45
10 30	17·04	17·43	17·68	17·62	16·87	16·42	16·60	17·05	17·76	17·63	17·52	17·07	17·22
11 30	17·46	18·10	18·34	17·91	17·12	16·65	17·03	17·76	18·11	18·45	17·67	17·28	17·66

Moon's Hor. Par. 57′.

TABLE IV. (*d.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, for every minute of her Horizontal Parallax.

Hor. Par. 54′.					Hor. Par. 55′.				
Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
	h m	h m	ft. in.	°		h m	h m	ft. in.	°
201	0 29·8	12 12·3	16 4·0	14·5	159	0 29·4	12 13·2	16 7·8	14·8
191	1 29·9	11 53·9	15 7·7	15·0	169	1 29·6	11 56·1	15 11·2	14·8
191	2 29·1	11 39·6	14 8·4	15·3	175	2 30·3	11 39·7	14 10·9	15·0
167	3 28·6	11 30·2	13 4·2	15·1	193	3 28·3	11 31·5	13 7·6	14·8
150	4 27·7	11 32·2	11 11·5	15·5	236	4 30·4	11 34·1	12 3·0	15·8
108	5 29·0	11 58·0	10 10·4	15·3	242	5 30·2	12 0·2	11 3·0	14·8
120	6 30·1	12 41·6	11 4·0	16·3	266	6 29·1	12 40·7	11 6·9	15·3
133	7 30·8	13 8·5	12 8·6	14·8	214	7 28·5	13 6·7	12 10·9	15·3
174	8 29·3	13 12·7	14 2·3	15·8	208	8 29·1	13 13·7	14 7·3	15·5
179	9 29·3	13 4·9	15 5·1	14·8	186	9 29·8	13 4·6	15 9·7	15·1
196	10 29·9	12 46·4	16 2·9	15·1	169	10 29·4	12 46·4	16 6·4	14·4
190	11 29·6	12 29·9	16 6·9	14·8	168	11 29·6	12 29·4	16 9·1	14·7
Sun's Declination 15°.					Sun's Declination 15°.				
Hor. Par. 56′.					Hor. Par. 57′.				
116	0 30·1	12 12·7	16 11·4	14·4	105	0 32·0	12 12·4	17 3·3	14·1
118	1 30·8	11 56·2	16 2·8	14·6	103	1 31·2	11 57·2	16 9·3	14·1
119	2 28·8	11 37·4	15 6·2	13·9	106	2 30·6	11 43·9	15 11·3	13·6
147	3 30·5	11 35·2	14 0·0	14·6	118	3 30·3	11 39·5	14 5·2	14·4
141	4 31·4	11 38·9	12 6·8	14·6	128	4 29·4	11 41·9	13 0·4	14·3
143	5 29·1	12 0·6	11 7·1	15·1	144	5 29·5	12 2·2	12 1·6	14·9
153	6 29·5	12 38·4	12 0·4	14·8	136	6 31·9	12 38·4	12 6·6	14·9
143	7 29·5	13 3·6	13 4·2	14·7	136	7 29·5	13 1·0	13 9·8	14·5
139	8 29·7	13 7·3	14 9·5	14·5	113	8 29·0	13 5·9	15 3·0	14·4
125	9 30·0	12 59·2	16 0·8	14·5	102	9 27·5	12 55·5	16 3·4	14·0
124	10 30·0	12 44·2	16 10·0	14·6	99	10 29·2	12 43·0	17 1·9	14·1
107	11 30·9	12 29·3	17 2·7	14·6	109	11 30·4	12 29·0	17 6·3	14·3
Sun's Declination 15°.					Sun's Declination 15°.				

Table IV. (*d.*) (Continued.)

Hor. Par. 58′.					Hor. Par. 59′.				
Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
	h m	h m	ft. in.	°		h m	h m	ft. in.	°
92	0 27·4	12 13·7	17 6·2	13·5	94	0 29·1	12 13·3	18 4·9	14·0
101	1 28·6	11 58·0	17 2·2	13·8	98	1 31·7	11 58·4	17 8·2	14·1
101	2 30·0	11 46·1	16 0·1	14·0	114	2 30·9	11 47·3	16 6·5	14·6
126	3 30·3	11 38·9	14 9·6	14·4	133	3 30·2	11 41·0	15 2·2	14·2
133	4 31·7	11 43·5	13 4·6	14·5	204	4 31·7	11 46·1	13 10·9	15·0
154	5 31·9	12 5·5	12 8·4	14·5	285	5 31·0	12 7·1	13 2·4	15·4
147	6 29·0	12 36·5	13 0·2	14·9	301	6 28·1	12 34·9	13 6·6	15·0
144	7 30·0	12 58·1	14 2·9	14·3	196	7 27·9	12 55·7	14 6·9	15·3
114	8 28·8	13 2·9	15 8·0	13·7	143	8 28·5	13 2·8	16 0·8	14·7
114	9 30·9	12 56·6	16 8·9	13·9	113	9 26·7	12 54·2	17 2·5	14·3
104	10 29·9	12 43·3	17 6·2	13·4	97	10 28·3	12 44·0	18 0·8	14·4
88	11 28·7	12 28·6	17 8·9	14·1	98	11 29·5	12 29·0	18 6·1	13·8
Sun's Declination 15°.					Sun's Declination 15°.				
Hor. Par. 60′.					Hor. Par. 61′.				
113	0 31·6	12 13·3	18 8·9	14·8	191	0 29·7	12 13·2	19 3·0	14·7
112	1 30·3	11 59·8	18 1·8	14·4	213	1 27·6	12 1·2	18 6·1	15·1
161	2 31·3	11 49·3	16 11·4	14·2	140	2 26·5	11 50·3	17 5·1	15·6
219	3 30·0	11 43·7	15 7·7	15·5	16	3 12·4	11 46·1	16 1·1	18·6
150	4 25·7	11 47·2	14 4·7	16·1					
33	5 15·3	11 58·6	13 8·4	15·2					
30	6 43·6	12 41·4	13 9·6	15·9					
150	7 33·9	12 55·6	15 0·9	15·4					
212	8 29·7	12 58·9	16 5·7	15·3	19	8 48·9	12 57·9	16 9·1	19·8
162	9 27·4	12 53·8	17 9·3	14·6	127	9 35·6	12 52·6	18 0·8	15·1
123	10 29·6	12 42·2	18 7·1	14·4	201	10 31·4	12 40·3	18 10·9	14·9
106	11 30·2	12 27·0	19 0·0	14·1	195	11 28·7	12 27·8	19 5·1	15·1
Sun's Declination 15°.					Sun's Declination 15°.				

Table V. (*e.*)

Interpolated from Table IV., and reduced to Moon's Declination 15°.

Apparent Solar Time of Moon's Transit A.	H. P. 54′.		H. P. 55′.		H. P. 56′.		H. P. 57′.	
	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.
h m	h m	feet.	h m	feet.	h m	feet.	h m	feet.
0 30	12 12·1	16·31	12 13·0	16·64	12 12·6	16·92	12 12·8	17·23
1 30	11 53·9	15·64	11 56·0	15·91	11 56·3	16·22	11 57·3	16·74
2 30	11 39·5	14·70	11 39·6	14·93	11 36·9	15·42	11 43·6	15·86
3 30	11 30·2	13·32	11 31·4	13·58	11 35·0	13·99	11 39·3	14·40
4 30	11 33·0	11·94	11 34·4	12·32	11 38·4	12·58	11 41·7	12·98
5 30	11 58·6	10·88	12 0·1	11·24	12 1·1	11·59	12 2·5	12·12
6 30	12 41·4	11·42	12 40·3	11·61	12 38·7	12·03	12 37·4	12·53
7 30	13 8·4	12·69	13 6·8	12·96	13 3·8	13·35	13 1·3	13·80
8 30	13 12·3	14·27	13 13·4	14·66	13 7·5	14·77	13 6·1	15·24
9 30	13 4·9	15·43	13 4·6	15·82	12 59·4	16·04	12 55·3	16·27
10 30	12 46·4	16·25	12 46·4	16·52	12 44·3	16·81	12 43·0	17·11
11 30	12 29·8	16·57	11 29·3	16·75	12 29·5	17·21	12 29·2	17·49
	H. P. 58′.		H. P. 59′.		H. P. 60′.		H. P. 61′.	
0 30	12 12·9	17·41	12 13·0	18·34	12 13·7	18·74	12 13·1	19·16
1 30	11 57·5	17·08	11 58·6	17·64	11 59·8	18·12	12 0·6	18·49
2 30	11 45·8	15·95	11 47·4	16·54	11 49·3	16·93	11 49·9	17·40
3 30	11 38·7	14·77	11 40·7	15·16	11 44·0	15·67		
4 30	11 43·0	13·39	11 45·8	13·95	11 45·8	14·37		
5 30	12 4·4	12·69	12 6·7	13·24				
6 30	12 37·0	13·03	12 35·9	13·58				
7 30	12 58·4	14·20	12 55·9	14·65	12 54·6	15·02		
8 30	13 3·3	15·02	13 3·0	16·10	12 58·7	16·51		
9 30	12 57·1	16·65	12 53·8	17·24	12 53·4	17·81	12 53·6	17·96
10 30	12 43·6	17·42	12 43·7	18·06	12 42·2	18·57	12 40·6	18·89
11 30	12 28·3	17·69	12 29·0	18·43	12 27·2	18·94	12 27·4	19·45

TABLE VI. (*f*.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Upper and Lower Transit A, P.M. and A.M.

January.

Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 30·1	57·6	12 11·3	18 7·6	S. 18·4	0 28·7	57·8	12 12·6	17 2·7	S. 19·2
1 30·4	58·0	11 56·3	18 5·3	S. 14·7	1 28·4	57·0	11 57·9	16 7·8	S. 14·7
2 29·7	57·2	11 45·7	17 0·3	S. 10·0	2 29·9	57·8	11 45·1	15 10·9	S. 9·9
3 29·9	56·9	11 38·3	15 9·7	S. 4·2	3 29·4	57·0	11 39·8	14 4·7	S. 3·9
4 29·4	56·9	11 42·9	14 3·1	N. 2·1	4 29·6	56·6	11 41·9	12 8·2	N. 2·2
5 30·6	56·5	12 3·8	12 10·1	N. 7·9	5 29·8	56·8	12 3·2	12 1·0	N. 7·6
6 33·5	56·7	12 39·0	12 10·7	N. 13·8	6 27·3	56·5	12 35·7	12 2·9	N. 13·5
7 28·9	56·8	13 1·9	13 9·5	N. 17·8	7 27·0	56·7	13 2·0	13 7·4	N. 17·9
8 29·3	56·9	13 8·5	15 4·1	N. 20·3	8 28·7	56·7	13 6·1	15 3·7	N. 20·6
9 31·2	57·2	12 59·9	16 1·6	N. 23·1	9 24·2	57·6	12 59·8	16 5·1	N. 22·5
10 34·2	57·2	12 45·4	16 11·2	N. 22·5	10 25·6	57·3	12 46·5	17 4·7	N. 22·2
11 32·1	57·8	12 29·2	17 4·6	N. 21·5	11 30·7	57·5	12 30·0	17 7·1	N. 22·2
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 29·1	57·4	12 13·5	17 2·5	N. 18·5	0 32·1	57·4	12 11·2	18 2·1	N. 18·2
1 33·2	56·9	11 57·0	16 8·7	N. 14·9	1 29·4	57·6	11 57·3	17 8·5	N. 15·2
2 30·7	57·3	11 46·1	15 8·8	N. 10·1	2 29·5	57·2	11 45·5	16 6·4	N. 8·6
3 29·0	57·0	11 39·7	14 7·1	N. 3·7	3 30·5	57·1	11 37·3	15 4·5	N. 4·0
4 30·8	56·6	11 43·1	12 11·5	S. 2·5	4 29·8	56·9	11 42·8	13 10·9	N. 0·5
5 29·8	57·2	12 5·0	12 6·8	S. 7·8	5 29·6	56·8	12 3·0	12 7·9	S. 8·9
6 28·9	56·9	12 34·8	12 8·3	S. 13·4	6 31·0	56·7	12 35·6	12 5·4	S. 13·1
7 33·1	56·9	13 2·3	14 0·7	S. 18·2	7 23·3	57·2	13 0·4	13 8·5	S. 17·7
8 35·1	57·4	13 5·9	15 10·2	S. 20·9	8 24·8	57·2	13 8·5	14 10·5	S. 20·5
9 32·1	57·8	12 58·7	16 4·7	S. 22·4	9 31·9	57·3	12 58·9	15 10·1	S. 22·8
10 26·6	57·8	12 45·1	17 9·5	S. 22·7	10 34·1	57·9	12 42·8	17 1·5	S. 22·9
11 26·1	57·5	12 28·9	18 1·0	S. 22·1	11 34·2	58·2	12 26·9	17 10·6	S. 21·5

February.

Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 32·1	56·7	12 12·1	18 4·1	S. 9·9	0 30·9	57·9	12 12·1	17 11·2	S. 10·2
1 31·5	58·2	11 58·0	18 4·6	S. 3·8	1 30·8	57·1	11 57·9	16 9·8	S. 3·4
2 30·2	57·2	11 44·6	16 10·7	N. 2·3	2 28·9	57·3	11 46·3	15 7·8	N. 0·6
3 30·8	56·9	11 37·0	15 0·7	N. 8·6	3 27·7	57·3	11 36·2	14 2·8	N. 8·7
4 30·1	56·9	11 42·5	13 8·0	N. 13·4	4 28·8	56·8	11 41·5	12 8·8	N. 13·4
5 28·8	56·5	12 0·6	11 11·9	N. 18·1	5 29·8	56·6	12 3·9	11 9·0	N. 18·6
6 29·9	56·6	12 39·8	11 10·5	N. 21·3	6 29·9	56·7	12 40·0	12 1·2	N. 21·0
7 32·2	56·7	13 4·1	13 3·6	N. 22·6	7 26·7	56·7	13 3·3	13 9·8	N. 21·9
8 34·1	57·2	13 5·6	15 1·2	N. 22·8	8 25·5	56·7	13 8·2	14 11·8	N. 23·2
9 32·8	57·1	12 57·7	16 6·2	N. 21·2	9 28·1	57·0	12 58·8	16 9·6	N. 21·6
10 31·4	57·4	12 42·8	17 9·9	N. 18·9	10 28·8	57·6	12 40·9	17 7·7	N. 18·8
11 29·4	56·9	12 26·2	17 10·8	N. 15·1	11 27·8	57·2	12 27·9	18 7·2	N. 15·2
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 28·9	57·9	12 15·0	18 5·1	N. 9·8	0 33·3	57·1	12 12·6	18 5·9	N. 9·6
1 29·7	57·0	11 57·9	17 4·5	N. 3·3	1 31·0	57·8	11 56·8	18 0·9	N. 3·8
2 31·7	57·5	11 47·9	16 3·7	S. 2·3	2 28·7	56·9	11 49·6	16 8·3	S. 2·9
3 30·1	57·3	11 40·5	14 6·4	S. 7·5	3 27·5	57·1	11 39·6	15 0·4	S. 7·3
4 29·9	56·8	11 43·3	12 6·5	S. 13·4	4 28·7	57·3	11 41·5	13 0·9	S. 13·9
5 32·2	57·1	12 7·1	11 8·7	S. 18·9	5 27·4	57·4	12 3·4	11 7·8	S. 18·1
6 28·3	56·9	12 41·7	12 0·2	S. 20·8	6 27·9	56·7	12 40·9	11 8·9	S. 20·7
7 28·2	57·4	13 7·6	13 10·7	S. 22·3	7 30·3	57·3	13 6·1	13 4·4	S. 22·4
8 30·0	57·2	13 10·7	15 4·4	S. 23·2	8 26·9	57·2	13 10·2	14 8·6	S. 22·6
9 32·6	56·9	13 0·0	16 7·7	S. 21·8	9 25·6	57·6	13 1·2	16 1·3	S. 22·3
10 33·2	57·7	12 45·3	18 2·9	S. 18·2	10 28·6	57·2	12 46·2	17 0·9	S. 19·6
11 30·7	57·9	12 28·5	18 9·4	S. 15·3	11 32·8	57·1	12 29·9	17 9·7	S. 14·1

Table VI. (*f*.) (Continued.)

March.

Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 30·7	57·8	12 12·3	18 10·4	N. 1·2	0 25·5	57·3	12 14·1	17 7·5	N. 1·2
1 31·0	57·2	11 58·3	17 5·1	N. 8·7	1 26·1	58·0	11 59·5	17 1·9	N. 6·7
2 29·9	57·6	11 46·0	16 4·6	N. 13·6	2 28·2	57·0	11 44·4	15 6·1	N. 12·9
3 29·1	56·6	11 35·4	14 2·3	N. 17·4	3 29·9	56·9	11 36·4	13 9·3	N. 18·0
4 30·1	56·8	11 35·7	12 2·8	N. 20·9	4 28·6	56·7	11 41·0	12 1·9	N. 20·5
5 28·6	56·7	11 56·6	10 10·7	N. 22·8	5 29·3	56·6	12 1·6	11 1·7	N. 21·9
6 31·8	56·5	12 42·5	11 6·8	N. 22·6	6 31·0	56·4	12 42·8	11 8·2	N. 22·6
7 29·4	56·6	13 5·9	13 1·4	N. 21·4	7 32·2	56·6	13 6·9	13 4·6	N. 21·9
8 28·5	56·6	13 9·2	14 10·1	N. 20·0	8 28·9	56·9	13 8·4	15 3·0	N. 19·4
9 31·1	56·9	12 57·8	16 6·2	N. 15·1	9 30·8	57·4	12 57·9	16 10·8	N. 15·4
10 29·5	57·6	12 43·5	17 10·8	N. 10·3	10 35·1	57·0	12 43·2	17 11·1	N. 10·2
11 29·4	57·2	12 29·9	18 1·0	N. 4·8	11 31·3	57·8	12 27·5	18 9·8	N. 4·8
Upper Transits A, A.M.					**Lower (Interpolated) Transits A, P.M.**				
0 31·7	58·1	12 12·2	18 3·9	S. 1·4	0 27·6	57·7	12 13·7	18 7·1	S. 0·6
1 28·9	57·8	11 58·7	17 3·9	S. 7·1	1 30·1	57·5	11 59·2	17 8·6	S. 8·3
2 31·9	57·3	11 45·2	15 11·4	S. 13·1	2 29·2	57·6	11 46·9	16 3·7	S. 12·7
3 35·0	57·3	11 34·6	14 1·1	S. 18·2	3 26·7	57·1	11 35·9	14 3·3	S. 17·4
4 30·1	56·9	11 37·2	12 5·8	S. 20·8	4 28·6	56·9	11 36·7	12 3·5	S. 20·8
5 30·5	56·7	12 2·1	11 6·8	S. 22·3	5 30·4	56·9	12 0·8	11 3·2	S. 22·2
6 32·4	56·8	12 44·4	12 1·9	S. 23·2	6 29·7	56·7	12 39·2	11 6·5	S. 22·9
7 31·2	57·1	13 5·2	13 8·7	S. 21·6	7 27·6	56·9	13 6·9	12 11·5	S. 21·9
8 32·5	56·8	13 7·9	15 4·8	S. 19·1	8 29·1	56·8	13 8·5	14 9·0	S. 20·1
9 27·6	57·6	12 58·9	17 2·2	S. 15·6	9 30·6	57·1	12 59·2	16 2·6	S. 14·9
10 29·0	57·3	12 42·7	18 2·0	S. 11·2	10 29·7	57·9	12 43·8	17 5·1	S. 10·3
11 30·9	58·0	12 27·4	19 1·1	S. 5·7	11 30·5	57·8	12 28·2	18 0·9	S. 5·7

April.

Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 29·5	57·6	12 14·3	17 9·7	N. 13·6	0 27·7	57·2	12 12·7	17 1·4	N. 11·9
1 30·1	57·0	11 58·3	16 7·2	N. 16·6	1 31·6	57·7	11 56·3	16 7·9	N. 17·0
2 32·1	57·5	11 43·2	15 4·3	N. 20·7	2 30·3	57·2	11 42·7	15 2·8	N. 20·2
3 33·1	57·1	11 33·6	13 7·5	N. 22·6	3 27·6	57·0	11 33·4	13 9·4	N. 22·0
4 34·0	56·8	11 38·8	11 10·0	N. 22·7	4 28·6	56·6	11 36·7	12 3·2	N. 22·9
5 34·7	56·6	12 5·3	11 1·4	N. 22·0	5 28·1	57·0	12 0·9	11 6·3	N. 22·4
6 30·9	56·7	12 40·3	11 9·5	N. 20·1	6 30·8	56·7	12 39·5	12 0·9	N. 19·0
7 32·6	56·8	13 2·8	13 9·4	N. 15·3	7 31·9	57·0	13 2·2	13 9·1	N. 16·1
8 31·9	57·0	13 5·3	15 6·7	N. 10·7	8 30·7	57·0	13 3·6	15 7·1	N. 11·7
9 26·6	57·3	12 58·4	16 11·6	N. 5·7	9 29·8	57·3	12 56·0	17 0·2	N. 5·1
10 27·5	57·1	12 42·9	17 8·6	S. 0·8	10 32·6	57·4	12 42·7	17 10·0	S. 0·5
11 26·6	57·9	12 29·6	18 3·6	S. 6·8	11 32·6	57·3	12 27·1	18 1·1	S. 7·4
Upper Transits A, A.M.					**Lower (Interpolated) Transits A, P.M.**				
0 26·9	57·3	12 13·7	17 7·3	S. 11·5	0 33·0	58·0	12 11·3	17 10·3	S. 13·0
1 29·2	58·0	11 58·3	17 1·4	S. 17·4	1 28·8	58·0	11 58·0	16 10·5	S. 16·4
2 28·4	57·9	11 43·5	15 10·9	S. 20·6	2 31·4	57·6	11 43·2	15 1·7	S. 20·8
3 30·7	57·3	11 34·3	14 2·8	S. 21·9	3 31·1	57·3	11 35·3	13 8·5	S. 22·4
4 31·6	57·3	11 37·5	12 10·2	S. 23·4	4 28·8	57·3	11 37·7	12 2·2	S. 22·9
5 31·8	57·2	12 1·6	12 0·6	S. 22·1	5 27·8	57·2	12 1·0	11 2·6	S. 22·3
6 32·5	56·7	12 41·3	12 3·1	S. 19·6	6 27·3	56·8	12 37·5	11 6·0	S. 20·5
7 31·4	57·2	12 59·8	13 11·9	S. 15·7	7 25·4	57·3	13 1·3	13 2·3	S. 16·1
8 27·2	57·2	13 3·9	15 7·6	S. 11·9	8 28·4	57·2	13 5·0	15 0·7	S. 10·8
9 27·1	56·7	12 57·0	16 10·7	S. 6·0	9 30·4	57·4	12 56·4	16 5·1	S. 5·0
10 26·4	57·5	12 44·1	18 0·1	N. 0·6	10 29·4	57·0	12 44·4	17 3·5	N. 0·5
11 28·3	56·9	12 30·2	18 0·1	N. 6·7	11 29·7	57·6	12 29·5	17 9·5	N. 7·3

TABLE VI. (*f*.) (Continued.)

May.

Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 27·2	57·1	12 13·4	16 5·9	N. 20·9	0 29·9	57·4	12 12·9	17 0·3	N. 20·2
1 30·4	57·1	11 57·6	15 9·9	N. 22·3	1 31·1	57·1	11 56·0	16 3·2	N. 21·8
2 31·4	57·5	11 43·2	14 10·7	N. 22·8	2 32·3	57·2	11 41·5	15 4·7	N. 23·3
3 29·4	57·0	11 35·7	13 7·9	N. 21·7	3 29·1	57·1	11 35·3	14 2·4	N. 21·9
4 26·3	57·0	11 39·2	12 8·0	N. 20·6	4 30·7	56·9	11 38·1	13 3·3	N. 19·6
5 28·9	56·9	12 2·8	12 2·2	N. 15·9	5 28·9	56·9	12 1·9	12 7·6	N. 16·2
6 29·2	56·9	12 36·3	12 11·7	N. 11·5	6 27·7	57·0	12 33·6	13 1·3	N. 12·5
7 26·9	57·2	12 57·5	14 2·2	N. 5·4	7 29·9	57·1	12 57·6	14 3·1	N. 5·9
8 28·9	56·8	13 2·5	15 6·2	S. 0·8	8 29·6	57·4	13 0·8	15 5·8	N. 0·4
9 29·9	57·8	12 56·0	16 8·6	S. 6·9	9 29·5	57·2	12 55·0	16 4·6	S. 6·8
10 31·1	57·1	12 43·9	17 4·0	S. 11·9	10 31·6	57·7	12 42·9	17 1·1	S. 12·2
11 32·5	57·2	12 28·2	17 8·7	S. 17·5	11 26·8	57·1	12 30·9	17 4·8	S. 16·4
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 29·8	57·6	12 12·2	17 3·9	S. 19·9	0 27·9	57·2	12 12·7	16 8·4	S. 20·3
1 27·3	57·5	11 56·6	16 8·1	S. 21·9	1 30·3	57·8	11 58·5	15 11·7	S. 22·5
2 28·8	57·7	11 43·1	15 9·6	S. 22·8	2 30·4	57·7	11 44·4	14 11·8	S. 22·6
3 29·7	57·5	11 33·7	14 7·7	S. 22·6	3 28·8	57·7	11 36·2	13 8·5	S. 22·2
4 31·0	57·3	11 38·9	13 7·5	S. 19·6	4 29·2	57·2	11 39·2	12 3·5	S. 20·3
5 30·4	57·4	12 3·4	12 10·7	S. 16·3	5 30·6	57·1	12 4·9	11 9·4	S. 15·4
6 26·9	56·9	12 34·4	13 2·6	S. 12·2	6 31·5	57·4	12 38·1	12 4·4	S. 11·4
7 28·1	57·0	12 55·9	14 3·0	S. 5·9	7 33·6	57·4	12 57·4	13 9·7	S. 6·3
8 29·6	57·1	13 0·7	15 5·8	N. 0·5	8 31·9	56·9	13 1·7	15 0·0	N. 0·9
9 32·5	56·9	12 54·8	16 4·3	N. 6·9	9 29·9	57·2	12 56·5	16 0·2	N. 6·4
10 34·1	57·1	12 40·3	16 7·0	N. 12·0	10 28·7	56·9	12 44·4	16 10·0	N. 11·8
11 30·1	57·0	12 29·7	17 0·5	N. 16·6	11 29·9	56·9	12 28·8	16 11·9	N. 17·0

June.

Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 30·8	57·4	12 12·1	16 3·7	N. 23·0	0 28·9	57·1	12 11·5	16 11·5	N. 23·1
1 26·1	57·2	11 57·7	15 10·9	N. 22·1	1 32·6	57·1	11 55·1	16 5·7	N. 21·6
2 29·4	56·8	11 43·4	14 10·9	N. 19·9	2 32·3	57·2	11 43·2	15 9·2	N. 18·8
3 31·7	56·8	11 37·6	13 10·9	N. 15·2	3 27·3	56·9	11 37·5	14 9·5	N. 16·4
4 30·9	57·1	11 45·0	13 2·3	N. 10·9	4 30·7	56·9	11 43·8	13 9·2	N. 11·4
5 29·4	56·7	12 5·0	12 6·9	N. 4·9	5 32·2	57·3	12 3·0	13 3·1	N. 4·9
6 29·5	57·0	12 37·5	13 1·8	S. 0·9	6 28·0	56·9	12 32·6	13 2·5	S. 0·8
7 31·8	57·2	12 57·8	14 1·7	S. 7·5	7 29·4	57·3	12 56·9	13 11·8	S. 7·2
8 33·3	57·4	13 2·2	15 6·9	S. 12·4	8 26·4	57·6	13 1·9	15 0·6	S. 11·9
9 34·9	57·7	12 56·9	16 7·8	S. 18·0	9 26·0	57·6	12 56·8	15 10·7	S. 16·4
10 28·6	57·9	12 45·3	17 3·0	S. 20·3	10 29·4	57·6	12 44·1	16 4·8	S. 20·5
11 26·6	57·4	12 30·5	17 4·3	S. 22·3	11 31·9	57·6	12 28·6	16 5·7	S. 22·4
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 25·3	57·3	12 10·9	17 3·0	S. 23·2	0 30·3	57·9	12 9·7	16 5·1	S. 22·9
1 27·3	57·8	11 57·0	16 10·7	S. 22·2	1 29·3	57·6	11 57·6	15 9·2	S. 22·5
2 27·4	57·6	11 44·1	16 4·8	S. 19·5	2 28·5	57·4	11 44·8	15 0·5	S. 19·7
3 25·6	57·3	11 37·8	15 4·4	S. 16·7	3 31·9	57·3	11 38·8	13 11·3	S. 15·7
4 26·3	57·2	11 44·3	14 5·6	S. 10·8	4 32·7	57·4	11 45·3	13 0·4	S. 10·9
5 27·5	57·1	12 4·2	13 7·9	S. 5·8	5 29·9	56·9	12 6·0	12 4·1	S. 5·2
6 31·4	57·2	12 35·9	13 10·3	N. 1·1	6 31·7	56·9	12 35·6	12 8·8	N. 1·0
7 33·9	56·9	12 57·5	14 2·4	N. 7·0	7 28·5	57·3	12 56·4	13 9·7	N. 6·9
8 30·2	56·9	13 4·1	15 2·0	N. 12·3	8 29·5	57·0	13 2·3	15 0·0	N. 12·4
9 28·9	57·0	12 58·4	15 11·8	N. 16·4	9 32·7	57·4	12 55·9	16 4·4	N. 17·2
10 31·0	57·0	12 45·9	16 3·8	N. 20·6	10 30·1	57·9	12 44·9	16 9·9	N. 20·4
11 32·7	57·1	12 29·2	16 5·7	N. 22·1	11 26·8	56·9	12 29·4	16 9·9	N. 21·9

Table VI. (*f*.) (Continued.)

July.

Upper Transits A, P.M.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°
0 34·5	56·9	12 13·2	16 10·2	N. 18·9
1 34·5	57·2	11 58·2	16 5·5	N. 15·6
2 31·3	57·0	11 44·0	15 6·7	N. 10·8
3 30·4	56·6	11 41·4	14 4·8	N. 4·9
4 30·8	56·9	11 45·3	13 4·5	S. 0·9
5 28·8	56·7	12 5·5	12 6·1	S. 7·6
6 28·8	56·9	12 37·8	12 9·0	S. 13·2
7 25·7	57·3	12 59·5	13 11·8	S. 17·4
8 25·1	57·2	13 4·9	15 3·3	S. 20·0
9 26·7	57·9	13 0·6	16 5·2	S. 22·6
10 30·9	57·8	12 44·5	17 5·2	S. 22·9
11 31·8	57·9	12 28·0	17 10·1	S. 21·5

Lower (Interpolated) Transits A, A.M.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°
0 28·6	57·4	12 12·8	17 9·9	N. 19·7
1 29·5	56·9	11 56·7	17 3·5	N. 16·5
2 29·6	56·8	11 42·8	16 5·8	N. 10·9
3 29·0	57·0	11 40·8	15 1·9	N. 5·0
4 28·4	56·7	11 44·3	13 11·9	S. 0·7
5 30·5	57·0	12 5·3	13 0·6	S. 7·6
6 30·9	57·2	12 35·1	12 10·1	S. 12·1
7 31·9	57·2	13 0·7	13 7·4	S. 17·2
8 34·1	57·2	13 6·8	14 7·7	S. 21·3
9 31·0	57·7	12 59·3	15 9·6	S. 22·1
10 26·2	57·9	12 46·7	16 3·5	S. 23·2
11 26·9	58·1	12 30·8	16 11·3	S. 22·3

Upper Transits A, A.M.

0 25·7	57·8	12 14·5	17 11·8	S. 19·7
1 25·4	57·2	11 59·2	17 4·2	S. 16·8
2 29·2	57·4	11 48·4	16 8·4	S. 11·0
3 31·2	57·3	11 39·5	15 6·4	S. 4·9
4 29·0	56·6	11 44·7	14 0·8	N. 1·7
5 29·4	56·8	12 5·6	13 2·0	N. 6·2
6 26·9	56·8	12 35·9	12 9·4	N. 12·7
7 27·6	56·9	13 0·1	13 7·4	N. 17·0
8 29·5	57·1	13 6·7	14 8·2	N. 20·7
9 28·3	57·3	13 2·6	15 9·7	N. 21·9
10 28·9	57·6	12 45·9	16 5·7	N. 22·6
11 31·6	57·2	12 28·6	16 8·9	N. 22·1

Lower (Interpolated) Transits A, P.M.

0 31·5	57·4	12 12·9	16 5·4	S. 19·3
1 31·4	57·8	12 0·4	16 2·2	S. 15·6
2 29·3	57·2	11 45·7	15 5·6	S. 11·4
3 32·4	56·9	11 44·7	14 2·3	S. 4·6
4 31·2	57·2	11 47·5	13 1·0	N. 1·8
5 27·4	56·7	12 4·4	12 1·6	N. 6·8
6 29·6	56·7	12 38·8	12 3·7	N. 12·9
7 33·1	57·1	13 3·1	13 9·3	N. 17·1
8 34·1	56·8	13 6·6	15 0·6	N. 21·0
9 32·6	57·3	12 57·9	16 3·9	N. 22·2
10 30·2	57·2	12 40·3	17 1·1	N. 22·9
11 29·5	57·2	12 29·6	17 8·4	N. 21·6

August.

Upper Transits A, P.M.

0 31·7	57·5	12 14·1	17 6·7	N. 10·8
1 31·4	57·3	11 59·7	16 10·6	N. 4·4
2 28·2	57·3	11 46·1	16 2·6	S. 1·1
3 26·2	56·7	11 41·4	14 7·7	S. 6·3
4 27·4	56·9	11 42·5	13 1·8	S. 13·4
5 24·5	56·7	12 1·0	12 0·1	S. 17·0
6 24·9	56·7	12 37·3	12 2·7	S. 20·5
7 26·3	57·2	13 4·3	13 9·4	S. 22·4
8 29·8	56·9	13 9·3	15 2·8	S. 22·9
9 29·0	57·6	13 1·6	16 11·6	S. 21·8
10 26·6	57·8	12 46·8	18 0·9	S. 19·8
11 26·2	57·8	12 28·0	18 8·6	S. 15·7

Lower (Interpolated) Transits A, A.M.

0 29·4	57·3	12 13·8	18 2·2	N. 10·6
1 24·9	57·7	12 1·0	18 0·0	N. 5·6
2 26·6	57·1	11 47·8	16 7·6	S. 0·7
3 31·1	57·1	11 39·3	14 11·6	S. 6·4
4 31·5	56·8	11 41·9	13 2·9	S. 12·1
5 31·2	56·9	12 2·0	11 9·5	S. 18·2
6 31·6	56·9	12 41·8	11 11·3	S. 20·7
7 31·3	57·3	13 4·2	13 2·1	S. 22·5
8 30·6	56·9	13 10·7	14 2·2	S. 22·8
9 32·8	57·6	12 58·9	15 10·0	S. 22·1
10 34·5	57·4	12 44·7	16 8·3	S. 18·9
11 35·3	58·2	12 28·2	17 5·5	S. 14·9

Upper Transits A, A.M.

0 30·7	57·8	12 13·6	18 9·3	S. 10·5
1 29·3	58·0	11 59·4	18 4·0	S. 5·3
2 28·6	56·9	11 46·7	16 9·0	N. 0·9
3 28·9	57·1	11 40·2	15 3·7	N. 6·7
4 26·9	56·9	11 41·2	13 4·1	N. 12·5
5 28·4	56·5	12 2·8	11 9·3	N. 18·1
6 28·6	56·5	12 41·2	11 11·7	N. 20·4
7 24·9	56·6	13 4·9	12 11·9	N. 21·8
8 26·3	56·8	13 9·2	14 4·5	N. 22·9
9 27·4	56·9	13 1·0	15 9·2	N. 21·9
10 29·6	56·9	12 42·9	16 8·6	N. 19·5
11 30·7	57·8	12 29·1	17 7·4	N. 15·3

Lower (Interpolated) Transits A, P.M.

0 25·8	58·1	12 15·7	17 5·6	S. 10·9
1 27·2	57·3	12 0·1	16 7·9	S. 5·2
2 28·9	58·1	11 47·5	15 10·9	N. 1·3
3 27·6	56·8	11 39·4	14 4·0	N. 6·4
4 28·9	56·9	11 39·6	12 8·1	N. 13·4
5 30·8	56·8	12 3·6	11 7·2	N. 16·7
6 31·4	56·8	12 38·4	11 11·6	N. 20·8
7 33·6	56·6	13 6·2	13 6·9	N. 22·4
8 32·3	56·9	13 9·3	15 2·1	N. 23·2
9 32·2	56·7	12 59·5	16 6·2	N. 20·9
10 31·0	57·4	12 45·0	17 8·5	N. 19·3
11 29·7	57·3	12 28·9	18 3·4	N. 15·9

Table VI. (*f*.) (Continued.)

September.

Upper Transits A, P.M.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°
0 30·0	57·7	12 14·3	18 2·1	S. 0·9
1 29·8	57·5	11 58·7	17 4·2	S. 6·9
2 30·5	57·7	11 46·4	16 0·2	S. 12·7
3 26·8	56·9	11 36·9	14 6·8	S. 16·8
4 25·9	57·0	11 38·1	12 10·7	S. 19·5
5 23·9	57·1	11 56·9	11 11·1	S. 22·3
6 26·7	56·8	12 34·1	12 5·0	S. 23·1
7 28·4	57·3	13 3·4	13 11·9	S. 22·1
8 25·6	56·9	13 8·3	15 7·9	S. 20·1
9 26·3	57·8	12 58·5	17 6·1	S. 15·9
10 29·2	57·2	12 44·9	18 5·8	S. 11·2
11 29·8	57·8	12 25·4	19 2·9	S. 5·2

Lower (Interpolated) Transits A, A.M.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°
0 31·9	57·3	12 12·2	18 6·6	S. 0·9
1 34·7	57·8	11 53·6	17 9·4	S. 7·1
2 31·4	57·4	11 45·2	16 3·6	S. 11·9
3 30·6	57·3	11 36·3	14 5·2	S. 16·9
4 32·4	56·9	11 37·2	12 3·8	S. 20·7
5 34·4	56·9	12 0·8	11 4·0	S. 22·1
6 32·7	57·0	12 42·1	12 3·5	S. 23·1
7 32·6	57·3	13 5·2	13 4·5	S. 22·2
8 33·9	57·0	13 9·2	14 11·8	S. 19·9
9 33·7	57·4	12 57·7	16 8·1	S. 13·4
10 29·5	57·8	12 41·5	17 5·1	S. 11·1
11 29·6	57·4	12 29·5	17 10·1	S. 6·4

Upper Transits A, A.M.

0 28·0	57·5	12 13·9	18 11·9	S. 0·3
1 28·0	57·3	11 59·7	18 1·0	N. 6·4
2 27·5	57·7	11 45·0	16 8·8	N. 12·0
3 29·5	56·8	11 35·9	14 5·9	N. 16·8
4 28·5	56·8	11 38·8	12 8·0	N. 19·1
5 25·7	56·4	11 59·2	11 4·1	N. 21·7
6 27·9	56·7	12 42·3	11 8·8	N. 23·7
7 27·6	56·8	13 5·4	13 3·4	N. 22·0
8 26·8	56·8	13 8·4	14 8·1	N. 20·4
9 29·1	56·8	12 59·3	16 1·8	N. 15·9
10 28·3	57·6	12 45·6	17 7·0	N. 11·2
11 30·0	56·9	12 30·4	17 8·2	N. 5·9

Lower (Interpolated) Transits A, P.M.

0 31·9	57·8	12 14·1	17 9·9	N. 1·6
1 31·4	57·3	12 2·5	16 11·9	N. 6·6
2 32·3	57·7	11 45·3	15 9·8	N. 12·7
3 32·5	57·1	11 37·1	14 1·9	N. 16·6
4 31·8	56·8	11 37·7	12 4·1	N. 20·2
5 29·3	56·9	12 0·3	10 5·4	N. 22·4
6 29·9	56·5	12 40·3	12 0·4	N. 23·1
7 32·5	56·6	13 5·6	13 8·2	N. 22·0
8 32·5	56·8	13 7·8	15 6·1	N. 19·6
9 30·3	56·9	12 59·7	16 9·9	N. 16·6
10 31·3	56·9	12 43·8	18 0·7	N. 11·0
11 31·9	57·9	12 28·8	18 9·9	N. 5·1

October.

Upper Transits A, P.M.

0 26·7	56·9	12 13·9	17 4·9	S. 11·2
1 26·5	57·7	11 58·5	17 4·2	S. 16·4
2 24·1	57·3	11 44·1	16 1·1	S. 19·8
3 22·6	57·6	11 36·3	14 9·5	S. 21·9
4 24·8	57·0	11 34·8	13 0·9	S. 23·2
5 27·8	57·3	11 59·0	12 3·2	S. 22·1
6 27·8	57·0	12 35·2	12 7·7	S. 20·1
7 27·3	57·3	12 59·8	14 3·6	S. 16·9
8 30·3	56·9	13 6·1	15 11·8	S. 12·8
9 29·3	57·2	12 58·8	17 5·7	S. 5·6
10 25·7	57·6	12 43·7	18 7·6	S. 1·0
11 26·9	56·8	12 29·1	19 0·2	N. 5·6

Lower (Interpolated) Transits A, A.M.

0 30·8	57·6	12 12·2	18 5·2	S. 11·6
1 28·2	57·5	11 56·4	17 4·5	S. 15·8
2 30·9	57·4	11 43·1	15 6·3	S. 20·3
3 31·3	57·7	11 35·5	14 1·5	S. 22·3
4 33·1	57·5	11 34·4	12 2·9	S. 22·6
5 32·2	57·2	12 1·4	11 3·0	S. 22·1
6 30·7	56·9	12 38·5	11 9·8	S. 20·6
7 33·5	57·1	13 3·9	13 5·4	S. 16·9
8 33·4	57·1	13 4·9	15 0·6	S. 12·4
9 29·9	57·3	12 56·8	16 5·7	S. 6·5
10 28·8	56·9	12 44·5	17 6·7	S. 0·4
11 30·2	57·8	12 27·6	18 2·9	N. 5·9

Upper Transits A, A.M.

0 28·4	57·7	12 12·1	18 8·4	N. 10·9
1 27·0	57·3	11 57·4	17 4·6	N. 16·4
2 28·9	57·1	11 40·9	15 6·2	N. 20·5
3 27·4	57·1	11 34·5	13 10·8	N. 21·5
4 26·9	57·2	11 34·3	12 5·9	N. 23·1
5 29·9	56·9	11 56·1	11 1·6	N. 22·1
6 25·5	56·6	12 35·9	11 10·5	N. 20·4
7 27·4	56·7	13 3·4	13 6·3	N. 17·1
8 29·0	56·9	13 5·3	15 3·2	N. 13·2
9 28·9	56·9	12 57·2	16 10·6	N. 6·9
10 29·7	57·2	12 43·3	17 4·6	S. 0·5
11 26·4	57·7	12 29·7	18 5·2	S. 4·9

Lower (Interpolated) Transits A, P.M.

0 29·4	56·9	12 13·2	17 8·1	N. 11·1
1 31·1	57·3	11 56·2	16 10·9	N. 16·6
2 30·9	57·4	11 36·6	15 6·6	N. 20·2
3 31·5	57·4	11 33·7	14 3·3	N. 22·2
4 34·1	56·9	11 35·9	12 7·5	N. 23·2
5 37·5	56·8	12 5·6	11 11·4	N. 21·5
6 34·8	56·8	12 41·2	12 3·9	N. 19·9
7 30·3	56·9	13 2·8	14 5·4	N. 17·2
8 30·5	56·7	13 6·1	15 9·7	N. 12·2
9 31·7	57·2	12 56·5	17 4·6	N. 6·0
10 32·7	57·3	12 42·3	17 6·2	S. 0·7
11 30·9	57·0	12 28·8	18 5·7	S. 6·5

Table VI. (*f*.) (Continued.)

November.

Upper Transits A, P.M.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°
0 31·5	57·4	12 11·3	17 9·8	S. 19·9
1 28·9	57·1	11 56·1	16 11·5	S. 21·7
2 28·4	57·2	11 41·8	16 0·9	S. 23·1
3 29·6	57·3	11 33·8	14 9·8	S. 22·0
4 28·4	57·3	11 38·7	14 0·7	S. 20·1
5 27·3	57·3	11 59·5	13 1·9	S. 16·5
6 29·4	56·9	12 36·2	13 4·6	S. 12·0
7 29·4	57·2	12 56·5	14 7·0	S. 6·6
8 27·3	57·3	13 1·0	16 0·1	S. 0·9
9 29·8	57·1	12 50·8	17 1·3	N. 5·5
10 30·7	57·5	12 43·1	17 9·5	N. 11·5
11 30·4	57·3	12 28·5	17 8·6	N. 17·0

Lower (Interpolated) Transits A, A.M.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°
0 29·5	57·7	12 12·4	17 5·1	S. 19·8
1 32·0	57·6	11 55·8	16 5·2	S. 22·2
2 29·6	57·5	11 40·9	15 3·3	S. 22·9
3 28·3	57·3	11 33·7	13 9·9	S. 22·6
4 30·9	57·2	11 38·4	12 5·2	S. 20·3
5 36·9	57·1	12 6·3	11 11·6	S. 16·2
6 32·0	57·2	12 37·8	12 2·7	S. 12·5
7 29·5	57·1	12 57·0	13 9·4	S. 6·6
8 30·5	57·0	13 2·7	15 3·9	S. 0·8
9 27·8	57·4	12 55·3	16 4·1	N. 4·5
10 27·2	57·3	12 44·1	17 2·3	N. 11·3
11 30·6	57·3	12 28·8	17 7·6	N. 16·3

Upper Transits A, A.M.

0 35·6	57·1	12 11·2	17 2·5	N. 19·8
1 33·2	57·4	11 55·7	16 4·9	N. 22·0
2 29·4	57·0	11 42·4	15 0·3	N. 22·9
3 25·5	56·9	11 34·0	13 9·7	N. 22·2
4 30·1	56·6	11 36·7	12 5·0	N. 20·5
5 31·5	57·0	12 4·2	12 0·0	N. 16·1
6 28·4	57·1	12 38·2	12 10·7	N. 12·4
7 29·2	56·7	12 58·6	14 1·2	N. 7·2
8 28·5	57·0	13 2·5	15 10·1	N., 0·6
9 28·6	57·5	12 55·6	17 1·9	S. 4·8
10 30·0	57·0	12 44·3	17 9·4	S. 12·2
11 32·9	57·9	12 28·4	18 2·5	S. 16·6

Lower (Interpolated) Transits A, P.M.

0 27·8	57·3	12 12·7	17 3·1	N. 19·3
1 28·5	57·0	11 56·5	16 6·1	N. 22·7
2 32·2	57·1	11 41·7	15 8·1	N. 22·9
3 34·2	56·8	11 34·7	14 5·3	N. 21·9
4 36·4	56·9	11 34·3	12 11·0	N. 19·6
5 28·2	56·8	11 59·3	12 8·7	N. 16·7
6 31·2	56·7	12 38·0	13 1·8	N. 12·5
7 32·3	57·3	12 58·7	14 7·7	N. 5·8
8 28·3	56·9	13 3·5	16 0·8	S. 0·2
9 27·2	57·1	12 56·6	17 2·0	S. 4·7
10 25·2	57·8	12 44·1	18 0·2	S. 11·2
11 26·5	57·2	12 29·5	17 10·3	S. 16·2

December.

Upper Transits A, P.M.

0 32·8	57·6	12 10·1	17 8·7	S. 23·2
1 29·2	57·6	11 55·8	17 3·6	S. 22·2
2 28·4	57·2	11 43·0	16 5·6	S. 19·7
3 29·5	56·9	11 35·2	15 0·0	S. 15·1
4 31·8	57·3	11 41·0	14 5·6	S. 11·4
5 31·8	56·9	12 3·8	13 8·1	S. 4·8
6 31·6	56·8	12 36·6	13 9·3	N. 1·1
7 30·8	57·3	12 58·1	14 7·5	N. 6·7
8 29·0	57·0	13 3·5	15 9·8	N. 12·4
9 29·2	57·4	12 57·2	16 9·3	N. 16·7
10 26·8	57·5	12 48·8	17 1·1	N. 19·9
11 27·5	57·6	12 29·8	17 4·1	N. 22·0

Lower (Interpolated) Transits A, A.M.

0 25·9	57·3	12 12·8	16 11·3	S. 23·2
1 31·4	57·3	11 54·6	16 2·4	S. 21·9
2 33·4	57·4	11 42·6	15 3·1	S. 19·1
3 27·4	57·7	11 42·9	14 3·7	S. 16·2
4 27·0	56·8	11 41·0	12 11·9	S. 11·6
5 29·9	57·5	12 5·2	12 10·3	S. 5·5
6 28·3	57·1	12 34·4	12 8·3	N. 0·3
7 27·7	57·0	12 57·1	14 1·0	N. 5·8
8 30·2	57·3	13 1·5	15 5·9	N. 12·2
9 29·5	57·6	12 55·9	16 5·7	N. 15·5
10 30·6	57·1	12 44·1	17 0·0	N. 20·8
11 34·1	57·2	12 26·6	17 6·0	N. 21·9

Upper Transits A, A.M.

0 27·0	57·2	12 14·0	16 8·2	N. 23·2
1 29·4	56·9	11 55·8	15 10·6	N. 22·5
2 29·3	56·8	11 42·9	14 11·2	N. 19·0
3 29·3	56·9	11 42·5	14 0·2	N. 15·8
4 30·6	56·6	11 42·5	12 8·2	N. 10·6
5 30·0	56·7	12 4·2	12 3·5	N. 6·2
6 31·4	57·0	12 36·3	12 11·7	S. 1·2
7 30·3	56·7	12 58·2	14 1·7	S. 6·1
8 28·6	57·7	13 1·9	15 9·3	S. 12·3
9 24·7	57·5	12 58·6	16 11·5	S. 15·8
10 27·5	57·6	12 45·2	17 0·9	S. 19·7
11 34·4	57·7	12 26·5	17 10·3	S. 22·9

Lower (Interpolated) Transits A, P.M.

0 33·9	57·1	12 10·8	17 1·7	N. 22·7
1 29·2	57·3	11 54·8	16 7·8	N. 21·6
2 25·2	56·9	11 42·3	15 11·5	N. 19·6
3 29·5	56·4	11 34·6	14 9·4	N. 16·1
4 32·2	57·0	11 42·1	14 0·5	N. 10·5
5 28·7	56·6	12 2·5	12 7·7	N. 4·9
6 29·4	56·8	12 35·6	13 1·4	S. 0·5
7 32·0	57·3	12 57·6	14 4·3	S. 5·6
8 30·0	57·3	13 2·4	15 6·8	S. 12·1
9 32·6	57·5	12 56·2	16 5·0	S. 17·3
10 32·2	58·1	12 44·1	17 4·7	S. 20·1
11 23·7	57·7	12 30·7	17 4·5	S. 21·8

Table VII. (*g*.)

Showing the Diurnal Inequality at Liverpool, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table II., and the Difference between the Height of High Water and the Height in Table III.

Apparent Solar Time of Moon's Transit A.	Diurnal Inequality. January. Interval.	Height.	Moon's Declination.	February. Interval.	Height.	Moon's Declination.	March. Interval.	Height.	Moon's Declination.
	h m	m	feet. °	m	feet. °		m	feet. °	
P.M.	0 30	+ 1·4	+ ·57 S. 18·3	− 0·2	+ ·29 S. 9·8		− 0·1	+ ·42 N. 0·9
	1 30	− 0·8	+ ·60 S. 15·0	− 0·5	+ ·50 S. 3·8		− 1·6	+ ·25 N. 8·5
	2 30	+ 0·4	+ ·50 S. 9·3	+ 0·2	+ ·45 N. 2·7		+ 0·3	+ ·35 N. 12·7
	3 30	− 1·1	+ ·55 S. 5·0	+ 0·2	+ ·37 N. 8·0		+ 0·5	+ ·13 N. 17·4
	4 30	− 0·3	+ ·60 N. 1·3	− 0·8	+ ·30 N. 13·7		− 1·7	− ·06 N. 20·8
	5 30	− 1·3	+ ·27 N. 8·4	− 1·0	+ ·03 N. 18·1		− 1·7	− ·23 N. 22·5
	6 30	− 0·1	+ ·08 N. 13·5	0·0	− ·13 N. 21·0		− 0·7	− ·10 N. 22·8
	7 30	− 1·1	+ ·02 N. 18·9	− 0·5	− ·28 N. 22·5		+ 1·9	− ·24 N. 21·7
	8 30	+ 1·3	− ·16 N. 20·4	− 0·4	− ·27 N. 22·7		+ 1·4	− ·22 N. 20·0
	9 30	0·0	− ·14 N. 22·9	+ 0·2	− ·27 N. 21·7		+ 0·1	− ·26 N. 15·0
	10 30	+ 0·3	− ·32 N. 22·7	+ 1·5	− ·22 N. 19·3		− 0·4	− ·35 N. 10·3
	11 30	+ 0·1	− ·27 N. 21·5	+ 0·1	− ·33 N. 14·6		+ 0·6	− ·35 N. 5·1
A.M.	0 30	0·0	− ·63 N. 18·8	+ 0·1	− ·38 N. 10·0		− 0·1	− ·35 S. 1·3
	1 30	+ 0·9	− ·45 N. 14·9	+ 0·4	− ·36 N. 3·4		− 0·2	− ·29 S. 6·9
	2 30	− 0·2	− ·57 N. 10·0	− 0·2	− ·49 S. 1·5		− 0·4	− ·21 S. 13·4
	3 30	+ 1·0	− ·56 N. 3·8	− 0·5	− ·43 S. 8·1		− 0·6	− ·18 S. 18·1
	4 30	0·0	− ·53 S. 2·3	+ 0·3	− ·28 S. 13·4		+ 1·5	+ ·03 S. 20·7
	5 30	+ 0·3	− ·32 S. 7·7	+ 1·1	+ ·01 S. 18·8		+ 1·5	+ ·17 S. 22·1
	6 30	+ 0·1	− ·07 S. 13·5	+ 0·5	+ ·10 S. 20·9		+ 1·5	+ ·02 S. 22·9
	7 30	− 0·9	− ·03 S. 18·0	+ 0·7	+ ·09 S. 22·2		+ 0·1	+ ·22 S. 21·8
	8 30	− 1·4	− ·19 S. 20·8	+ 1·4	+ ·43 S. 23·2		+ 0·4	+ ·20 S. 19·3
	9 30	+ 0·4	− ·17 S. 22·4	− 0·2	+ ·27 S. 21·7		+ 0·3	+ ·08 S. 15·5
	10 30	+ 0·2	− ·37 S. 22·4	− 0·6	+ ·25 S. 18·5		− 1·1	+ ·26 S. 10·7
	11 30	+ 0·2	− ·18 S. 22·2	− 0·2	+ ·25 S. 15·3		− 0·7	+ ·20 S. 5·3
		Sun's Declination 21° S.		Sun's Declination 13° S.			Sun's Declination 2° S.		
		April.		May.			June.		
P.M.	0 30	+ 0·6	− ·15 N. 13·3	0·0	− ·31 N. 20·6		+ 0·3	− ·54 N. 23·0	
	1 30	+ 0·4	− ·05 N. 16·5	+ 1·8	− ·19 N. 22·4		+ 0·2	− ·50 N. 22·3	
	2 30	+ 0·2	− ·13 N. 20·7	− 0·4	− ·43 N. 22·7		+ 0·6	− ·44 N. 19·8	
	3 30	+ 0·1	− ·15 N. 22·5	+ 0·6	− ·43 N. 22·0		+ 0·6	− ·42 N. 15·5	
	4 30	0·0	− ·29 N. 22·8	+ 0·6	− ·55 N. 18·5		− 0·1	− ·54 N. 11·0	
	5 30	+ 0·6	− ·27 N. 22·1	+ 0·7	− ·30 N. 15·7		+ 0·7	− ·40 N. 5·0	
	6 30	− 0·3	− ·31 N. 20·3	+ 0·8	− ·33 N. 11·4		+ 1·9	− ·32 S. 1·0	
	7 30	+ 0·5	− ·10 N. 15·6	+ 0·5	− ·18 N. 5·8		+ 0·4	− ·09 S. 7·2	
	8 30	+ 0·6	− ·17 N. 10·8	+ 0·3	− ·03 S. 0·9		− 0·6	+ ·07 S. 12·4	
	9 30	+ 0·8	− ·20 N. 5·4	+ 1·1	+ ·03 S. 6·6		+ 1·9	+ ·26 S. 17·6	
	10 30	− 0·2	− ·14 S. 0·7	+ 0·7	+ ·26 S. 11·9		+ 0·2	+ ·26 S. 20·4	
	11 30	+ 0·2	− ·15 S. 7·0	− 0·7	+ ·18 S. 17·3		− 0·1	+ ·38 S. 22·1	
A.M.	0 30	− 0·1	− ·12 S. 11·7	+ 0·1	+ ·55 S. 19·5		− 0·3	+ ·34 S. 23·2	
	1 30	− 0·6	+ ·01 S. 17·2	− 0·3	+ ·31 S. 21·9		− 0·4	+ ·47 S. 21·9	
	2 30	+ 0·7	+ ·06 S. 20·4	− 1·8	+ ·27 S. 23·0		− 0·7	+ ·51 S. 19·2	
	3 30	− 2·1	− ·12 S. 21·9	− 0·7	+ ·35 S. 22·3		− 1·1	+ ·58 S. 16·6	
	4 30	− 0·6	+ ·26 S. 23·2	− 0·7	+ ·50 S. 19·6		0·0	+ ·43 S. 11·1	
	5 30	− 1·0	+ ·14 S. 22·3	− 0·5	+ ·38 S. 16·3		− 1·1	+ ·47 S. 5·4	
	6 30	+ 1·7	+ ·26 S. 19·3	− 1·0	+ ·32 S. 12·4		− 0·9	+ ·30 N. 0·9	
	7 30	− 0·7	+ ·16 S. 15·9	− 0·5	+ ·18 S. 5·7		− 0·6	+ ·07 N. 7·1	
	8 30	− 0·6	+ ·21 S. 11·8	+ 0·2	·00 N. 0·5		+ 0·5	− ·05 N. 12·1	
	9 30	+ 0·1	+ ·16 N. 5·6	− 1·0	·00 N. 6·9		+ 0·1	− ·15 N. 16·4	
	10 30	+ 0·4	+ ·19 N. 0·6	− 0·8	− ·03 N. 12·1		− 0·5	− ·10 N. 20·6	
	11 30	− 0·2	+ ·10 N. 7·0	− 3·4	− ·12 N. 18·7		+ 0·2	− ·35 N. 22·3	
		Sun's Declination 10° N.		Sun's Declination 19° N.			Sun's Declination 23° N.		

Table VII. (*g*.) (Continued.)

Diurnal Inequality.

Apparent Solar Time of Moon's Transit A.	July.			August.			September.		
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.
	h m	m	°	m	feet.	°	m	feet.	°
P.M.	0 30	+ 0·5	− ·42	N. 19·1	+ 0·4	− ·57	N. 10·9	+ 0·3	− ·49	S. 1·3
	1 30	+ 1·3	− ·56	N. 15·6	+ 0·1	− ·53	N. 4·9	+ 1·7	− ·31	S. 6·8
	2 30	− 0·5	− ·58	N. 11·1	− 0·9	− ·52	S. 1·2	+ 0·2	− ·35	S. 12·7
	3 30	+ 1·7	− ·40	N. 4·8	+ 0·9	− ·36	S. 6·4	+ 0·6	− ·03	S. 16·5
	4 30	− 1·8	− ·50	S. 1·4	− 0·4	− ·24	S. 13·4	+ 0·3	+ ·05	S. 19·8
	5 30	+ 1·4	− ·36	S. 7·2	+ 0·3	− ·04	S. 18·9	0·0	+ ·09	S. 22·4
	6 30	− 4·7	− ·17	S. 13·0	− 1·3	+ ·08	S. 20·7	− 2·7	+ ·15	S. 23·1
	7 30	+ 0·8	− ·08	S. 17·3	+ 0·2	+ ·27	S. 22·4	− 0·6	+ ·28	S. 22·0
	8 30	− 0·7	+ ·32	S. 20·5	− 0·2	+ ·45	S. 23·0	− 0·1	+ ·39	S. 19·9
	9 30	− 1·3	+ ·33	S. 22·4	+ 0·1	+ ·49	S. 21·3	+ 0·3	+ ·40	S. 16·3
	10 30	− 1·6	+ ·43	S. 20·7	+ 0·9	+ ·61	S. 19·6	+ 0·2	+ ·48	S. 11·2
	11 30	− 0·9	+ ·53	S. 21·5	− 0·6	+ ·53	S. 15·8	− 1·3	+ ·58	S. 5·1
A.M.	0 30	− 0·6	+ ·60	S. 19·7	− 0·5	+ ·49	S. 10·6	− 0·4	+ ·44	S. 0·6
	1 30	− 1·1	+ ·51	S. 16·6	− 0·4	+ ·69	S. 5·5	− 1·9	+ ·37	N. 6·8
	2 30	+ 0·3	+ ·50	S. 11·0	+ 0·7	+ ·38	N. 0·8	− 0·9	+ ·25	N. 11·9
	3 30	− 2·7	+ ·37	S. 4·9	− 0·7	+ ·28	N. 6·6	− 0·4	+ ·10	N. 16·8
	4 30	− 0·3	+ ·37	N. 1·3	+ 0·1	+ ·21	N. 12·3	− 0·2	− ·06	N. 17·6
	5 30	+ 0·7	+ ·37	N. 6·9	− 0·4	+ ·01	N. 18·1	+ 0·1	− ·10	N. 21·9
	6 30	− 1·4	+ ·19	N. 12·4	+ 1·1	− ·12	N. 20·6	+ 2·4	− ·22	N. 23·4
	7 30	− 0·7	− ·24	N. 17·1	− 0·1	− ·28	N. 24·6	+ 0·7	− ·29	N. 22·1
	8 30	+ 0·7	− ·34	N. 21·0	+ 0·5	− ·40	N. 22·9	+ 0·1	− ·40	N. 20·1
	9 30	− 0·9	− ·28	N. 21·9	0·0	− ·55	N. 22·0	− 0·2	− ·30	N. 15·5
	10 30	+ 0·3	− ·48	N. 22·9	− 0·7	− ·46	N. 19·2	− 0·4	− ·54	N. 11·1
	11 30	− 0·7	− ·44	N. 22·2	− 0·7	− ·61	N. 15·1	+ 1·1	− ·61	N. 6·2
		Sun's Declination 21° N.			Sun's Declination 13° N.			Sun's Declination 3° N.		

	October.			November.			December.		
P.M.	0 30	+ 0·6	− ·34	S. 11·1	− 0·1	+ ·11	S. 19·6	− 0·6	+ ·34	S. 22·9
	1 30	+ 0·9	− ·15	S. 16·5	+ 0·1	+ ·19	S. 22·3	− 0·2	+ ·45	S. 22·0
	2 30	− 1·3	+ ·09	S. 20·0	+ 0·1	+ ·36	S. 23·0	− 2·4	+ ·58	S. 19·6
	3 30	− 0·3	+ ·20	S. 22·1	− 0·5	+ ·51	S. 21·9	− 2·4	+ ·62	S. 15·7
	4 30	+ 0·7	+ ·20	S. 23·2	− 1·0	+ ·59	S. 19·9	− 1·2	+ ·69	S. 10·8
	5 30	+ 1·4	+ ·53	S. 21·8	− 1·3	+ ·46	S. 16·6	− 0·6	+ ·35	S. 4·9
	6 30	− 0·3	+ ·31	S. 20·0	− 0·6	+ ·35	S. 12·3	0·0	+ ·29	N. 0·8
	7 30	− 2·0	+ ·50	S. 17·0	− 1·9	+ ·29	S. 6·2	+ 0·9	+ ·05	N. 6·2
	8 30	0·0	+ ·38	S. 12·5	− 0·1	+ ·27	S. 0·6	+ 0·4	− ·09	N. 12·4
	9 30	+ 0·5	+ ·33	S. 5·8	− 0·1	+ ·24	N. 5·1	− 0·4	− ·09	N. 17·0
	10 30	− 0·5	+ ·35	S. 0·9	− 0·3	+ ·17	N. 11·4	+ 0·6	− ·12	N. 20·0
	11 30	+ 0·2	+ ·35	N. 6·0	− 0·2	+ ·00	N. 16·6	+ 0·8	− ·19	N. 21·9
A.M.	0 30	− 0·4	+ ·54	N. 11·3	+ 0·5	− ·10	N. 19·8	+ 0·3	− ·29	N. 23·2
	1 30	− 0·2	+ ·16	N. 18·1	− 0·1	− ·26	N. 22·1	+ 0·3	− ·34	N. 22·2
	2 30	+ 0·9	− ·12	N. 20·4	+ 1·1	− ·11	N. 22·9	+ 0·4	− ·49	N. 19·0
	3 30	+ 0·1	− ·23	N. 21·9	− 0·3	− ·38	N. 22·4	+ 3·9	− ·70	N. 16·0
	4 30	− 1·2	− ·38	N. 22·9	+ 0·9	− ·54	N. 20·4	+ 3·4	− ·63	N. 11·2
	5 30	− 1·0	− ·43	N. 22·1	+ 1·4	− ·45	N. 16·2	+ 0·4	− ·36	N. 5·9
	6 30	+ 0·3	− ·26	N. 20·5	+ 0·8	− ·46	N. 12·5	0·0	− ·40	S. 0·8
	7 30	− 2·7	− ·54	N. 17·0	− 0·1	− ·43	N. 6·9	− 0·4	− ·07	S. 5·9
	8 30	− 0·4	− ·45	N. 12·3	− 0·1	− ·20	N. 0·8	− 0·3	− ·15	S. 12·3
	9 30	− 0·3	− ·39	N. 6·7	+ 2·4	− ·27	S. 4·7	+ 0·2	+ ·07	S. 15·7
	10 30	+ 0·3	− ·18	S. 0·9	+ 0·2	− ·09	S. 11·7	− 0·4	+ ·08	S. 20·3
	11 30	− 0·1	− ·45	S. 5·5	+ 0·2	− ·02	S. 16·5	− 0·6	+ ·17	S. 22·4
		Sun's Declination 9° S.			Sun's Declination 19° S.			Sun's Declination 23° S.		

Table VIII. (*h*.)

Showing a Comparison between the Semimenstrual Correction at Liverpool in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables II. and III.

Moon's Hor. Par. 57′, and Decl. 15°.

Apparent Solar Time of Moon's Transit A.	Interval. ψ + constant.		Height. h.	
	Theory.	Observation.	Theory.	Observation.
h m	h m	h m	feet.	feet.
0 0	12 21·2		17·67	
0 30	12 13·2	12 12·3	17·51	17·46
1 0	12 5·3		17·37	
1 30	11 58·7	11 57·0	17·00	16·78
2 0	11 51·0		16·58	
2 30	11 45·0	11 43·9	16·58	15·77
3 0	11 40·2		15·38	
3 30	11 38·2	11 37·0	14·70	14·40
4 0	11 38·2		13·97	
4 30	11 41·2	11 40·4	13·30	13·02
5 0	11 50·3		12·69	
5 30	12 2·8	12 2·8	12·29	12·14
6 0	12 21·2		12·15	
6 30	12 36·5	12 37·9	12·27	12·54
7 0	12 51·9		12·65	
7 30	13 1·5	13 1·7	13·25	13·81
8 0	13 4·4		13·92	
8 30	13 4·2	13 5·7	14·72	15·27
9 0	13 2·0		15·39	
9 30	12 58·2	12 58·2	16·00	16·45
10 0	12 51·6		16·59	
10 30	12 44·5	12 44·5	17·02	17·22
11 0	12 37·0		17·36	
11 30	12 29·0	12 29·0	17·52	17·66

α' = Moon's Right Ascension. α = Sun's Right Ascension.

μ = Sidereal Time.

$\mu - \alpha' = \psi$ $\alpha - \alpha' = \varphi$ $\mu - \alpha = \psi - \varphi$.

h = Height of High Water.

The columns headed "Theory" have been calculated from the expressions

$$\tan 2\psi = \frac{(A)\sin 2\varphi}{1 + (A)\cos 2\varphi}$$

$$h = D + (E)\{(A)\cos(2\psi - 2\varphi) + \cos 2\psi\}$$

$$\log(A) = 9{\cdot}56965 \qquad \log(E) = 0{\cdot}871{\cdot}30 \qquad D = 7{\cdot}46$$

Table IX. (*i.*)

Showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Tables II. and III. See Plate I.

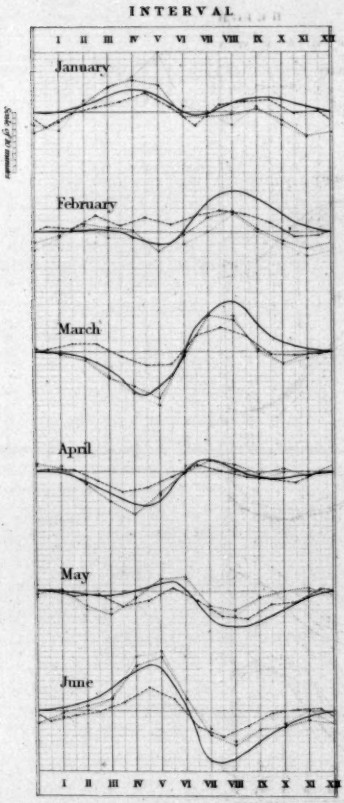
Apparent Solar Time of Moon's Transit A.	January.					February.					March.					Apparent Solar Time of Moon's Transit. A.
	d ↓		d h		Moon's Decli-nation.	d ↓		d h		Moon's Decli-nation.	d ↓		d h		Moon's Decli-nation.	
	Theory.	Obser-vation.	Theory.	Obser-vation.		Theory.	Obser-vation.	Theory.	Obser-vation.		Theory.	Obser-vation.	Theory.	Obser-vation.		
h m	m	m	feet.	feet.	°	m	m	feet.	feet.	°	m	m	feet.	feet.	°	h m
0 30	−0·2	−2·4	−·39	+·13	19	0·0	+0·9	+·44	+·72	10	−0·1	+0·3	+·74	+·60	5	0 30
1 30	+0·5	−0·2	−·05	+·37	15	+0·2	+0·5	+·60	+·65	5	−0·6	+1·1	+·59	+·34	8	1 30
2 30	+1·9	+0·9	+·26	+·42	10	+0·4	+2·9	+·55	+·58	5	−1·9	+1·1	+·26	+·12	13	2 30
3 30	+3·3	+1·8	+·44	+·66	6	0·0	+1·0	+·40	+·26	8	−4·5	−0·9	−·12	−·27	17	3 30
4 30	+3·4	+3·1	+·50	+·49	5	−1·7	+2·3	+·06	−·04	13	−7·2	−2·1	−·59	−·67	21	4 30
5 30	+1·3	+0·9	+·44	+·51	8	−1·9	+1·2	−·36	−·32	18	−3·9	−2·0	−·82	−·81	22	5 30
6 30	−0·5	−2·1	+·12	+·15	14	+2·8	+2·8	−·64	−·48	21	+4·2	+2·7	−·91	−·51	23	6 30
7 30	+0·7	+1·1	−·19	+·04	18	+6·3	+3·6	−·65	−·23	22	+7·0	+4·0	−·68	−·43	22	7 30
8 30	+2·1	+1·8	−·49	+·03	21	+6·4	+3·0	−·65	−·20	23	+6·1	+2·6	−·37	−·12	20	8 30
9 30	+2·4	+2·0	−·71	−·47	23	+4·2	+1·5	−·51	·00	22	+2·5	−0·5	+·13	+·15	15	9 30
10 30	+1·3	−0·2	−·75	−·18	23	+1·7	−0·9	−·20	+·21	19	+0·9	−0·7	+·45	+·46	11	10 30
11 30	+0·5	+0·1	−·67	−·20	22	+0·3	−0·6	+·14	+·44	15	+0·1	−0·3	+·72	+·68	6	11 30
	Sun's Decl. S. 21°, and Par. 8″·94.					Sun's Decl. S. 13°, and Par. 8″·90.					Sun's Decl. S. 2°, and Par. 8″·84.					

	April.					May.					June.					
0 30	0·0	+0·3	+·25	−·11	12	+0·1	0·0	−·63	−·35	20	+0·3	−1·7	−1·11	−·84	23	0 30
1 30	−0·8	+0·2	−·10	−·23	17	−0·2	−0·9	−·78	−·76	22	+1·0	−0·8	−·92	−·65	22	1 30
2 30	−2·8	−1·5	−·46	−·54	21	−0·7	−0·5	−·80	−·44	23	+2·5	−0·1	−·58	−·35	20	2 30
3 30	−4·6	−3·3	−·57	−·63	22	−0·4	−2·5	−·62	−·47	22	+5·6	+1·4	−·13	−·02	16	3 30
4 30	−5·8	−2·7	−·71	−·72	23	+0·7	−1·6	−·30	−·09	20	+7·7	+3·9	+·40	+·61	11	4 30
5 30	−2·4	−0·9	−·67	−·65	22	+1·6	+0·4	+·18	+·16	15	+4·1	+2·0	+·81	+·79	6	5 30
6 30	+1·7	+1·1	−·48	−·51	20	−2·1	−1·5	+·38	+·33	12	−4·1	−2·6	+·83	+·71	5	6 30
7 30	+1·5	0·0	−·10	−·17	16	−5·7	−4·0	+·51	+·25	7	−8·5	−4·2	+·54	+·13	8	7 30
8 30	−0·5	−1·0	+·24	+·13	11	−5·8	−4·4	+·42	+·10	5	−6·7	−2·4	+·07	−·18	13	8 30
9 30	−1·2	−1·3	+·48	+·34	6	−4·0	−2·1	+·29	−·10	7	−3·6	−0·3	−·33	−·44	17	9 30
10 30	−0·8	−1·9	+·55	+·40	4	−1·9	−1·9	+·02	−·35	12	−1·5	+0·2	−·73	−·80	20	10 30
11 30	−0·2	0·0	+·48	+·25	7	−0·3	+0·4	−·37	−·54	17	−0·3	+0·4	−1·00	−1.01	22	11 30
	Sun's Decl. N. 10°, and Par. 8″·76.					Sun's Decl. N. 19°, and Par. 8″·70.					Sun's Decl. N. 23°, and Par. 8″·66.					

	July.					August.					September.					
0 30	+0·4	+1·0	−·65	−·42	19	+0·3	+1·7	+·21	+·29	11	+0·1	+1·1	+·65	+·70	4	0 30
1 30	+1·9	+1·2	−·35	−·07	16	+1·4	+2·1	+·42	+·29	6	+0·1	+1·4	+·53	+·55	7	1 30
2 30	+4·5	+1·3	+·07	+·30	11	+2·3	+2·2	+·45	+·51	5	−0·6	+1·0	+·26	+·27	12	2 30
3 30	+7·3	+5·0	+·40	+·41	6	+2·9	+3·2	+·39	+·39	7	−3·0	−0·6	−·14	−·03	17	3 30
4 30	+7·7	+5·5	+·62	+·73	5	+2·1	+1·7	+·14	+·12	13	−5·0	−2·1	−·46	−·43	20	4 30
5 30	+3·4	+3·1	−·66	+·64	8	−0·2	+0·6	−·12	−·25	17	−3·0	−2·5	−·73	−·58	22	5 30
6 30	−2·7	−0·5	+·40	+·17	13	+1·3	+1·9	−·48	−·38	21	+3·3	+1·6	−·82	−·28	23	6 30
7 30	−4·1	−0·6	·00	+·06	17	+4·1	+3·1	−·67	−·36	23	+6·3	+3·1	−·64	−·23	22	7 30
8 30	−1·9	+0·7	−·53	−·40	21	+3·6	+3·8	−·67	−·46	23	+4·6	+2·3	−·39	−·01	20	8 30
9 30	−0·8	+2·9	−·76	−·55	22	+2·2	+2·3	−·61	−·22	22	+1·8	+0·8	−·01	+·24	16	9 30
10 30	−0·3	−0·7	−·97	−·62	23	+0·5	−0·1	−·36	−·17	19	+0·3	−1·1	+·36	+·54	11	10 30
11 30	−0·1	+1·2	−·93	−·63	22	0·0	−0·2	−·12	+·10	16	−0·1	−0·2	+·61	+·45	6	11 30
	Sun's Decl. N. 21°, and Par. 8″·66.					Sun's Decl. N. 13°, and Par. 8″·70.					Sun's Decl. N. 3°, and Par. 8″·76.					

	October.					November.					December.					
0 30	−0·2	+0·1	+·42	+·40	11	−0·3	−0·4	−·45	−·19	20	−1·0	−0·4	−·85	−·49	23	0 30
1 30	−1·3	−0·9	+·07	+·24	16	−1·4	−1·1	−·62	−·38	22	−0·5	−2·1	−·70	−·43	22	1 30
2 30	−3·6	−3·3	−·30	−·22	20	−2·7	−2·5	−·70	−·30	23	+0·1	−1·6	−·36	−·18	19	2 30
3 30	−6·3	−3·0	−·55	−·34	22	−3·2	−3·1	−·58	−·22	22	+1·5	+1·0	−·09	+·30	16	3 30
4 30	−7·6	−5·7	−·76	−·44	23	−2·4	−3·5	−·38	−·10	20	+2·4	+1·4	+·28	+·53	11	4 30
5 30	−3·3	−3·4	−·76	−·52	22	−0·1	−1·0	−·06	+·29	16	+1·9	+1·0	+·55	+·76	7	5 30
6 30	+2·6	−0·2	−·57	−·34	20	−0·6	−0·6	+·22	+·46	12	−2·0	−2·3	+·61	+·67	5	6 30
7 30	+3·8	+1·7	−·23	+·07	17	−2·6	−4·0	+·43	+·43	7	−4·3	−3·7	+·45	+·43	7	7 30
8 30	+1·5	−0·2	+·21	+·32	12	−3·0	−3·0	+·46	+·52	5	−3·1	−2·7	+·17	+·26	12	8 30
9 30	+0·2	−0·7	+·51	+·56	7	−2·2	−4·6	+·43	+·44	6	−1·2	−0·6	−·12	+·01	16	9 30
10 30	−0·1	−1·8	+·65	+·41	4	−0·7	−1·5	+·18	+·30	12	+0·1	−0·0	−·51	−·15	20	10 30
11 30	0·0	−0·5	+·60	+·79	7	−0·3	−0·1	−·19	+·01	17	+0·3	−0·5	−·74	−·38	22	11 30
	Sun's Decl. S. 9°, and Par. 8″·84.					Sun's Decl. S. 19°, and Par. 8″·90.					Sun's Decl. S. 23°, and Par. 8″·94.					

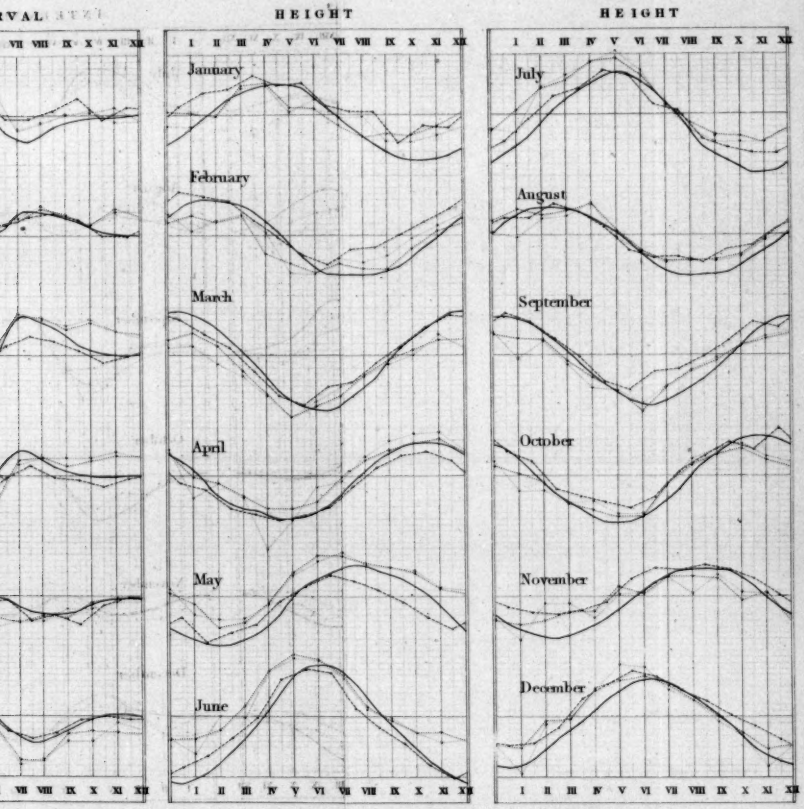
Diagram showing a comparison between the
deduced from Bernoullis' Theory and from



Theory ——— *Obs*

In this comparison of the London and Liverpool
and the London height corrections have been m

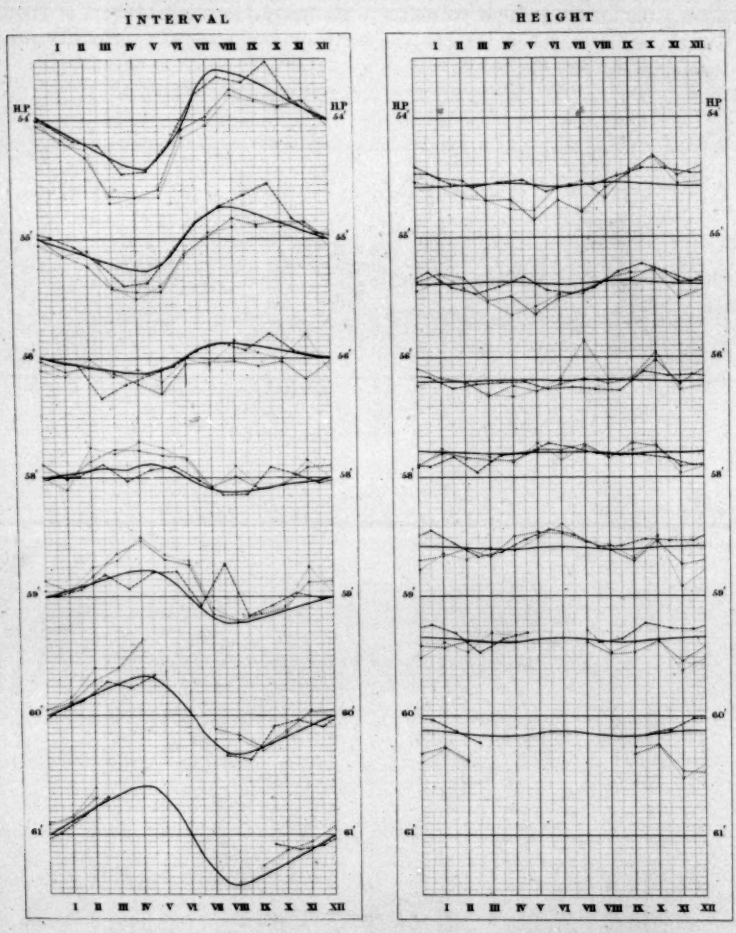
he Calendar Month Inequality in the Interval and in the Height of high water as
n Observations at the London and Liverpool Docks.—See Tables IX and XXIII. p. 118 and 192.



Observation { ..... London 19 Years (13,370 Observations)
..... 35 Years (24,392 Observations)
..... Liverpool 19 Years (13,391 Observations)

ol results, the London corrections have been shifted to the left half an hour agreeably to the remark p.100
n multiplied by 1·7. The abscissa represents the apparent solar time of moon's transit A.

Diagram showing a comparison between the Moon's Parallax inequality in the Interval and in the Height of high water as deduced from Bernoulli's Theory and from Observations at the London and Liverpool Docks. See Tables X and XXIV. p. 119. and p. 133.



Theory ——— *Observation*

London 19 Years
35 Years
Liverpool

In this comparison of the London and Liverpool results the London corrections have been shifted to the left half an hour agreeably to the remark in p. 100 and the London height corrections have been multiplied by 1.7. The abscissa represents the apparent solar time of moon's transit A.

Table X. (*j*.)

Showing the Moon's Parallax Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Table V. See Plate II.

Apparent Solar Time of Moon's Transit A.	H. P. 54′.				H. P. 55′.				Apparent Solar Time of Moon's Transit A.
	d ↓		d h		d ↓		d h		
	Theory.	Observation.	Theory.	Observation.	Theory.	Observation.	Theory.	Observation.	
h m	m	m	feet.	feet.	m	m	feet.	feet.	h m
0 30	− 1·0	− 0·7	−1·16	−0·92	− 0·7	+ 0·2	− ·79	− ·59	0 30
1 30	− 3·0	− 3·4	−1·14	−1·10	− 2·0	− 1·3	− ·77	− ·83	1 30
2 30	− 5·3	− 4·1	−1·11	−1·16	− 3·4	− 4·0	− ·75	− ·93	2 30
3 30	− 7·4	− 9·1	−1·09	−1·08	− 4·9	− 7·9	− ·74	− ·82	3 30
4 30	− 8·3	− 8·7	−1·10	−1·04	− 5·3	− 7·3	− ·75	− ·66	4 30
5 30	− 4·0	− 3·9	−1·15	−1·24	− 2·6	− 2·4	− ·78	− ·88	5 30
6 30	+ 4·0	+ 4·0	−1·15	−1·11	+ 2·6	+ 2·9	− ·78	− ·92	6 30
7 30	+ 8·3	+ 7·1	−1·10	−1·11	+ 5·3	+ 5·5	− ·75	− ·84	7 30
8 30	+ 7·4	+ 6·2	−1·09	−0·97	+ 4·9	+ 7·3	− ·74	− ·58	8 30
9 30	+ 5·3	+ 9·6	−1·11	−0·84	+ 3·4	+ 9·3	− ·75	− ·45	9 30
10 30	+ 3·0	+ 3·4	−1·14	−0·86	+ 2·0	+ 3·4	− ·77	− ·59	10 30
11 30	+ 1·0	+ 0·6	−1·16	−0·92	+ 0·7	+ 0·1	− ·79	− ·74	11 30
	H. P. 56′.				H. P. 57′.				
0 30	− 0·3	− 0·2	− ·40	− ·31	0·0	0·0	0·0	0·0	0 30
1 30	− 1·0	− 1·0	− ·40	− ·52	0·0	0·0	0·0	0·0	1 30
2 30	− 1·7	− 0·7	− ·39	− ·44	0·0	0·0	0·0	0·0	2 30
3 30	− 2·3	− 4·3	− ·38	− ·41	0·0	0·0	0·0	0·0	3 30
4 30	− 2·5	− 2·7	− ·38	− ·40	0·0	0·0	0·0	0·0	4 30
5 30	− 1·2	− 1·4	− ·39	− ·53	0·0	0·0	0·0	0·0	5 30
6 30	+ 1·2	+ 1·3	− ·39	− ·50	0·0	0·0	0·0	0·0	6 30
7 30	+ 2·5	+ 2·5	− ·38	− ·44	0·0	0·0	0·0	0·0	7 30
8 30	+ 2·3	+ 1·4	− ·38	− ·46	0·0	0·0	0·0	0·0	8 30
9 30	+ 1·7	+ 4·1	− ·39	− ·23	0·0	0·0	0·0	0·0	9 30
10 30	+ 1·0	+ 1·3	− ·40	− ·30	0·0	0·0	0·0	0·0	10 30
11 30	+ 0·3	+ 0·3	− ·40	− ·28	0·0	0·0	0·0	0·0	11 30
	H. P. 58′.				H. P. 59′.				
0 30	+ 0·3	+ 0·1	+ ·42	+ ·18	+ 0·6	+ 0·2	+ ·85	+1·11	0 30
1 30	+ 0·9	+ 0·2	+ ·41	+ ·34	+ 1·8	+ 1·3	+ ·84	+0·90	1 30
2 30	+ 1·6	+ 2·2	+ ·40	+ ·09	+ 3·0	+ 3·8	+ ·81	+0·68	2 30
3 30	+ 2·2	− 0·6	+ ·39	+ ·37	+ 4·2	+ 1·4	+ ·80	+0·76	3 30
4 30	+ 2·3	+ 1·3	+ ·41	+ ·40	+ 4·4	+ 4·1	+ ·81	+0·97	4 30
5 30	+ 1·1	+ 1·9	+ ·42	+ ·57	+ 2·0	+ 4·2	+ ·84	+1·12	5 30
6 30	− 1·1	− 0·4	+ ·42	+ ·50	− 2·0	− 1·5	+ ·84	+1·05	6 30
7 30	− 2·3	− 2·9	+ ·41	+ ·40	− 4·4	− 5·4	+ ·81	+0·85	7 30
8 30	− 2·2	− 2·8	+ ·39	+ ·38	− 4·2	− 3·1	+ ·80	+0·86	8 30
9 30	− 1·6	+ 1·8	+ ·40	+ ·38	− 3·0	− 1·5	+ ·81	+0·97	9 30
10 30	− 0·9	+ 0·6	+ ·41	+ ·31	− 1·8	+ 0·7	+ ·84	+0·95	10 30
11 30	− 0·3	− 0·9	+ ·42	+ ·20	− 0·6	− 0·2	+ ·85	+0·94	11 30
	H. P. 60′.				H. P. 61′.				
0 30	+ 0·9	+ 0·9	+1·30	+1·51	+ 1·2	+ 0·3	+1·76	+1·93	0 30
1 30	+ 2·7	+ 2·5	+1·28	+1·38	+ 3·5	+ 3·3	+1·73	+1·75	1 30
2 30	+ 4·5	+ 5·7	+1·24	+1·07	+ 5·9	+ 6·3	+1·68	+1·54	2 30
3 30	+ 6·1	+ 4·7	+1·22	+1·27	+ 7·9		+1·66		3 30
4 30	+ 6·2	+ 6·9	+1·24	+1·69	+ 8·0		+1·69		4 30
5 30	+ 2·9		+1·28		+ 3·7		+1·74		5 30
6 30	− 2·9		+1·28		− 3·7		+1·74		6 30
7 30	− 6·2	− 6·7	+1·24	+1·22	− 8·0		+1·69		7 30
8 30	− 6·1	− 7·4	+1·22	+1·27	− 7·9		+1·66		8 30
9 30	− 4·5	− 1·9	+1·24	+1·54	− 5·9	− 1·7	+1·68	+1·69	9 30
10 30	− 2·7	− 0·8	+1·28	+1·46	− 3·5	− 2·4	+1·73	+1·78	10 30
11 30	− 0·9	− 2·0	+1·30	+1·45	− 1·2	− 1·8	+1·76	+1·96	11 30

Table XI. (*k*.)

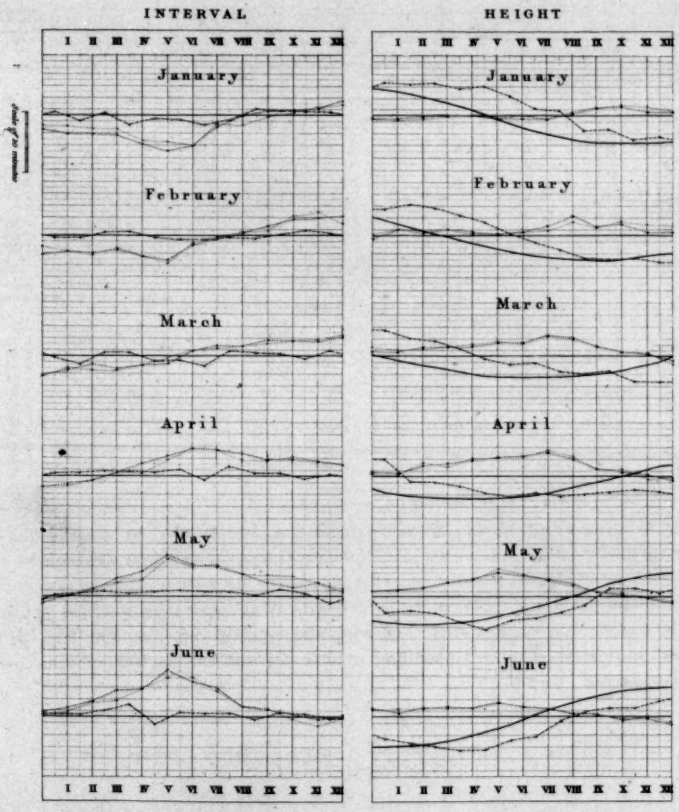
Showing the Diurnal Inequality in the Interval and in the Height in the first six months of the year, for the Moon's Transit A, P.M. See Plate III.

Apparent Solar Time of Moon's Transit A.	January.			February.			March.			Apparent Solar Time of Moon's Transit A.
	d ψ.	d h.	Moon's Declination.	d ψ.	d h.	Moon's Declination.	d ψ.	d h.	Moon's Declination.	
	Observation.	Observation.		Observation.	Observation.		Observation.	Observation.		
P.M.	m	feet.	°	m	feet.	°	m	feet.	°	h m
h m										
0 30	+ 0·6	+ ·56	S. 19	− 0·3	+ ·43	S. 10	− 0·2	+ ·43	N. 1	0 30
1 30	− 1·0	+ ·53	S. 15	− 0·4	+ ·52	S. 4	− 1·4	+ ·30	N. 7	1 30
2 30	+ 0·4	+ ·54	S. 10	+ 0·5	+ ·46	N. 2	+ 0·5	+ ·27	N. 13	2 30
3 30	− 1·6	+ ·47	S. 5	+ 0·6	+ ·34	N. 7	+ 0·5	+ ·11	N. 17	3 30
4 30	− 0·6	+ ·50	N. 2	− 0·4	+ ·21	N. 13	− 0·9	− ·05	N. 20	4 30
5 30	− 0·9	+ ·33	N. 8	− 0·7	+ ·02	N. 18	− 0·8	− ·15	N. 22	5 30
6 30	− 1·6	+ ·11	N. 13	− 0·7	− ·11	N. 21	− 1·8	− ·12	N. 23	6 30
7 30	− 0·9	+ ·09	N. 18	− 0·4	− ·23	N. 23	+ 0·8	− ·26	N. 22	7 30
8 30	+ 1·0	− ·25	N. 21	− 0·6	− ·39	N. 23	+ 0·5	− ·30	N. 20	8 30
9 30	+ 0·7	− ·23	N. 22	+ 0·1	− ·40	N. 22	+ 0·2	− ·26	N. 16	9 30
10 30	+ 0·6	− ·40	N. 22	+ 0·9	− ·39	N. 19	− 0·5	− ·41	N. 11	10 30
11 30	+ 0·5	− ·36	N. 22	+ 0·4	− ·43	N. 15	+ 0·9	− ·43	N. 5	11 30
	July.			August.			September.			
	April.			May.			June.			
0 30	+ 0·4	+ ·29	N. 12	+ 0·2	− ·27	N. 20	+ 0·4	− ·38	N. 23	0 30
1 30	+ 0·5	− ·09	N. 17	+ 0·6	− ·24	N. 22	+ 0·3	− ·44	N. 22	1 30
2 30	+ 0·8	− ·10	N. 20	+ 0·9	− ·29	N. 23	+ 1·0	− ·50	N. 19	2 30
3 30	+ 0·7	− ·18	N. 22	+ 0·5	− ·42	N. 22	+ 2·0	− ·58	N. 16	3 30
4 30	+ 0·6	− ·28	N. 23	+ 0·8	− ·55	N. 20	− 1·2	− ·57	N. 11	4 30
5 30	+ 1·0	− ·34	N. 22	+ 1·0	− ·40	N. 16	+ 0·7	− ·40	N. 5	5 30
6 30	− 0·7	− ·29	N. 20	+ 0·8	− ·37	N. 12	+ 0·5	− ·33	S. 1	6 30
7 30	+ 1·3	− ·33	N. 16	+ 0·8	− ·27	N. 6	+ 0·6	− ·07	S. 7	7 30
8 30	+ 0·4	− ·30	N. 12	+ 0·2	− ·13	S. 1	− 0·5	+ ·09	S. 12	8 30
9 30	+ 0·4	− ·27	N. 6	+ 1·2	+ ·14	S. 6	+ 0·7	+ ·14	S. 17	9 30
10 30	− 0·4	− ·22	S. 1	+ 0·5	+ ·14	S. 12	+ 0·4	+ ·14	S. 22	10 30
11 30	+ 0·2	− ·26	S. 6	− 1·1	+ ·08	S. 17	− 0·4	+ ·26	S. 22	11 30
	October.			November.			December.			

The tide depending on the Moon's Transit A.M. for the last six months has the same inequality and the same signs as the above; and in the first six months A.M. and the last six months P.M. the same values obtain, but with a contrary sign.

The quantities in the columns headed "Observation" have been obtained by taking the mean of January and July, February and August, &c. A.M. and P.M., as explained in p. 100. The corresponding moon's declination has been obtained in a similar manner.

Diagram showing the Diurnal inequality in the Interval and in the Height of high water as deduced from Observations at the London and Liverpool Docks. *See Tables XI. & XXV. p. 200 and p. 234.*



*Theory*_____ *Observation*

{ *London* 35 *Years*
{ *London* 19 *Years*
{ *Liverpool*

In this comparison of the London and Liverpool results the London corrections have been shifted to the left half an hour agreeably to the remark in p. 200 and the signs reversed. The abscissa represents the apparent solar time of moons transit A.P.M.

Diagram showing the Establishment of the Port of Liverpool. — *See Table XIV. p. 121.*

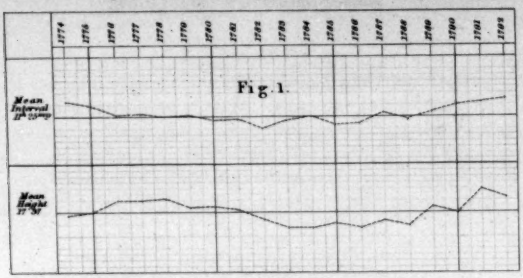


Diagram showing the Establishment of the Port of London. — *See Table XXX. p. 136.*

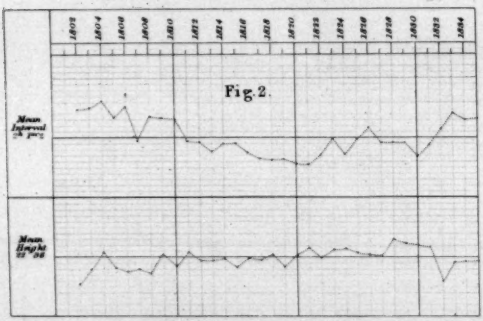


Diagram showing the errors of calculated Heights of High water for May & June and the corresponding Heights of the Barometer at Liverpool & London. — *See. p. 104.*

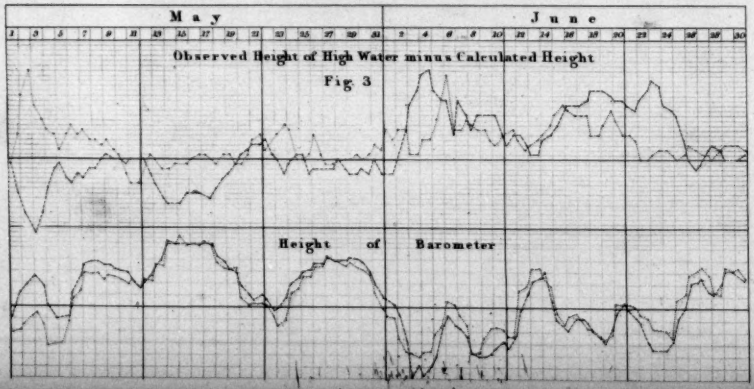


Table XII. (*l*.)

Showing the Interval and Height of High Water at the Liverpool Docks, with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax (for the mean of all the Moon's Transits A occurring between 0^h and 1^h) for every year from 1774 to 1792.

Year.	Number of Obser. vations.	Moon's Transit A.	Interval.	Height.	Moon's Declination.	Moon's Hor. Par.	Sun's Declination.
		h m	h m	ft. in.	°	′	°
1774	58	0 27·5	12 15·4	18 0·9	10·9	57·6	14·0
1775	57	0 29·3	12 14·3	17 10·7	11·6	57·4	14·0
1776	59	0 30·8	12 12·3	17 11·7	12·7	57·3	14·5
1777	59	0 32·2	12 12·1	17 10·1	14·0	57·3	14·6
1778	52	0 30·6	12 12·1	17 10·0	15·1	57·4	14·6
1779	58	0 29·3	12 12·7	17 8·9	15·8	57·6	14·4
1780	62	0 29·3	12 11·9	17 6·7	16·8	57·3	14·4
1781	59	0 29·5	12 11·9	17 6·0	16·9	57·3	14·2
1782	53	0 29·6	12 10·5	17 4·4	17·2	57·4	13·6
1783	57	0 30·3	12 11·6	17 4·7	16·8	57·7	13·7
1784	59	0 29·4	12 13·5	17 2·0	17·3	57·3	14·1
1785	55	0 31·3	12 11·4	17 4·1	16·5	57·4	14·3
1786	62	0 29·6	12 11·3	17 3·1	16·1	57·3	14·4
1787	62	0 28·3	12 13·1	17 7·0	15·1	57·6	14·1
1788	55	0 30·0	12 11·9	17 5·8	14·0	57·4	14·7
1789	54	0 28·1	12 13·7	17 11·1	12·4	57·4	14·0
1790	56	0 28·2	12 14·8	17 9·8	11·9	57·3	14·4
1791	55	0 29·6	12 14·8	18 3·8	11·7	57·6	14·9
1792	62	0 29·3	12 15·4	18 1·0	11·2	57·3	14·4

Table XIII. (*m*.)

Interpolated from Table XII. by reducing each quantity to Moon's Transit A ($0^h 30^m$), and correcting the quantities for deviation from mean Declinations and Parallax.

Moon's Transit A = 0^h 30^m.

Year.	Interval.	Height.
	h m	feet.
1774	12 14·7	17·40
1775	12 14·1	17·46
1776	12 12·5	17·65
1777	12 12·7	17·65
1778	12 13·3	17·67
1779	12 13·5	17·53
1780	12 11·7	17·55
1781	12 11·8	17·49
1782	12 10·4	17·34
1783	12 11·7	17·20
1784	12 12·3	17·19
1785	12 11·0	17·27
1786	12 11·2	17·20
1787	12 12·9	17·32
1788	12 11·9	17·24
1789	12 13·2	17·55
1790	12 14·3	17·45
1791	12 14·7	17·84
1792	12 15·2	17·69

Table XIV. (*n*.)

Showing the *Establishment* of the Port of Liverpool obtained from Table XIII. by altering the argument from Transit A to Transit D, and reducing it to $0^h 0^m$ from $0^h 30^m$. Moon's Hor. Par. 57′, and Decl. 15°.

Moon's Transit D = 0^h 0^m.

Year.	Interval.*	Height.
	h m	feet.
1774	11 28·3	17·31
1775	11 27·7	17·37
1776	11 26·1	17·56
1777	11 26·3	17·56
1778	11 25·9	17·58
1779	11 26·1	17·44
1780	11 25·3	17·46
1781	11 25·4	17·40
1782	11 24·0	17·25
1783	11 25·3	17·11
1784	11 25·9	17·10
1785	11 24·6	17·18
1786	11 24·8	17·11
1787	11 26·5	17·23
1788	11 25·5	17·15
1789	11 26·8	17·46
1790	11 27·9	17·36
1791	11 28·3	17·75
1792	11 28·8	17·60

* i. e. *Establishment*.

Results deduced from Observations made at

LONDON.

Table XV. (*a*.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B in each month of the year, from 24,592 observations made at the London Docks, between the 1st of September 1801 and the 31st of August 1836.

January.						February.					
Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.	Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
*	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
167	0 30·1	3 3·9	22 9·1	19	57·1	165	0 30·2	3 5·5	22 10·8	10	57·3
166	1 29·7	2 49·7	22 6·3	15	57·3	169	1 28·3	2 50·9	22 8·2	6	57·2
186	2 29·1	2 38·1	21 11·4	10	57·1	172	2 28·4	2 38·5	22 1·4	5	57·2
188	3 29·6	2 30·5	21 4·8	6	56·9	175	3 29·1	2 27·1	21 3·8	8	56·9
189	4 29·9	2 29·9	20 6·0	5	56·9	166	4 29·9	2 22·9	20 0·9	14	56·7
194	5 30·1	2 41·7	19 7·3	9	56·9	152	5 29·6	2 32·5	19 2·2	18	56·6
179	6 29·3	3 9·4	19 7·0	13	56·9	151	6 29·7	3 9·6	19 1·5	21	56·7
176	7 29·3	3 40·7	20 1·8	18	57·0	144	7 29·0	3 43·8	19 11·2	22	56·8
169	8 29·9	3 52·8	20 11·9	21	57·2	142	8 28·7	3 56·9	20 9·4	23	56·9
154	9 29·2	3 50·7	21 8·9	22	57·2	149	9 28·6	3 51·1	21 6·7	21	57·1
162	10 29·1	3 37·8	22 3·2	23	57·2	157	10 30·0	3 37·2	22 4·1	19	57·2
158	11 29·1	3 19·9	22 7·4	21	57·2	157	11 30·8	3 20·6	22 9·5	15	57·3
Sun's Declination 21°.						Sun's Declination 13°.					
March.						April.					
178	0 29·3	3 7·8	22 10·8	5	57·2	170	0 29·0	3 9·0	23 0·0	12	57·3
185	1 29·9	2 50·5	22 8·2	8	57·1	165	1 28·9	2 51·8	22 5·8	17	57·2
176	2 29·6	2 35·1	22 0·0	13	56·9	161	2 29·0	2 34·7	21 10·5	20	57·0
170	3 28·7	2 21·6	20 11·8	17	56·8	154	3 29·2	2 21·0	20 11·2	22	56·8
167	4 28·8	2 16·7	19 9·9	20	56·7	155	4 29·1	2 15·4	19 10·4	23	56·7
166	5 30·0	2 28·4	18 10·8	22	56·7	157	5 29·0	2 31·4	19 1·6	22	56·5
163	6 29·6	3 9·5	19 0·4	23	56·8	165	6 28·8	3 9·6	19 2·9	20	56·7
163	7 28·9	3 47·4	19 10·0	22	56·9	174	7 29·2	3 43·5	20 1·9	16	56·7
177	8 29·0	3 58·3	20 11·8	19	57·2	175	8 29·2	3 54·5	21 3·3	11	57·0
175	9 29·6	3 50·5	21 10·7	16	57·3	186	9 30·0	3 49·8	22 2·1	6	57·2
180	10 30·3	3 36·7	22 6·6	11	57·4	175	10 29·1	3 39·5	22 9·4	5	57·3
187	11 30·5	3 23·7	22 11·2	6	57·5	179	11 28·9	3 23·0	23 1·4	7	57·4
Sun's Declination 3°.						Sun's Declination 10°.					
May.						June.					
166	0 30·0	3 6·9	22 10·3	20	57·6	157	0 28·2	3 5·8	22 6·8	23	57·3
161	1 29·8	2 51·6	22 5·6	22	57·5	156	1 29·3	2 51·2	22 3·8	22	57·4
165	2 30·0	2 34·5	21 8·1	23	57·2	163	2 29·5	2 38·1	21 10·6	19	57·3
163	3 30·1	2 22·5	20 10·4	22	56·9	172	3 30·0	2 28·7	21 2·9	16	57·0
170	4 29·7	2 21·8	20 1·3	20	56·8	176	4 30·0	2 30·6	20 7·2	11	56·8
171	5 29·4	2 37·5	19 8·2	16	56·6	186	5 29·6	2 45·0	20 0·9	7	56·7
183	6 29·4	3 12·0	19 9·7	12	56·5	184	6 29·8	3 12·0	19 11·9	5	56·6
192	7 29·7	3 39·3	20 6·7	7	56·7	179	7 29·3	3 37·5	20 7·2	7	56·8
188	8 29·8	3 50·6	21 5·0	5	56·8	178	8 29·4	3 47·7	21 2·5	12	56·9
183	9 30·0	3 49·3	22 2·0	7	57·0	161	9 29·0	3 47·6	21 10·0	17	57·0
176	10 29·2	3 39·0	22 8·0	12	57·2	161	10 29·3	3 36·4	22 3·4	20	57·1
175	11 28·9	3 24·4	22 9·9	17	57·3	149	11 28·8	3 22·4	22 6·6	22	57·2
Sun's Declination 19°.						Sun's Declination 23°.					

TABLE XV. (*a*.) (Continued.)

July.

Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declina. tion.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
159	0 30·0	3 6·6	22 7·4	19	57·2
167	1 28·4	2 52·7	22 6·3	16	57·1
186	2 29·7	2 40·7	22 2·1	11	57·1
186	3 29·7	2 34·7	21 6·0	6	57·1
198	4 29·3	2 33·4	20 8·7	5	56·9
191	5 29·3	2 46·6	20 1·0	8	56·9
185	6 29·0	3 14·4	19 10·4	13	56·8
175	7 29·2	3 39·2	20 4·0	17	56·9
165	8 29·0	3 52·3	21 0·4	20	56·9
162	9 29·2	3 50·6	21 8·5	22	57·1
154	10 29·6	3 39·5	22 2·8	23	57·1
164	11 30·5	3 22·3	22 5·6	22	57·2

Sun's Declination 21°.

August.

Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declina. tion.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
179	0 30·6	3 10·0	22 10·3	11	57·0
178	1 29·7	2 53·4	22 7·9	6	56·9
181	2 28·8	2 42·0	22 1·6	5	57·0
184	3 29·0	2 31·2	21 4·4	8	56·9
182	4 29·4	2 27·5	20 5·4	13	56·7
174	5 29·0	2 38·5	19 6·4	17	56·7
175	6 29·2	3 11·7	19 4·4	20	56·9
168	7 30·2	3 43·8	20 0·2	22	57·0
162	8 30·0	3 57·7	20 11·2	23	57·1
165	9 29·9	3 53·7	21 8·4	22	57·2
165	10 30·0	3 40·6	22 3·6	19	57·3
171	11 29·8	3 28·2	22 8·5	16	57·3

Sun's Declination 14°.

September.

179	0 29·4	3 10·9	22 11·9	4	57·4
182	1 29·7	2 54·0	22 7·4	7	57·1
173	2 29·8	2 38·3	22 0·9	12	57·0
163	3 29·0	2 27·3	21 0·9	17	56·9
164	4 29·3	2 17·5	19 11·4	20	56·7
157	5 30·0	2 28·8	19 1·7	22	56·7
155	6 29·9	3 11·6	18 11·5	23	56·7
160	7 29·1	3 47·7	19 10·8	22	56·8
164	8 28·4	3 59·7	20 11·5	20	57·0
174	9 29·5	3 54·5	21 10·5	16	57·2
176	10 30·1	3 44·1	22 6·7	12	57·3
177	11 29·8	3 27·7	22 11·1	6	57·4

Sun's Declination 4°.

October.

175	0 30·0	3 10·6	22 11·1	12	57·5
173	1 29·7	2 53·0	22 6·4	16	57·4
166	2 29·8	2 34·9	21 10·3	20	57·0
164	3 28·9	2 21·3	20 10·5	22	56·9
157	4 28·9	2 12·5	19 10·8	23	56·8
164	5 29·2	2 24·8	19 0·8	22	56·6
170	6 30·6	3 5·6	19 1·0	20	56·6
167	7 30·6	3 44·6	20 1·2	17	56·6
180	8 29·7	3 56·5	21 2·5	12	56·9
187	9 28·7	3 52·3	22 1·0	7	57·1
190	10 29·0	3 41·4	22 8·9	5	57·4
189	11 30·1	3 26·8	22 11·3	7	57·5

Sun's Declination 9°.

November.

162	0 28·9	3 7·3	22 6·8	20	57·4
158	1 28·9	2 49·2	22 3·2	22	57·4
152	2 29·2	2 33·2	21 9·4	23	57·3
163	3 29·3	2 23·4	21 0·6	22	57·2
161	4 29·4	2 18·8	20 0·6	20	56·8
169	5 28·7	2 32·2	19 6·6	17	56·6
185	6 30·0	3 7·0	19 6·0	12	56·7
177	7 30·2	3 37·4	20 4·8	7	56·8
182	8 29·9	3 50·6	21 3·6	5	56·8
174	9 29·4	3 46·8	22 0·9	7	57·1
176	10 29·6	3 37·9	22 5·0	11	57·2
159	11 29·3	3 23·6	22 9·5	16	57·3

Sun's Declination 18°.

December.

157	0 29·4	3 4·6	22 6·2	22	57·3
159	1 29·6	2 47·6	22 2·8	22	57·3
165	2 30·0	2 33·4	21 11·1	20	57·2
175	3 30·4	2 27·9	21 1·7	16	57·1
185	4 29·7	2 26·7	20 5·6	11	56·9
197	5 29·8	2 38·9	19 10·6	7	56·9
181	6 29·2	3 8·2	19 10·2	5	56·7
193	7 28·4	3 34·3	20 5·2	8	56·7
184	8 29·2	3 46·4	21 3·3	12	56·9
167	9 29·3	3 47·6	21 10·7	17	57·1
160	10 28·6	3 36·9	22 3·5	20	57·2
163	11 28·5	3 21·5	22 5·1	22	57·2

Sun's Declination 23°.

TABLE XVI. (*b*.) (Interpolated from Table XV.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit B, and the Time of High Water at the London Docks for each month in the year.

Apparent Solar Time of Moon's Transit B.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 30	3 3·8	3 5·3	3 7·5	3 8·5	3 6·6	3 5·1	3 6·5	3 10·2	3 10·5	3 10·3	3 6·8	3 4·2	3 7·1
1 30	2 49·3	2 50·3	2 50·4	2 51·4	2 51·1	2 50·6	2 52·2	2 53·4	2 53·8	2 52·5	2 48·5	2 47·2	2 50·9
2 30	2 37·7	2 37·8	2 35·2	2 34·5	2 34·1	2 37·4	2 40·6	2 41·8	2 38·3	2 34·9	2 32·5	2 33·0	2 36·5
3 30	2 30·9	2 27·5	2 22·3	2 21·7	2 22·9	2 28·7	2 34·3	2 31·6	2 27·7	2 21·7	2 22·7	2 27·5	2 26·7
4 30	2 29·3	2 24·2	2 18·2	2 16·9	2 22·8	2 31·5	2 33·9	2 28·9	2 18·9	2 13·6	2 19·8	2 27·2	2 23·8
5 30	2 42·1	2 34·4	2 29·7	2 34·0	2 39·5	2 46·5	2 47·4	2 40·3	2 30·1	2 26·9	2 34·6	2 39·4	2 37·1
6 30	3 10·0	3 10·3	3 10·0	3 10·7	3 13·1	3 12·8	3 15·2	3 12·3	3 12·2	3 6·0	3 7·5	3 9·1	3 10·8
7 30	3 40·7	3 43·9	3 47·5	3 43·4	3 39·2	3 37·5	3 39·3	3 43·8	3 47·8	3 44·1	3 37·3	3 34·4	3 41·7
8 30	3 53·1	3 56·8	3 58·6	3 54·5	3 50·3	3 47·6	3 52·2	3 57·8	3 59·6	3 56·4	3 50·3	3 46·3	3 53·6
9 30	3 50·9	3 50·9	3 50·8	3 50·1	3 49·3	3 47·4	3 50·6	3 54·0	3 54·7	3 52·2	3 46·8	3 47·6	3 50·4
10 30	3 37·8	3 37·4	3 37·1	3 39·5	3 39·0	3 36·3	3 39·5	3 40·8	3 44·3	3 41·5	3 38·0	3 36·8	3 39·0
11 30	3 19·7	3 20·8	3 23·8	3 22·7	3 24·1	3 22·1	3 22·4	3 28·1	3 27·6	3 26·8	3 23·4	3 21·1	3 23·6

TABLE XVII. (*c.*) (Interpolated from Table XV.)

Showing the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, in each month of the year.

Apparent Solar Time of Moon's Transit B.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0 30	22·74	22·84	22·86	22·94	22·75	22·50	22·58	22·86	22·91	22·82	22·49	22·46	22·73
1 30	22·45	22·62	22·66	22·43	22·37	22·23	22·48	22·68	22·60	22·45	22·17	22·15	22·44
2 30	21·91	22·05	22·01	21·85	21·63	21·80	22·16	22·11	22·07	21·86	21·69	21·88	21·92
3 30	21·42	21·33	21·00	20·96	20·90	21·24	21·46	21·37	21·08	20·87	20·98	21·12	21·14
4 30	20·53	20·16	19·87	19·93	20·16	20·66	20·74	20·52	20·02	19·94	20·10	20·49	20·22
5 30	19·64	19·30	18·99	19·27	19·80	20·16	20·19	19·61	19·23	19·18	19·66	19·91	19·55
6 30	19·61	19·20	19·07	19·31	19·94	20·09	19·92	19·40	19·03	19·17	19·57	19·93	19·52
7 30	20·17	19·99	19·87	20·25	20·64	20·65	20·37	20·02	19·96	20·18	20·44	20·55	20·26
8 30	20·95	20·83	20·97	21·29	21·46	21·24	21·08	20·91	21·00	21·24	21·34	21·31	21·14
9 30	21·72	21·57	21·84	22·13	22·17	21·85	21·71	21·66	21·84	22·09	22·06	21·89	21·89
10 30	22·24	22·30	22·47	22·73	22·64	22·27	22·22	22·24	22·50	22·67	22·38	22·27	22·42
11 30	22·59	22·74	22·84	23·05	22·77	22·55	22·44	22·66	22·85	22·85	22·74	22·40	22·71

In reducing the above Tables from Table XV., the quantities have been corrected for the deviation from a mean Horizontal Parallax (57′).

TABLE XVIII. (*d.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, for every minute of her Horizontal Parallax.

Hor. Par. 54′.						Hor. Par. 55′.					
Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.	Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.
	h m	h m	ft. in.	°	°		h m	h m	ft. in.	°	°
409	0 29·2	3 6·2	22 2·7	14·5	14·3	280	0 30·3	3 6·5	22 3·4	14·4	14·5
417	1 29·3	2 48·3	21 10·2	14·5	14·3	289	1 29·2	2 48·9	22 1·4	14·4	14·9
383	2 29·0	2 30·7	21 4·0	14·9	14·7	322	2 29·9	3 32·3	21 6·3	14·0	14·5
375	3 28·9	2 14·5	20 5·8	14·7	15·0	343	3 29·2	2 19·1	20 7·8	14·8	15·0
331	4 29·2	2 10·7	19 6·9	14·9	14·9	385	4 30·0	2 15·0	19 8·0	14·8	15·0
293	5 28·9	2 25·4	18 8·4	15·5	14·9	412	5 30·1	2 29·2	18 11·3	14·9	15·2
298	6 30·0	3 7·3	18 8·7	15·0	15·3	412	6 28·9	3 6·0	18 11·1	15·0	15·2
334	7 30·6	3 42·9	19 4·2	15·1	15·5	385	7 29·3	3 42·6	19 8·6	14·9	14·7
364	8 29·6	3 57·5	20 5·7	14·8	14·9	347	8 29·6	3 55·7	20 8·9	14·4	15·1
402	9 29·3	3 54·3	21 3·8	14·7	14·8	293	9 29·1	3 53·5	21 5·3	14·6	14·3
406	10 29·3	3 41·5	21 11·0	14·3	14·4	294	10 30·1	3 42·1	21 11·9	14·5	14·3
410	11 29·8	3 24·8	22 2·4	14·1	14·2	290	11 29·8	3 24·6	22 3·9	14·2	14·3
Hor. Par. 56′.						Hor. Par. 57′.					
219	0 29·2	3 5·9	22 7·0	14·3	14·2	166	0 30·0	3 6·2	22 8·4	14·5	14·3
203	1 29·5	2 49·5	22 3·0	14·2	14·3	182	1 30·1	2 51·5	22 4·8	14·9	14·4
228	2 29·0	2 35·3	21 8·5	14·6	15·1	201	2 30·2	2 36·6	21 10·8	14·9	14·6
234	3 30·2	2 24·9	20 10·4	14·8	14·9	215	3 29·9	2 27·2	21 2·9	14·7	14·3
259	4 29·4	2 19·7	20 0·5	14·9	15·1	233	4 29·9	2 24·0	20 3·6	15·0	15·2
276	5 29·4	2 32·9	19 4·2	14·7	15·2	256	5 29·2	2 37·2	19 8·3	15·5	15·0
271	6 29·8	3 8·0	19 3·3	14·8	15·4	254	6 29·4	3 8·8	19 6·3	15·0	15·3
253	7 28·8	3 41·4	20 0·3	14·7	14·9	240	7 28·6	3 42·0	20 2·7	15·1	15·2
254	8 28·7	3 56·3	20 10·5	15·1	14·5	210	8 29·1	3 52·8	21 1·5	15·0	14·5
226	9 29·0	3 52·3	21 7·6	14·7	14·6	202	9 29·0	3 51·3	21 10·2	14·8	14·5
217	10 28·5	3 40·1	22 2·9	14·5	14·5	187	10 27·7	3 39·6	22 3·0	14·7	14·9
206	11 29·0	3 25·3	22 5·8	14·6	14·7	182	11 30·0	3 21·7	22 9·0	14·0	14·0

Table XVIII. (*d.*) (Continued.)

Hor. Par. 58′.						Hor. Par. 59′.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.
	h m	h m	ft. in.	°	°		h m	h m	ft. in.	°	°
158	0 31·1	3 8·0	22 9·9	14·6	14·5	180	0 28·2	3 9·4	23 0·8	14·2	14·3
173	1 29·1	2 51·5	22 7·9	14·5	14·2	182	1 30·0	2 52·4	22 10·0	14·1	14·4
197	2 29·7	2 39·7	22 1·3	14·7	14·9	204	2 30·4	2 40·1	22 4·5	14·7	14·8
208	3 29·6	2 32·0	21 4·8	15·0	14·4	282	3 31·0	2 34·3	21 7·8	14·8	14·4
268	4 28·9	2 28·3	20 5·9	14·8	15·1	404	4 32·3	2 33·6	20 10·0	14·9	14·6
288	5 29·9	2 41·7	19 11·0	14·9	14·8	533	5 30·0	2 44·1	20 3·5	15·0	15·1
290	6 28·5	3 11·7	19 8·8	15·4	14·9	518	6 29·6	3 14·2	20 1·5	15·1	15·0
254	7 28·1	3 40·2	20 6·2	14·6	15·0	381	7 27·6	3 38·6	20 8·6	14·7	15·0
220	8 27·9	3 53·0	21 3·2	15·3	14·7	258	8 27·8	3 49·3	21 6·9	14·2	14·7
184	9 28·3	3 49·4	22 1·7	14·9	14·8	216	9 28·6	3 48·4	22 2·5	14·6	14·2
171	10 30·0	3 38·1	22 6·9	14·7	14·5	176	10 29·7	3 47·5	22 10·0	14·2	14·2
174	11 29·4	3 23·7	22 10·0	14·7	14·7	164	11 28·0	3 24·6	23 0·4	14·4	14·4
Hor. Par. 60′.						Hor. Par. 61′.					
200	0 29·3	3 7·4	23 4·2	14·9	14·8	390	0 29·7	3 8·0	23 5·8	14·3	14·5
223	1 30·5	2 53·7	23 0·7	14·4	14·3	301	1 28·4	2 55·0	23 3·4	14·6	14·5
327	2 31·5	2 42·4	22 7·2	14·3	14·2	172	2 22·8	2 44·2	22 9·0	15·1	15·5
380	3 28·1	2 35·7	21 11·9	14·5	15·0	10	3 13·7	3 14·2	21 9·3	16·6	15·7
180	4 22·7	2 35·4	21 2·0	15·2	15·6						
12	5 7·6	2 50·2	20 4·5	17·2	16·2						
28	6 42·7	3 25·4	20 2·9	16·4	14·2						
207	7 35·0	3 41·0	21 0·1	15·1	15·5						
388	8 31·1	3 49·0	21 9·0	15·5	15·9	18	8 47·2	3 52·4	21 10·2	16·7	17·9
309	9 26·8	3 46·3	22 5·7	14·0	13·8	195	9 36·4	3 44·7	22 8·8	15·3	15·5
229	10 28·2	3 37·0	22 11·6	14·4	14·2	342	10 31·2	3 36·2	23 2·0	14·3	14·6
203	11 28·7	3 22·7	23 3·4	14·0	14·0	386	11 30·1	3 22·8	23 4·7	14·3	14·3

Table XIX. (*e.*) (Interpolated from Table XVIII.)

Apparent Solar Time of Moon's Transit B.	H. P. 54′.		H. P. 55′.		H. P. 56′.		H. P. 57′.	
	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.
h m	h m	feet.	h m	feet.	h m	feet.	h m	feet.
0 30	3 6·1	22·22	3 6·4	22·26	3 5·7	22·55	3 6·2	22·68
1 30	2 48·2	21·83	2 48·7	22·06	2 49·4	22·21	2 51·6	22·40
2 30	2 30·5	21·31	2 32·2	21·52	2 35·0	21·67	2 36·7	21·90
3 30	2 14·5	20·44	2 19·1	20·62	2 24·9	20·86	2 27·4	21·23
4 30	2 10·9	19·52	2 15·0	19·74	2 19·6	20·03	2 23·9	20·30
5 30	2 26·2	18·69	2 29·0	18·94	2 32·9	19·34	2 37·9	19·66
6 30	3 7·3	18·72	3 6·5	18·94	3 8·1	19·26	3 9·1	19·51
7 30	3 42·8	19·34	3 42·7	19·73	3 41·7	20·03	3 42·3	20·23
8 30	3 57·5	20·47	3 56·0	20·73	3 56·0	20·90	3 52·5	21·14
9 30	3 54·2	21·32	3 53·4	21·44	3 52·1	21·64	3 51·0	21·84
10 30	3 41·3	21·90	3 42·1	21·98	3 39·7	22·24	3 39·2	22·27
11 30	3 24·8	22·16	3 24·6	22·29	3 25·0	22·46	3 21·7	22·71
	H. P. 58′.		H. P. 59′.		H. P. 60′.		H. P. 61′.	
0 30	3 8·3	22·80	3 8·9	23·03	3 7·2	23·35	3 7·9	23·47
1 30	2 51·3	22·63	2 52·4	22·79	2 53·8	23·05	2 54·6	23·24
2 30	2 39·6	22·09	2 40·2	22·37	2 42·7	22·60	2 42·8	22·61
3 30	2 32·2	21·39	2 34·5	21·62	2 35·4	21·92		
4 30	2 28·4	20·46	2 33·3	20·88	2 36·7	21·00		
5 30	2 41·7	19·92	2 44·1	20·29				
6 30	3 12·5	19·77	3 14·4	20·12				
7 30	3 40·8	20·54	3 39·3	20·76	3 40·0	20·92		
8 30	3 52·6	21·33	3 49·4	21·60	3 49·2	21·74		
9 30	3 49·1	22·17	3 48·1	22·18	3 45·6	22·45	3 45·9	22·61
10 30	3 38·0	22·56	3 37·4	22·80	3 36·6	22·97	3 36·6	23·13
11 30	3 23·5	22·82	3 24·1	23·02	3 22·4	23·24	3 22·8	23·34

In forming the above Table, the quantities have been corrected for deviations from mean Declinations.

TABLE XX. (*f*.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, A.M. and P.M.

January.

A.M.						P.M.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.
	h m	h m	ft. in.	° ′	′		h m	h m	ft. in.	° ′	′
84	0 30·8	3 3·0	22 8·3	N. 19·0	57.1	83	0 29·3	3 5·0	22 10·2	S. 19·1	57·2
82	1 30·4	2 48·0	22 5·5	N. 15·3	57·4	84	1 29·0	2 51·3	22 7·2	S. 15·1	57·1
94	2 29·3	2 35·5	21 11·2	N. 10·0	57·1	92	2 28·8	2 40·8	21 11·7	S. 9·9	57·2
91	3 29·2	2 28·2	21 5·3	N. 4·1	56·9	97	3 29·9	2 32·7	21 4·7	S. 4·0	56·9
97	4 29·6	2 24·2	20 5·9	S. 1·7	56·8	92	4 30·3	2 33·6	20 6·1	N. 2·1	56·9
95	5 29·2	2 35·5	19 6·3	S. 8·2	56·9	89	5 31·0	2 47·7	19 8·1	N. 8·2	56·8
91	6 28·9	3 3·9	19 5·8	S. 13·1	56·9	88	6 29·8	3 15·0	19 8·2	N. 13·5	56·9
89	7 30·1	3 38·0	20 0·9	S. 17·7	57·0	87	7 28·5	3 43·4	20 2·7	N. 17·5	57·0
83	8 30·4	3 51·2	21 0·1	S. 20·8	57·2	86	8 29·5	3 54·3	20 11·7	N. 20·7	57·1
73	9 28·1	3 50·7	21 10·8	S. 22·4	57·2	81	9 30·2	3 50·7	21 7·1	N. 22·5	57·2
83	10 28·0	3 38·0	22 4·9	S. 22·5	57·1	79	10 30·3	3 37·5	22 1·4	N. 22·9	57·4
79	11 28·3	3 19·2	22 8·0	S. 21·2	57·2	79	11 30·0	3 20·5	22 6·8	N. 21·1	57·2
Sun's Declination S. 21°.											

February.

86	0 29·2	3 3·5	22 9·2	N. 10·0	57·2	79	0 31·2	3 7·8	23 0·4	S. 9·8	57·3
81	1 27·5	2 48·6	22 10·0	N. 4·3	57·2	88	1 29·1	2 52·9	22 6·3	S. 4·0	57·2
91	2 28·0	2 36·0	22 1·5	S. 1·8	57·1	81	2 28·9	3 41·1	22 1·3	N. 2·2	57·2
86	3 28·9	2 27·1	21 4·6	S. 7·8	57·0	89	3 29·3	2 27·1	21 3·2	N. 8·0	56·9
84	4 30·1	2 19·8	20 1·1	S. 13·6	56·6	82	4 29·7	2 26·4	20 0·7	N. 13·8	56·8
77	5 30·0	2 28·2	19 1·3	S. 18·1	56·5	75	5 29·0	2 37·1	19 3·1	N. 18·1	56·6
73	6 29·6	3 6·9	19 1·6	S. 20·6	56·7	78	6 29·9	3 12·0	19 1·3	N. 20·6	56·7
74	7 28·5	3 41·3	20 0·2	S. 22·4	56·7	70	7 29·7	3 46·2	19 10·1	N. 22·5	56·9
73	8 29·4	3 54·4	20 11·9	S. 22·5	56·8	69	8 28·0	3 59·5	20 6·9	N. 22·8	56·9
71	9 28·5	3 50·9	21 8·6	S. 21·4	57·1	78	9 28·7	3 51·2	21 5·0	N. 21·4	57·1
80	10 29·9	3 37·8	22 6·6	S. 18·5	57·3	77	10 30·1	3 36·6	22 1·5	N. 18·5	57·2
81	11 31·6	3 22·1	22 9·8	S. 15·0	57·3	76	11 30·0	3 19·0	22 9·2	N. 14·7	57·3
Sun's Declination S. 13°.											

March.

91	0 29·1	3 5·7	22 11·6	S. 1·1	57·0	87	0 29·5	3 9·4	22 9·9	N. 1·7	57·3
89	1 28·7	2 50·6	22 9·0	S. 6·9	57·1	96	1 31·0	2 50·5	22 7·5	N. 7·4	57·1
94	2 28·8	2 34·8	22 1·4	S. 12·6	56·9	82	2 30·5	2 35·6	21 10·3	N. 13·0	56·9
84	3 29·1	2 20·9	21 1·8	S. 17·3	56·8	86	3 28·4	2 22·2	20 9·8	N. 17·3	56·8
83	4 29·1	2 15·9	20 0·5	S. 20·2	56·7	84	4 28·4	2 17·6	19 7·3	N. 20·4	56·6
84	5 30·0	2 27·1	19 1·6	S. 21·9	56·7	82	5 30·0	2 29·9	18 7·9	N. 22·3	56·7
84	6 30·2	3 9·0	19 2·6	S. 22·8	56·8	79	6 28·9	3 10·2	18 10·5	N. 22·7	56·8
80	7 29·7	3 46·9	20 2·3	S. 21·8	56·9	83	7 28·1	3 47·9	19 6·0	N. 21·7	56·9
86	8 28·7	3 57·8	21 3·0	S. 19·2	57·2	91	8 29·3	3 58·7	20 8·8	N. 19·0	57·2
89	9 28·9	3 51·9	22 0·4	S. 15·7	57·3	86	9 30·3	3 49·0	21 9·0	N. 15·3	57·3
89	10 30·5	3 38·2	22 7·8	S. 10·6	57·6	91	10 30·2	3 35·2	22 5·4	N. 10·4	57·3
96	11 30·5	3 24·4	22 11·9	S. 4·7	57·4	91	11 30·3	3 23·0	22 10·5	N. 4·7	57·6
Sun's Declination S. 2°.											

April.

84	0 29·0	3 8·2	23 0·8	S. 12·2	57·3	86	0 29·2	3 9·6	22 11·3	N. 12·4	57·3
83	1 29·0	2 51·7	22 6·9	S. 16·4	57·3	82	1 28·8	2 52·1	22 4·6	N. 17·0	57·2
79	2 28·9	2 35·0	22 0·6	S. 20·0	57·0	82	2 29·0	2 34·6	21 8·5	N. 19·9	57·0
78	3 29·0	2 22·6	21 1·4	S. 21·8	56·8	76	3 29·1	2 19·8	20 9·0	N. 21·9	56·9
79	4 30·2	2 17·9	20 1·8	S. 22·6	56·7	76	4 28·0	2 12·6	19 6·8	N. 22·6	56·7
79	5 30·8	2 34·7	19 5·1	S. 21·9	56·6	78	5 27·2	2 28·0	18 10·0	N. 21·8	56·5
78	6 29·1	3 11·6	19 7·3	S. 20·0	56·7	87	6 28·4	3 7·9	18 10·7	N. 19·7	56·7
89	7 28·9	3 44·6	20 6·0	S. 15·7	56·7	85	7 29·6	3 42·1	19 9·7	N. 15·9	56·7
87	8 29·3	3 56·4	21 6·3	S. 11·5	57·0	88	8 29·1	3 52·6	21 0·3	N. 11·0	56·9
95	9 29·9	3 50·4	22 4·0	S. 5·4	57·2	91	9 30·0	3 49·3	22 0·2	N. 5·3	57·1
83	10 28·2	3 40·6	22 10·5	N. 0·6	57·4	92	10 29·9	3 38·6	27 8·4	S. 0·8	57·3
92	11 28·5	3 23·9	23 1·3	N. 6·3	57·4	87	11 29·3	3 22·1	23 1·4	S. 6·9	57·4
Sun's Declination N. 19°.											

TABLE XX. (*f.*) (Continued.)

May.

A.M. Number of Obser-vations.	A.M. Apparent Solar Time of Moon's Transit B.	A.M. Interval between the Moon's Transit and the Time of high water.	A.M. Height of Tide.	A.M. Moon's Declina-tion.	A.M. Horizontal Parallax.	P.M. Number of Obser-vations.	P.M. Apparent Solar Time of Moon's Transit B.	P.M. Interval between the Moon's Transit and the Time of high water.	P.M. Height of Tide.	P.M. Moon's Declina-tion.	P.M. Horizontal Parallax.
	h m	h m	ft. in.	° ′	′ ″		h m	h m	ft. in.	° ′	′ ″
81	0 29·3	3 7·5	22 11·3	S. 19·7	57·7	85	0 30·7	3 6·4	22 9·3	N. 19·9	57·5
82	1 28·7	2 51·7	22 7·4	S. 21·9	57·5	79	1 31·0	2 51·5	22 3·8	N. 21·7	57·5
83	2 29·4	2 37·2	21 10·3	S. 22·5	57·2	82	2 30·6	2 31·8	21 5·9	N. 22·8	57·2
80	3 29·4	2 26·6	21 2·1	S. 21·8	56·9	83	3 30·9	2 18·4	20 7·6	N. 21·8	56·9
86	4 29·2	2 24·8	20 4·4	S. 19·9	56·8	84	4 30·3	2 18·6	19 10·1	N. 19·7	56·9
87	5 29·4	2 43·5	20 0·2	S. 16·3	56·7	84	5 29·3	2 31·4	19 4·2	N. 16·4	56·6
90	6 29·7	3 16·3	20 1·8	S. 11·7	56·5	93	6 29·2	3 7·7	19 5·8	N. 11·4	56·5
95	7 29·0	3 42·5	20 9·8	S. 5·9	56·7	97	7 30·5	3 36·1	20 3·5	N. 5·7	56·7
94	8 28·7	3 52·9	21 7·0	N. 0·0	56·8	94	8 30·9	3 48·4	21 3·1	S. 0·2	56·8
94	9 30·2	3 50·8	22 3·3	N. 6·1	57·0	89	9 29·8	3 47·7	22 0·7	S. 6·4	57·0
91	10 30·4	3 39·2	22 8·2	N. 12·0	57·3	85	10 28·0	3 38·7	22 7·7	S. 11·6	57·2
85	11 30·1	3 25·5	22 9·8	N. 16·9	57·4	90	11 27·6	3 23·6	22 9·9	S. 16·6	57·3

Sun's Declination N. 19°.

June.

77	0 26·3	3 7·7	22 8·3	S. 23·0	57·3	80	0 30·1	3 3·8	22 5·3	N. 22·7	57·3
79	1 27·4	2 54·3	22 5·3	S. 21·9	57·5	77	1 31·2	2 48·0	22 2·2	N. 21·8	57·2
83	2 28·4	2 41·7	22 0·3	S. 19·2	57·3	80	2 30·7	2 34·3	21 9·0	N. 19·3	57·3
86	3 29·3	2 33·6	21 4·5	S. 16·3	56·9	86	3 30·7	2 23·9	21 1·4	N. 15·8	57·1
90	4 30·2	2 35·9	20 9·4	S. 11·6	56·9	86	4 29·7	2 25·2	20 4·8	N. 11·0	56·8
94	5 30·4	2 52·3	20 1·9	S. 5·9	56·6	99	5 28·8	2 37·5	19 11·8	N. 5·9	56·8
90	6 30·4	3 18·2	20 2·2	N. 0·7	56·7	94	6 29·2	3 6·1	19 9·7	S. 0·3	56·5
91	7 29·5	3 41·3	20 8·5	N. 6·3	56·7	88	7 29·1	3 33·7	20 5·8	S. 6·8	56·9
87	8 29·1	3 47·9	21 3·0	N. 12·1	56·9	92	8 29·7	3 47·5	21 2·1	S. 11·9	56·9
82	9 28·4	3 47·4	21 10·0	N. 16·5	57·0	79	9 29·7	3 47·7	21 10·0	S. 17·2	57·0
81	10 28·9	3 36·1	22 3·0	N. 20·0	57·1	80	10 29·7	3 36·7	22 3·9	S. 19·9	57·1
76	11 29·2	3 21·6	22 6·4	N. 22·1	57·4	73	11 28·3	3 23·3	22 6·7	S. 21·9	57·1

Sun's Declination N. 23°.

July.

82	0 30·0	3 9·9	22 7·6	S. 19·1	57·1	77	0 30·2	3 2·9	22 7·2	N. 19·3	57·3
80	1 28·3	2 57·8	22 7·0	S. 15·9	57·3	87	1 28·5	2 48·0	22 5·8	N. 15·4	57·0
97	2 30·3	2 44·3	22 2·4	S. 11·0	57·1	89	2 28·9	2 36·8	22 1·8	N. 11·0	57·1
90	3 30·3	2 39·5	21 6·1	S. 4·8	57·1	96	3 29·1	2 30·2	21 5·9	N. 4·9	57·1
98	4 28·8	2 38·6	20 8·8	N. 0·9	56·9	100	4 29·8	2 28·3	20 8·7	S. 1·4	56·9
96	5 28·3	2 52·3	20 1·4	N. 7·0	56·9	95	5 30·4	2 40·8	20 0·7	S. 7·1	56·9
92	6 28·3	3 18·9	19 10·6	N. 12·6	56·8	93	6 29·7	3 9·9	19 10·3	S. 12·6	56·8
88	7 28·1	3 40·3	20 4·2	N. 16·4	56·8	87	7 30·0	3 38·4	20 3·8	S. 17·1	57·0
82	8 28·2	3 51·5	20 11·7	N. 20·1	56·9	83	8 29·7	3 53·1	21 1·3	S. 20·1	56·9
80	9 28·2	3 49·2	21 7·7	N. 22·3	57·1	82	9 30·1	3 52·0	21 9·3	S. 22·3	57·1
79	10 29·2	3 37·6	22 1·5	N. 23·1	57·1	75	10 30·1	3 41·4	22 4·2	S. 22·8	57·1
83	11 31·1	3 19·4	22 4·6	N. 21·4	57·1	81	11 29·9	3 25·2	22 6·6	S. 21·6	57·3

Sun's Declination N. 21°.

August.

92	0 30·1	3 13·9	22 10·0	S. 10·9	57·0	87	0 31·1	3 6·0	22 10·7	N. 10·4	57·0
87	1 29·1	2 57·1	22 7·3	S. 5·2	56·9	91	1 30·3	2 49·9	22 8·6	N. 4·8	56·9
91	2 28·2	2 45·3	22 0·5	N. 0·8	57·0	90	2 29·4	2 38·6	22 2·8	S. 1·2	57·0
92	3 27·9	2 36·1	21 3·0	N. 7·1	56·8	92	3 30·2	2 26·2	21 5·8	S. 7·4	56·9
93	4 29·2	2 31·7	20 4·0	N. 12·4	56·8	89	4 29·6	2 23·3	20 6·9	S. 12·6	56·7
88	5 29·2	2 43·1	19 5·7	N. 17·2	56·9	86	5 28·9	2 33·8	19 7·2	S. 17·4	56·5
87	6 29·6	3 13·0	19 3·2	N. 20·2	56·9	88	6 28·7	3 10·5	19 5·6	S. 20·3	56·9
86	7 31·4	3 42·6	19 9·7	N. 22·2	57·0	82	7 29·0	3 44·7	20 2·8	S. 22·3	57·0
81	8 31·9	3 54·6	20 9·0	N. 22·3	57·1	81	8 28·3	4 1·1	21 1·6	S. 22·7	57·1
82	9 31·5	3 50·2	21 6·6	N. 21·7	57·3	83	9 28·3	3 57·2	21 10·2	S. 21·7	57·2
80	10 31·4	3 35·2	22 1·7	N. 19·4	57·3	85	10 28·8	3 45·7	22 5·4	S. 19·3	57·3
87	11 30·5	3 24·0	22 7·3	N. 15·8	57·2	84	11 29·1	3 32·5	22 9·8	S. 15·7	57·3

Sun's Declination N. 14°.

TABLE XX. (*f*.) (Continued.)

September.

A.M.						P.M.					
Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.	Number of Obser. vations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.
	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
83	0 50·0	3 15·7	22 11·0	S. 0·3	57·4	96	0 28·8	3 6·7	23 0·8	N. 0·1	57·4
93	1 29·0	2 58·7	22 6·3	N. 6·6	57·1	89	1 30·5	2 49·1	22 8·5	S. 6·6	57·1
86	2 29·4	2 40·7	21 11·1	N. 11·9	57·1	87	2 30·2	2 36·0	22 2·7	S. 12·3	56·9
83	3 29·0	2 31·0	20 11·2	N. 16·5	56·9	80	3 29·1	2 23·5	21 2·7	S. 16·8	56·9
82	4 30·0	2 19·8	19 9·2	N. 20·1	56·7	82	4 28·7	2 15·2	20 1·6	S. 20·1	56·7
81	5 31·7	2 30·2	18 10·7	N. 22·1	56·6	76	5 28·3	2 27·0	19 4·8	S. 22·0	56·7
76	6 30·9	3 10·0	18 7·8	N. 22·8	56·7	79	6 27·8	3 14·4	19 3·1	S. 23·0	56·7
78	7 29·5	3 43·1	19 6·4	N. 22·1	56·9	82	7 28·7	3 52·1	20 2·9	S. 22·0	56·7
82	8 28·4	3 55·8	20 7·2	N. 19·7	57·1	82	8 28·4	4 3·6	21 3·8	S. 19·9	56·9
88	9 29·9	3 50·5	21 8·6	N. 16·4	57·2	86	9 29·1	3 58·6	22 0·5	S. 16·1	57·2
89	10 30·6	3 41·0	22 5·7	N. 11·0	57·3	87	10 29·5	3 47·5	22 7·6	S. 11·9	57·3
82	11 29·3	3 23·7	22 10·7	N. 6·1	57·3	95	11 30·2	3 31·1	22 11·4	S. 5·4	57·5

Sun's Declination N. 3°.

October.

86	0 29·8	3 14·1	22 10·1	N. 11·5	57·5	89	0 30·2	3 7·5	23 0·1	S. 11·5	57·5
89	1 29·9	2 55·0	22 6·4	N. 15·6	57·4	84	1 29·5	2 51·0	22 6·4	S. 16·3	57·4
84	2 30·9	2 36·2	21 7·5	N. 19·8	56·9	82	2 28·5	2 33·6	22 1·2	S. 19·2	57·1
80	3 30·0	2 21·8	20 7·5	N. 22·0	56·9	84	3 27·8	2 20·9	21 1·3	S. 21·8	56·9
77	4 29·3	2 11·2	19 7·6	N. 22·7	56·8	80	4 28·4	2 13·8	20 1·8	S. 22·7	56·8
81	5 29·0	2 20·4	18 9·0	N. 22·2	56·7	83	5 29·3	2 29·1	19 4·4	S. 22·0	56·6
87	6 30·9	2 58·0	18 9·3	N. 19·6	56·7	83	6 30·4	3 13·3	19 4·8	S. 20·0	56·6
83	7 31·6	3 37·8	19 8·1	N. 16·9	56·6	84	7 29·7	3 51·3	20 6·3	S. 16·6	56·6
92	8 30·3	3 51·4	20 10·6	N. 12·6	56·8	88	8 29·1	4 1·9	21 6·7	S. 12·4	56·9
90	9 28·8	3 47·8	21 11·3	N. 7·0	57·0	97	9 28·5	3 56·5	22 2·5	S. 7·0	57·1
98	10 28·8	3 36·6	22 8·2	N. 0·9	57·4	92	10 29·2	3 46·5	22 9·9	S. 0·3	57·4
95	11 29·7	3 23·4	22 11·1	S. 5·6	57·6	94	11 30·3	3 30·6	22 11·6	N. 5·7	57·3

Sun's Declination S. 9°.

November.

81	0 30·5	3 8·5	22 5·2	N. 19·8	57·5	81	0 27·2	3 6·2	22 8·4	S. 19·2	57·4
80	1 31·5	2 48·7	22 1·7	N. 21·9	57·4	78	1 26·3	2 49·7	22 4·7	S. 21·7	57·4
75	2 31·8	2 32·6	21 7·3	N. 22·6	57·3	77	2 26·7	2 33·7	21 11·5	S. 22·5	57·3
81	3 30·9	2 21·4	20 9·1	N. 21·8	57 ·7	82	3 27·7	2 25·4	21 4·0	S. 22·1	57·2
79	4 30·5	2 14·4	19 8·1	N. 20·4	56·9	82	4 28·4	1 23·1	20 4·9	S. 19·8	56·8
85	5 29·5	2 24·7	19 1·0	N. 17·0	56·6	84	5 27·9	2 39·8	20 0·2	S. 17·0	56·6
91	6 30·2	3 0·4	19 1·6	N. 12·1	56·7	94	6 29·8	3 13·3	19 10·3	S. 12·1	56·7
87	7 29·7	3 30·6	20 0·7	N. 6·4	56·8	90	7 30·6	3 44·1	20 8·5	S. 6·2	56·8
94	8 29·7	3 45·8	21 0·7	N. 0·5	56·8	88	8 30·1	3 55·9	21 6·8	S. 0·1	56·8
85	9 29·6	3 43·7	22 0·1	S. 5·6	57·1	89	9 29·3	3 49·7	22 1·6	N. 5·8	57·1
89	10 29·0	3 35·6	22 4·4	S. 11·2	57·2	87	10 30·2	3 40·3	22 5·6	N. 11·6	57·2
78	11 28·3	3 20·6	22 9·7	S. 16·2	57·4	81	11 30·2	3 26·6	22 9·3	N. 16·1	57·3

Sun's Declination S. 18°.

December.

81	0 28·6	3 4·9	22 4·7	N. 22·4	57·4	76	0 30·2	3 4·4	22 7·9	S. 22·4	57·2
78	1 29·6	2 47·3	22 1·9	N. 23·2	57·3	81	1 29·6	2 48·0	22 3·7	S. 21·8	57·3
82	2 29·2	2 32·2	21 9·3	N. 19·6	57·3	83	2 30·7	2 34·6	22 0·8	S. 19·6	57·1
89	3 29·5	2 24·3	21 0·1	N. 16·0	57·1	86	3 31·4	2 31·6	21 3·4	S. 16·0	57·1
91	4 29·2	2 22·7	20 4·5	N. 11·5	57·0	94	4 30·2	2 30·6	20 6·8	S. 10·9	56·9
101	5 29·9	2 30·9	19 6·7	N. 5·5	56·8	96	5 29·6	2 47·3	20 9·7	S. 5·3	56·9
89	6 29·7	3 2·0	19 7·7	S. 0·2	56·7	92	6 28·7	3 14·3	20 0·7	N. 0·7	56·7
97	7 28·4	3 28·4	20 3·7	S. 6·6	56·7	96	7 28·3	3 40·4	20 6·8	N. 6·7	56·7
89	8 28·5	3 43·0	21 2·6	S. 12·1	57·0	95	8 29·8	3 49·6	21 4·1	N. 12·3	56·9
87	9 28·7	3 45·5	21 9·7	S. 16·4	57·1	80	9 29·9	3 49·9	21 11·8	N. 16·9	57·1
82	10 29·9	3 35·4	22 4·5	S. 19·7	57·1	78	10 27·3	3 38·4	22 2·5	N. 19·7	57·2
81	11 30·0	3 21·2	22 5·3	S. 22·3	57·3	82	11 27·0	3 21·7	22 4·9	N. 22·1	57·1

Sun's Declination S. 23°.

In the above Table the Lower Transits have been incorporated with the upper, the declinations are those corresponding to the Upper Transits.

Table XXI. (*g*.)

Showing the Diurnal Inequality at London, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table XVI., and the Difference between the Height of High Water and the Height in Table XVII.

Diurnal Inequality.										
Apparent Solar Time of Moon's Transit B.	January.			February.			March.			
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	
h m	m	feet.	°	m	feet.	°	m	feet.	°	
P.M. 0 30	+ 0·9	+ ·07	S. 19·1	+ 2·6	+ ·14	S. 9·8	+ 1·6	− ·09	N. 1·7	
1 30	+ 1·7	+ ·12	S. 15·1	+ 2·2	− ·15	S. 4·0	+ 0·2	− ·05	N. 7·4	
2 30	+ 2·5	·00	S. 9·9	+ 2·7	·00	N. 2·2	+ 0·7	− ·12	N. 13·0	
3 30	+ 2·2	·00	S. 4·0	0·0	− ·05	N. 8·0	+ 0·5	− ·20	N. 17·3	
4 30	+ 4·6	+ ·02	N. 2·1	+ 3·1	− ·04	N. 13·8	+ 1·5	− ·19	N. 20·4	
5 30	+ 5·9	+ ·12	N. 8·2	+ 4·9	+ ·07	N. 18·1	+ 1·5	− ·24	N. 22·3	
6 30	+ 5·3	+ ·09	N. 13·5	+ 2·1	− ·02	N. 20·6	+ 1·0	− ·15	N. 22·7	
7 30	+ 3·0	+ ·08	N. 17·5	+ 2·3	− ·12	N. 22·5	+ 0·7	− ·31	N. 21·7	
8 30	+ 1·3	+ ·02	N. 20·7	+ 2·5	− ·19	N. 22·8	+ 0·4	− ·26	N. 19·0	
9 30	+ 0·1	− ·17	N. 22·5	+ 0·2	− ·14	N. 21·4	− 1·3	− ·16	N. 15·3	
10 30	+ 0·1	− ·20	N. 22·9	− 0·6	− ·22	N. 18·5	− 1·7	− ·08	N. 10·4	
11 30	+ 0·8	− ·05	N. 21·1	− 1·8	− ·02	N. 14·7	− 0·5	− ·07	N. 4·7	
A.M. 0 30	− 0·7	− ·07	N. 19·0	− 2·1	− ·11	N. 10·0	− 2·0	+ ·11	S. 1·1	
1 30	− 1·6	− ·06	N. 15·3	− 2·5	+ ·14	N. 4·3	− 0·2	+ ·05	S. 6·9	
2 30	− 2·5	− ·01	N. 10·0	− 2·4	+ ·01	S. 1·8	− 0·4	+ ·10	S. 12·6	
3 30	− 2·3	+ ·03	N. 4·1	− 0·4	+ ·03	S. 7·8	− 0·7	+ ·18	S. 17·3	
4 30	− 4·2	+ ·01	S. 1·7	− 2·6	+ ·04	S. 13·6	− 0·8	+ ·23	S. 20·2	
5 30	− 5·8	− ·09	S. 8·2	− 4·1	− ·04	S. 18·1	− 1·3	+ ·23	S. 21·9	
6 30	− 5·3	− ·10	S. 13·1	− 2·7	·00	S. 20·6	− 0·8	+ ·19	S. 22·8	
7 30	− 2·7	− ·10	S. 17·7	− 2·5	+ ·13	S. 22·4	− 0·6	+ ·35	S. 21·8	
8 30	− 1·6	− ·01	S. 20·8	− 2·7	+ ·21	S. 22·5	− 0·5	+ ·27	S. 19·2	
9 30	− 0·2	+ ·18	S. 22·4	− 0·2	+ ·16	S. 21·4	+ 1·3	+ ·15	S. 15·7	
10 30	− 0·1	+ ·18	S. 22·5	+ 0·6	+ ·19	S. 18·5	+ 1·7	+ ·05	S. 10·6	
11 30	− 1·0	+ ·06	S. 21·2	+ 1·7	+ ·02	S. 15·0	+ 0·8	+ ·05	S. 4·7	
	April.			May.			June.			
P.M. 0 30	+ 0·7	− ·06	N. 12·4	− 0·3	− ·08	N. 19·9	− 1·5	− ·12	N. 22·7	
1 30	+ 0·3	− ·11	N. 17·0	+ 0·1	− ·14	N. 21·7	− 2·5	− ·07	N. 21·8	
2 30	− 0·1	− ·16	N. 19·9	− 2·6	− ·16	N. 22·8	− 3·6	− ·11	N. 19·3	
3 30	− 1·5	− ·20	N. 21·9	− 4·1	− ·22	N. 21·8	− 5·2	− ·13	N. 15·8	
4 30	− 2·6	− ·32	N. 22·6	− 3·9	− ·25	N. 19·7	− 5·4	− ·20	N. 11·0	
5 30	− 2·5	− ·32	N. 21·8	− 6·1	− ·34	N. 16·4	− 7·6	− ·13	N. 5·9	
6 30	− 1·5	− ·35	N. 19·7	− 4·2	− ·33	N. 11·4	− 5·5	− ·08	S. 0·5	
7 30	− 1·4	− ·36	N. 15·9	− 3·4	− ·29	N. 5·7	− 3·7	− ·13	S. 6·8	
8 30	− 2·0	− ·23	N. 11·0	− 2·2	− ·18	S. 0·2	− 0·2	− ·04	S. 11·9	
9 30	− 0·7	− ·13	N. 5·3	− 1·6	− ·11	S. 6·4	+ 0·2	− ·01	S. 17·2	
10 30	− 0·7	− ·09	S. 0·8	− 0·6	− ·01	S. 11·6	+ 0·4	+ ·03	S. 19·9	
11 30	− 0·8	·00	S. 6·9	− 1·1	+ ·01	S. 16·6	+ 0·8	+ ·02	S. 21·9	
A.M. 0 30	− 0·8	+ ·07	S. 12·2	+ 0·3	+ ·06	S. 19·7	+ 1·4	+ ·11	S. 23·0	
1 30	− 0·2	+ ·06	S. 16·4	− 0·2	+ ·13	S. 21·9	+ 2·8	+ ·07	S. 21·9	
2 30	+ 0·3	+ ·18	S. 20·0	+ 2·6	+ ·18	S. 22·5	+ 3·4	+ ·13	S. 19·2	
3 30	+ 1·6	+ ·18	S. 21·8	+ 4·1	+ ·29	S. 21·8	+ 4·5	+ ·14	S. 16·3	
4 30	+ 2·3	+ ·30	S. 22·6	+ 3·1	+ ·25	S. 19·9	+ 4·8	+ ·15	S. 11·6	
5 30	+ 2·0	+ ·28	S. 21·9	+ 5·6	+ ·31	S. 16·3	+ 7·3	+ ·12	S. 5·9	
6 30	+ 1·9	+ ·37	S. 20·0	+ 4·1	+ ·33	S. 11·7	+ 5·7	+ ·16	N. 0·7	
7 30	+ 1·2	+ ·34	S. 15·7	+ 3·3	+ ·27	S. 5·9	+ 3·7	+ ·14	N. 6·3	
8 30	+ 1·9	+ ·25	S. 11·5	+ 2·3	+ ·19	0·0	+ 0·2	+ ·05	N. 12·1	
9 30	+ 0·6	+ ·16	S. 5·4	+ 1·5	+ ·10	N. 6·1	− 0·3	+ ·01	N. 16·5	
10 30	+ 1·0	+ ·08	N. 0·6	+ 0·5	− ·03	N. 12·0	− 0·4	− ·03	N. 20·0	
11 30	+ 0·8	·00	N. 6·3	+ 1·4	− ·02	N. 16·9	− 0·7	− ·02	N. 22·1	

TABLE XXI. (*g*.) (Continued.)

Diurnal Inequality.

Apparent Solar Time of Moon's Transit B.	July.			August.			September.		
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.
h m	m	feet.	°	m	feet.	°	m	feet.	°
P.M. 0 30	− 3·8	− ·04	N. 19·3	− 3·9	+ ·03	N. 10·4	− 4·3	+ ·08	N. 0·1
1 30	− 4·6	− ·02	N. 15·4	− 3·3	+ ·06	N. 4·8	− 4·7	+ ·10	S. 6·6
2 30	− 4·0	− ·03	N. 11·0	− 3·3	+ ·11	S. 1·2	− 2·1	+ ·17	S. 12·3
3 30	− 4·5	− ·02	N. 4·9	− 5·0	+ ·15	S. 7·4	− 3·8	+ ·15	S. 16·8
4 30	− 5·2	+ ·01	S. 1·4	− 4·2	+ ·12	S. 12·6	− 2·1	+ ·16	S. 20·1
5 30	− 6·4	− ·01	S. 7·1	− 3·8	+ ·13	S. 17·4	− 0·9	+ ·24	S. 22·0
6 30	− 4·9	− ·02	S. 12·6	− 1·0	+ ·10	S. 20·3	+ 3·3	+ ·31	S. 23·0
7 30	− 0·9	− ·05	S. 17·1	+ 1·1	+ ·23	S. 22·3	+ 4·4	+ ·38	S. 22·0
8 30	+ 0·8	+ ·06	S. 20·1	+ 3·3	+ ·24	S. 22·7	+ 3·8	+ ·38	S. 19·9
9 30	+ 1·5	+ ·04	S. 22·3	+ 3·2	+ ·18	S. 21·7	+ 4·0	+ ·18	S. 16·1
10 30	+ 2·0	+ ·11	S. 22·8	+ 4·8	+ ·17	S. 19·3	+ 3·3	+ ·07	S. 11·9
11 30	+ 2·8	+ ·06	S. 21·6	+ 4·2	+ ·11	S. 15·7	+ 3·5	+ ·01	S. 5·4
A.M. 0 30	+ 3·3	+ ·03	S. 19·1	+ 3·7	− ·03	S. 10·9	+ 5·0	− ·07	S. 0·3
1 30	+ 4·9	+ ·01	S. 15·9	+ 3·6	− ·06	S. 5·2	+ 4·6	− ·11	N. 6·6
2 30	+ 3·8	+ ·05	S. 11·0	+ 3·2	− ·11	N. 0·8	+ 2·1	− ·18	N. 11·9
3 30	+ 4·8	+ ·03	S. 4·8	+ 5·1	− ·12	N. 7·1	+ 3·7	− ·14	N. 16·5
4 30	+ 5·3	·00	N. 0·9	+ 3·9	− ·15	N. 12·4	+ 2·2	− ·17	N. 20·1
5 30	+ 6·2	+ ·03	N. 7·0	+ 3·6	− ·12	N. 17·2	+ 0·9	− ·20	N. 22·1
6 30	+ 4·9	+ ·02	N. 12·6	+ 1·1	− ·11	N. 20·2	− 2·2	− ·32	N. 22·8
7 30	+ 1·3	+ ·06	N. 16·4	− 1·5	− ·25	N. 22·2	− 4·7	− ·40	N. 22·1
8 30	− 0·9	− ·05	N. 20·1	− 3·0	− ·23	N. 22·3	− 3·8	− ·38	N. 19·7
9 30	− 1·6	− ·05	N. 22·3	− 3·1	− ·20	N. 21·7	− 3·9	− ·16	N. 16·4
10 30	− 2·0	− ·11	N. 23·1	− 5·1	− ·18	N. 19·4	− 3·0	− ·09	N. 11·0
11 30	− 2·7	− ·08	N. 21·4	− 4·0	− ·08	N. 15·8	− 4·1	− ·01	N. 6·1

	October.			November.			December.		
P.M. 0 30	− 3·1	+ ·09	S. 11·5	− 1·5	+ ·12	S. 19·2	+ 0·2	+ ·16	S. 22·4
1 30	− 2·0	·00	S. 16·3	− 0·1	+ ·08	S. 21·7	+ 0·4	+ ·09	S. 21·8
2 30	− 1·7	+ ·19	S. 19·2	− 0·9	+ ·13	S. 22·5	+ 1·5	+ ·19	S. 19·6
3 30	− 0·5	+ ·21	S. 21·8	+ 1·9	+ ·24	S. 22·1	+ 3·7	+ ·16	S. 16·0
4 30	+ 1·4	+ ·24	S. 22·7	+ 4·5	+ ·34	S. 19·8	+ 3·8	+ ·11	S. 10·9
5 30	+ 4·2	+ ·30	S. 22·0	+ 8·0	+ ·46	S. 17·0	+ 8·5	+ ·34	S. 5·3
6 30	+ 7·8	+ ·33	S. 20·0	+ 6·4	+ ·36	S. 12·1	+ 6·3	+ ·21	N. 0·7
7 30	+ 6·9	+ ·44	S. 16·6	+ 6·6	+ ·30	S. 6·2	+ 6·1	+ ·12	N. 6·7
8 30	+ 5·4	+ ·36	S. 12·4	+ 5·3	+ ·27	S. 0·1	+ 3·2	+ ·06	N. 12·3
9 30	+ 4·1	+ ·13	S. 7·0	+ 2·9	+ ·06	N. 5·8	+ 2·4	+ ·07	N. 16·9
10 30	+ 5·1	+ ·08	S. 0·3	+ 2·5	+ ·05	N. 11·6	+ 1·2	− ·06	N. 19·7
11 30	+ 3·9	+ ·07	N. 5·7	+ 3·3	− ·02	N. 16·1	− 0·2	+ ·01	N. 22·1
A.M. 0 30	+ 3·5	− ·08	N. 11·5	+ 1·5	− ·16	N. 19·8	+ 0·1	− ·16	N. 22·4
1 30	+ 2·1	·00	N. 15·6	+ 0·2	− ·09	N. 21·9	− 0·3	− ·07	N. 22·2
2 30	+ 1·6	− ·20	N. 19·8	− 0·2	− ·11	N. 22·6	− 1·5	− ·20	N. 19·6
3 30	+ 0·5	− ·21	N. 22·0	− 2·0	− ·25	N. 21·8	− 3·6	− ·13	N. 16·0
4 30	− 1·4	− ·26	N. 22·7	− 5·1	− ·39	N. 20·4	− 4·3	− ·14	N. 11·5
5 30	− 3·7	− ·35	N. 22·2	− 7·9	− ·46	N. 17·0	− 7·6	− ·29	N. 5·5
6 30	− 8·0	− ·34	N. 19·6	− 6·7	− ·37	N. 12·1	− 6·4	− ·22	S. 0·2
7 30	− 7·0	− ·45	N. 16·9	− 6·7	− ·33	N. 6·4	− 5·9	− ·14	S. 6·6
8 30	− 5·3	− ·33	N. 12·6	− 4·8	− ·23	N. 0·5	− 3·4	− ·06	S. 12·1
9 30	− 4·6	− ·13	N. 7·0	− 3·1	− ·06	S. 5·6	− 2·2	− ·07	S. 16·4
10 30	− 4·9	− ·05	N. 0·9	− 2·4	− ·04	S. 11·2	− 1·3	+ ·08	S. 19·7
11 30	− 3·4	− ·03	S. 5·6	− 3·3	+ ·01	S. 16·2	+ 0·1	− ·01	S. 22·3

Table XXII. (*h*.)

Showing a Comparison between the Semimenstrual Inequality at London in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables XVI. and XVII.

Moon's Hor. Par. 57′, and Decl. 15°.

Apparent Solar Time of Moon's Transit B.	Interval. ψ + a constant.		Height. A.	
	Theory.	Observation.	Theory.	Observation.
h m	h m	h m	feet.	feet.
0 0	3 15·3		22·76	
0 30	3 7·1	3 7·1	22·77	22·72
1 0	2 58·8		22·70	
1 30	2 51·3	2 50·9	22·58	22·44
2 0	2 43·1		22·35	
2 30	2 36·8	2 36·5	22·00	21·92
3 0	2 30·8		21·73	
3 30	2 26·8	2 26·7	21·35	21·14
4 0	2 24·8		20·90	
4 30	2 23·3	2 24·0	20·47	20·23
5 0	2 29·8		20·10	
5 30	2 37·8	2 37·5	19·75	19·57
6 0	2 52·8		19·58	
6 30	3 10·8	3 10·8	19·47	19·55
7 0	3 25·8		19·64	
7 30	3 42·8	3 41·5	19·85	20·26
8 0	3 48·8		20·25	
8 30	3 53·8	3 53·4	20·63	21·15
9 0	3 51·8		21·10	
9 30	3 49·8	3 50·4	21·50	21·89
10 0	3 44·8		21·89	
10 30	3 38·8	3 39·0	22·22	22·42
11 0	3 30·8		22·47	
11 30	3 23·8	3 23·6	22·66	22·70

The above Inequalities from theory are the same as for the preceding London Discussion*, excepting that the constant applied to ψ is now $3^h\ 8^m{\cdot}4$, formerly it was $3^h\ 6^m{\cdot}6$. They have been calculated from the expressions (See p. 117.)

$$\tan 2\psi = \frac{(A)\sin 2\varphi}{1 + (A)\cos 2\varphi}$$

$$h = D + (E)\{(A)\cos(2\psi - 2\varphi) + \cos 2\psi\}$$

$$\log(A) = 9{\cdot}56284 \qquad \log(E) = 0{\cdot}63749 \qquad D = 16{\cdot}79$$

The columns headed "Observation" have been deduced from the quantities headed "Mean" in Tables II. and III., by applying to them proper corrections for the deviations from declination 15°.

* Philosophical Transactions, 1836.

Table XXIII. (*i.*)

Showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Tables XVI. and XVII. See Plate I.

Apparent Solar Time of Moon's Transit B.	January.					February.					March.					Apparent Solar Time of Moon's Transit. B.
	d ☬		d h		Moon's Decli-nation.	d ☬		d h		Moon's Decli-nation.	d ☬		d h		Moon's Decli-nation.	
	Theory.	Obser-vation.	Theory.	Obser-vation.		Theory.	Obser-vation.	Theory.	Obser-vation.		Theory.	Obser-vation.	Theory.	Obser-vation.		
h m	m	m	feet.	feet.	°	m	m	feet.	feet.	°	m	m	feet.	feet.	°	h m
0 30	0·0	−3·3	−·49	+·02	19	0·0	−1·8	+·08	+·12	10	0·0	+0·4	+·32	+·14	5	0 30
1 30	+0·3	−1·6	−·36	+·01	16	+0·1	−0·6	+·16	+·18	6	−0·4	−0·5	+·25	+·22	8	1 30
2 30	+1·2	+1·2	−·13	−·01	11	+0·2	+1·3	+·16	+·13	5	−1·7	−1·3	+·10	+·09	13	2 30
3 30	+2·8	+4·2	+·03	+·28	6	0·0	+0·8	+·10	+·19	8	−4·1	−4·4	−·11	−·14	17	3 30
4 30	+3·5	+5·3	+·10	+·30	5	−2·2	+0·2	−·09	−·17	14	−7·2	−5·8	−·36	−·36	22	4 30
5 30	+2·5	+4·6	+·08	+·07	9	−3·6	−3·1	−·28	−·27	18	−7·8	−7·8	−·52	−·58	23	5 30
6 30	0·0	−0·8	+·01	+·06	13	0·0	−0·5	−·45	−·35	21	0·0	−0·8	−·58	−·48	23	6 30
7 30	+0·9	−0·8	−·24	−·09	18	+7·0	+2·4	−·54	−·27	23	+7·8	+6·0	−·49	−·39	22	7 30
8 30	+3·0	−0·3	−·45	−·20	21	+6·9	+3·4	−·50	−·32	22	+6·5	+5·2	−·26	−·18	19	8 30
9 30	+3·3	+0·5	−·57	−·17	23	+5·2	+0·5	−·45	−·32	22	+3·7	+0·4	−·06	−·05	16	9 30
10 30	+2·1	−1·2	−·62	−·18	23	+2·4	−1·6	−·29	−·12	19	+1·7	−1·9	+·16	+·05	11	10 30
11 30	+0·1	−3·9	−·60	−·11	21	+0·7	−2·8	−·07	+·04	14	+0·3	+0·2	+·29	+·14	6	11 30
	Sun's Decl. 21°, and Par. 8″·94.					Sun's Decl. 13°, and Par. 8″·90.					Sun's Decl. 3°, and Par. 8″·84.					
	April.					May.					June.					
0 30	0·0	+1·4	+·22	+·22	13	0·0	−0·5	−·17	+·03	20	0·0	−2·0	−·34	−·22	23	0 30
1 30	−0·5	+0·5	+·04	−·01	17	−0·2	+0·2	−·28	−·07	22	+0·7	−0·3	−·29	−·21	22	1 30
2 30	−2·0	−2·0	−·12	−·07	20	−0·7	−2·4	−·30	−·29	23	+2·2	+0·9	−·14	−·12	20	2 30
3 30	−4·1	−5·0	−·24	−·18	22	−0·6	−3·8	−·22	−·24	22	+5·1	+2·0	+·11	+·10	16	3 30
4 30	−6·4	−7·1	−·31	−·30	23	+0·2	−1·2	−·07	−·07	20	+7·8	+7·5	+·36	+·43	11	4 30
5 30	−5·3	−3·5	−·32	−·30	22	+1·8	+2·0	+·10	+·23	17	+8·5	+9·0	+·55	+·59	7	5 30
6 30	0·0	−0·1	−·22	−·24	20	0·0	+2·3	+·34	+·39	12	0·0	+2·0	+·67	+·54	5	6 30
7 30	+1·9	+1·9	−·02	−·01	16	−5·4	−2·3	+·42	+·38	7	−8·2	−4·0	+·55	+·39	8	7 30
8 30	−0·1	+1·1	+·20	+·14	11	−6·4	−3·1	+·41	+·31	5	−8·2	−5·8	+·34	+·09	12	8 30
9 30	−1·2	−0·3	+·34	+·24	6	−4·7	−1·1	+·34	+·28	7	−4·7	−3·0	+·06	−·04	17	9 30
10 30	−0·6	+0·5	+·38	+·31	5	−2·4	0·0	+·20	+·22	12	−2·2	−2·7	−·14	−·15	20	10 30
11 30	−0·3	−0·9	+·37	+·35	7	−0·7	+0·5	−·02	+·07	17	−0·6	−1·5	−·29	−·15	22	11 30
	Sun's Decl. 10°, and Par. 8″·76.					Sun's Decl. 19°, and Par. 8″·70.					Sun's Decl. 23°, and Par. 8″·66.					
	July.					August.					September.					
0 30	0·0	−0·6	−·14	−·14	20	0·0	+3·1	+·30	+·14	11	0·0	+3·4	+·46	+·19	5	0 30
1 30	+1·3	+1·3	+·05	+·04	16	+0·9	+2·5	+·41	+·24	7	0·0	+2·9	+·39	+·16	8	1 30
2 30	+2·8	+4·1	+·26	+·24	11	+2·2	+5·3	+·44	+·19	4	−0·9	+1·8	+·26	+·15	12	2 30
3 30	+6·8	+7·6	+·41	+·32	6	+2·9	+4·9	+·35	+·23	8	−2·7	+1·0	+·02	−·06	17	3 30
4 30	+8·7	+9·9	+·48	+·51	5	+2·2	+4·9	+·22	+·29	13	−5·4	−5·1	−·18	−·21	20	4 30
5 30	+7·1	+9·9	+·50	+·53	8	−0·3	+2·8	·00	+·04	18	−6·4	−7·4	−·36	−·34	22	5 30
6 30	0·0	+4·4	+·42	+·37	13	0·0	+1·5	−·16	−·15	21	0·0	+1·4	−·45	−·52	23	6 30
7 30	−3·7	−2·2	+·20	+·11	17	+3·0	+2·3	−·21	−·24	22	+6·4	+6·3	−·36	−·30	22	7 30
8 30	−2·2	−1·2	−·01	−·07	20	+3·7	+4·4	−·24	−·24	23	+4·7	+6·2	−·18	−·15	20	8 30
9 30	−0·7	+0·2	−·22	−·18	23	+2·3	+3·6	−·19	−·23	22	+2·3	+4·3	+·02	−·05	17	9 30
10 30	−0·5	+0·5	−·27	−·20	23	+0·4	+1·8	−·03	−·18	19	+0·9	+5·3	+·26	+·08	12	10 30
11 30	−0·5	−1·2	−·25	−·26	22	−0·2	+4·5	+·10	−·04	16	−0·1	+4·0	+·41	+·15	7	11 30
	Sun's Decl. 21°, and Par. 8″·66.					Sun's Decl. 14°, and Par. 8″·70.					Sun's Decl. 4°, and Par. 8″·76.					
	October.					November.					December.					
0 30	0·0	+ 3·2	+·13	+·10	12	0·0	−0·3	−·42	−·23	20	0·0	−2·9	−·77	−·26	23	0 30
1 30	−0·9	+ 1·6	−·05	+·01	16	−1·0	−2·4	−·53	−·27	22	−0·5	−3·7	−·70	−·29	22	1 30
2 30	−2·8	− 1·6	−·24	−·06	20	−2·3	−4·0	−·56	−·23	23	−0·2	−3·5	−·53	−·04	20	2 30
3 30	−5·3	− 5·0	−·37	−·27	22	−3·2	−4·0	−·47	−·16	22	+1·3	+0·8	−·27	−·02	16	3 30
4 30	−8·2	−10·4	−·45	−·29	23	−3·4	−4·8	−·34	−·13	20	+2·6	+3·2	−·01	+·26	11	4 30
5 30	−6·7	−10·6	−·46	−·39	22	−1·2	−2·9	−·22	+·09	18	+4·1	+1·9	+·15	+·34	7	5 30
6 30	0·0	− 4·8	−·37	−·38	20	0·0	−3·3	+·05	+·02	12	0·0	−1·7	+·25	+·38	5	6 30
7 30	+3·3	+ 2·6	−·21	−·08	17	−2·4	−4·2	+·14	+·18	7	−3·8	−7·1	+·13	+·29	8	7 30
8 30	+1·7	+ 3·0	+·04	+·09	12	−2·8	−3·1	+·15	+·19	5	−3·0	−7·1	−·04	+·16	12	8 30
9 30	0·0	+ 1·8	+·19	+·20	7	−2·1	−3·6	+·09	+·17	7	+0·9	−2·8	−·32	·00	17	9 30
10 30	+0·2	+ 2·5	+·26	+·25	5	−0·8	−1·0	·00	−·04	12	+0·2	−2·2	−·53	−·15	20	10 30
11 30	+0·1	+ 3·2	+·24	+·15	7	−0·2	−0·2	−·23	+·04	16	+0·6	−2·5	−·70	−·30	23	11 30
	Sun's Decl. 9°, and Par. 8″·84.					Sun's Decl. 18°, and Par. 8″·90.					Sun's Decl. 23°, and Par. 8″·94.					

Table XXIV. (*j*.)

Showing the Moon's Parallax Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Table XIX. See Plate II.

Apparent Solar Time of Moon's Transit B.	H. P. 54′.				H. P. 55′.				Apparent Solar Time of Moon's Transit B.
	d ψ		d h		d ψ		d h		
	Theory.	Obser-vation.	Theory.	Obser-vation.	Theory.	Obser-vation.	Theory.	Obser-vation.	
h m	m	m	feet.	feet.	m	m	feet.	feet.	h m
0 30	0·0	− 0·1	− ·66	− ·46	0·0	+ 0·2	− ·45	− ·42	0 30
1 30	− 2·0	− 3·4	− ·66	− ·57	− 1·3	− 2·9	− ·45	− ·34	1 30
2 30	− 4·2	− 6·2	− ·64	− ·59	− 2·7	− 4·5	− ·44	− ·38	2 30
3 30	− 6·5	−12·9	− ·62	− ·79	− 4·2	− 8·3	− ·42	− ·61	3 30
4 30	− 8·7	−13·0	− ·61	− ·78	− 5·6	− 8·9	− ·42	− ·56	4 30
5 30	− 8·4	−11·7	− ·64	− ·97	− 5·2	− 8·9	− ·44	− ·72	5 30
6 30	0·0	− 1·8	− ·66	− ·79	0·0	− 2·6	− ·45	− ·57	6 30
7 30	+ 8·4	+ 0·5	− ·64	− ·89	+ 5·2	+ 0·4	− ·44	− ·50	7 30
8 30	+ 8·7	+ 5·0	− ·61	− ·67	+ 5·6	+ 3·5	− ·42	− ·41	8 30
9 30	+ 6·5	+ 3·2	− ·62	− ·52	+ 4·2	+ 2·4	− ·42	− ·40	9 30
10 30	+ 4·2	+ 2·1	− ·64	− ·37	+ 2·7	+ 2·9	− ·44	− ·29	10 30
11 30	+ 2·0	+ 3·1	− ·66	− ·55	+ 1·3	+ 2·9	− ·45	− ·42	11 30
	H. P. 56′.				H. P. 57′.				
0 30	0·0	− 0·5	− ·23	− ·10	0·0	0·0	·00	·00	0 30
1 30	− 0·6	− 2·2	− ·23	− ·19	0·0	0·0	·00	·00	1 30
2 30	− 1·3	− 1·7	− ·23	− ·23	0·0	0·0	·00	·00	2 30
3 30	− 2·0	− 2·5	− ·21	− ·37	0·0	0·0	·00	·00	3 30
4 30	− 2·7	− 4·3	− ·21	− ·27	0·0	0·0	·00	·00	4 30
5 30	− 2·5	− 6·0	− ·22	− ·32	0·0	0·0	·00	·00	5 30
6 30	0·0	− 1·0	− ·23	− ·25	0·0	0·0	·00	·00	6 30
7 30	+ 2·5	− 0·6	− ·22	− ·20	0·0	0·0	·00	·00	7 30
8 30	+ 2·7	+ 0·5	− ·21	− ·24	0·0	0·0	·00	·00	8 30
9 30	+ 2·0	+ 1·1	− ·21	− ·20	0·0	0·0	·00	·00	9 30
10 30	+ 1·3	+ 0·5	− ·23	− ·03	0·0	0·0	·00	·00	10 30
11 30	+ 0·6	+ 3·3	− ·23	− ·25	0·0	0·0	·00	·00	11 30
	H. P. 58′.				H. P. 59′.				
0 30	0·0	+ 2·1	+ ·24	+ ·12	0·0	+ 2·7	+ ·49	+ ·35	0 30
1 30	+ 0·6	− 0·3	+ ·24	+ ·23	+ 1·2	+ 0·8	+ ·48	+ ·39	1 30
2 30	+ 1·3	+ 2·9	+ ·23	+ ·19	+ 2·5	+ 3·5	+ ·47	+ ·47	2 30
3 30	+ 2·0	+ 4·8	+ ·22	+ ·16	+ 3·8	+ 7·1	+ ·45	+ ·39	3 30
4 30	+ 2·5	+ 4·5	+ ·22	+ ·16	+ 4·8	+ 9·4	+ ·45	+ ·58	4 30
5 30	+ 2·0	+ 3·8	+ ·23	+ ·26	+ 4·2	+ 6·2	+ ·46	+ ·63	5 30
6 30	0·0	+ 3·4	+ ·24	+ ·26	0·0	+ 5·3	+ ·49	+ ·61	6 30
7 30	− 2·0	− 1·5	+ ·23	+ ·31	− 4·2	− 3·0	+ ·46	+ ·53	7 30
8 30	− 2·5	+ 0·1	+ ·22	+ ·19	− 4·8	− 3·1	+ ·45	+ ·46	8 30
9 30	− 2·0	− 1·9	+ ·22	+ ·33	− 3·8	− 2·9	+ ·45	+ ·34	9 30
10 30	− 1·3	− 1·2	+ ·23	+ ·29	− 2·5	1·8	+ ·47	+ ·53	10 30
11 30	− 0·6	+ 1·8	+ ·24	+ ·11	− 1·2	+ 2·4	+ ·48	+ ·31	11 30
	H. P. 60′.				H. P. 61′.				
0 30	0·0	+ 1·0	+ ·75	+ ·67	0·0	+ 1·7	+1·01	+ ·79	0 30
1 30	+ 1·8	+ 2·2	+ ·73	+ ·65	+ 2·3	+ 3·0	+0·99	+ ·84	1 30
2 30	+ 3·6	+ 6·0	+ ·72	+ ·70	+ 4·7	+ 6·1	+0·97	+ ·71	2 30
3 30	+ 5·5	+ 8·0	+ ·70	+ ·69	+ 7·1		+0·95		3 30
4 30	+ 6·9	+12·8	+ ·69	+ ·70	+ 8·9		+0·94		4 30
5 30	+ 6·1		+ ·71		+ 7·6		+0·97		5 30
6 30	0·0		+ ·75		0·0		+1·01		6 30
7 30	− 6·1	− 2·3	+ ·71	+ ·69	− 7·6		+0·97		7 30
8 30	− 6·9	− 3·3	+ ·69	+ ·60	− 8·9		+0·94		8 30
9 30	− 5·5	− 5·4	+ ·70	+ ·61	− 7·1	− 5·1	+0·95	+ ·77	9 30
10 30	− 3·6	− 2·6	+ ·72	+ ·70	− 4·7	− 2·6	+0·97	+ ·86	10 30
11 30	− 1·8	+ 0·7	+ ·73	+ ·53	− 2·3	− 1·1	+0·99	+ ·63	11 30

Table XXV. (*k*.)

Showing the Diurnal Inequality in the Interval and in the Height of High Water for the first six months of the year, for the Moon's Transit B, P.M. See Plate III.

Apparent Solar Time of Moon's Transit B.	January.		February.		March.		Apparent Solar Time of Moon's Transit B.
	d ☿.	d ♄.	d ☿.	d ♄.	d ☿.	d ♄.	
	Observation.	Observation.	Observation.	Observation.	Observation.	Observation.	
P.M.							
h m	m	feet.	m	feet.	m	feet.	h m
0 30	+ 2·3	+ ·05	+ 3·2	+ ·05	+ 3·2	− ·09	0 30
1 30	+ 3·2	+ ·05	+ 2·8	− ·10	+ 2·5	− ·08	1 30
2 30	+ 3·2	+ ·02	+ 3·0	− ·05	+ 1·4	− ·15	2 30
3 30	+ 3·3	·00	+ 2·5	− ·10	+ 2·1	− ·16	3 30
4 30	+ 4·9	·00	+ 3·5	− ·08	+ 1·6	− ·17	4 30
5 30	+ 6·0	+ ·05	+ 4·1	− ·03	+ 1·2	− ·22	5 30
6 30	+ 5·1	+ ·05	+ 1·6	− ·06	− 0·9	− ·23	6 30
7 30	+ 2·0	+ ·06	+ 0·6	− ·17	− 1·8	− ·36	7 30
8 30	+ 0·2	− ·02	− 0·4	− ·22	− 1·7	− ·32	8 30
9 30	− 0·7	− ·11	− 1·5	− ·16	− 2·7	− ·17	9 30
10 30	− 0·9	− ·15	− 2·7	− ·20	− 2·5	− ·08	10 30
11 30	− 1·0	− ·07	− 2·9	− ·06	− 2·2	− ·04	11 30
	April.		May.		June.		
0 30	+ 1·9	− ·08	+ 0·6	− ·10	− 0·8	− ·14	0 30
1 30	+ 1·2	− ·05	0·0	− ·11	− 1·4	− ·08	1 30
2 30	+ 0·8	− ·18	− 1·1	− ·15	− 2·6	− ·15	2 30
3 30	− 0·5	− ·20	− 3·0	− ·23	− 4·3	− ·15	3 30
4 30	− 2·0	− ·28	− 4·2	− ·30	− 4·6	− ·15	4 30
5 30	− 3·3	− ·31	− 7·0	− ·40	− 7·8	− ·23	5 30
6 30	− 4·7	− ·34	− 5·3	− ·35	− 5·9	− ·17	6 30
7 30	− 4·2	− ·40	− 5·0	− ·30	− 4·9	− ·12	7 30
8 30	− 2·7	− ·30	− 3·8	− ·21	− 1·7	− ·05	8 30
9 30	− 3·4	− ·13	− 2·2	− ·09	− 1·1	− ·04	9 30
10 30	− 2·9	− ·08	− 1·5	− ·03	− 0·4	+ ·04	10 30
11 30	− 2·3	− ·03	− 2·2	+ ·01	+ 0·5	+ ·02	11 30

The tide depending on the Moon's Transit A.M. for the last six months has the same inequality and the same signs as the above; and in the first six months A.M. and the last six months P.M. the same values obtain, but with a contrary sign.

The quantities in the columns headed "Observation" have been obtained by taking the mean of January and July, February and August, &c., A.M. and P.M., as explained in p. 100.

Table XXVI.

Showing that part of the Diurnal Inequality in the Height of High Water depending on the Moon, calculated from the expression $\mathrm{d}\,h = B \sin 2\,\delta$, assuming for Parallax 57′, $B = 0{\cdot}5$ feet.

d *h*.

Moon's Declination.	Moon's Horizontal Parallax.				
	54′.	55′.	57′.	59′.	61′.
3	·04	·05	·05	·06	·06
6	·09	·09	·10	·11	·12
9	·13	·14	·15	·17	·18
12	·17	·18	·20	·22	·24
15	·21	·22	·25	·27	·30
18	·25	·26	·29	·32	·35
21	·28	·30	·33	·37	·40
24	·31	·33	·37	·40	·44
27	·34	·36	·40	·44	·48
30	·37	·39	·43	·47	·51

For Moon's Upper Transit the correction in the above Table has the same sign as the Declination. For the Lower Transit it has a contrary sign.

This Table was originally calculated for an hourly variation of φ, and for a Mean Parallax from the expression $\mathrm{d}\,h = B \sin 2\,\delta \cos \psi$; but it being found that as long as the factor B was less than unity, $\cos \psi$ might always be considered equal to 1, it was thought preferable to Tabulate it in the above form, which admits of B varying as the cube of the Moon's Parallax.

Table XXVII.

The following Table contains the part of the Diurnal Inequality in the Height depending upon the Sun's Declination, calculated from the expression

$$(A)\ B \sin 2\,\delta \cos \varphi \qquad \log (A) = 9{\cdot}56965.$$

φ.	Sun's Declination.								φ.
	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	
0	+ ·02	+ ·04	+ ·06	+ ·08	+ ·10	+ ·11	+ ·13	+ ·14	360
15	+ ·02	+ ·04	+ ·06	+ ·08	+ ·09	+ ·11	+ ·13	+ ·14	345
30	+ ·02	+ ·04	+ ·05	+ ·07	+ ·09	+ ·10	+ ·12	+ ·13	330
45	+ ·02	+ ·03	+ ·05	+ ·06	+ ·08	+ ·09	+ ·11	+ ·12	315
60	+ ·01	+ ·03	+ ·04	+ ·05	+ ·06	+ ·07	+ ·08	+ ·09	300
75	+ ·0	+ ·02	+ ·02	+ ·03	+ ·04	+ ·04	+ ·05	+ ·06	285
90	·00	·00	·00	·00	·00	·00	·00	·00	270
105	− ·01	− ·02	− ·02	− ·03	− ·04	− ·04	− ·05	− ·06	255
120	− ·01	− ·03	− ·04	− ·05	− ·06	− ·07	− ·08	− ·09	240
135	− ·02	− ·03	− ·05	− ·06	− ·08	− ·09	− ·11	− ·12	225
150	− ·02	− ·04	− ·05	− ·07	− ·09	− ·10	− ·12	− ·13	210
165	− ·02	− ·04	− ·06	− ·08	− ·09	− ·11	− ·13	− ·14	195
180	− ·02	− ·04	− ·06	− ·08	− ·10	− ·11	− ·13	− ·14	180

Table XXVIII. (*l*.)

Showing the Interval and Height of High Water at the London Docks, with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax, (for the Mean of all the Moon's Transits B occurring between 0^h and 1^h) for every year from 1802 to 1835.

Year.	Number of Observations.	Moon's Transit B.	Interval.	Height of Tide.	Moon's Declination.	Moon's Hor. Par.	Sun's Declination.
		h m	h m	ft. in.	°	′	°
1802	58	0 29·4	3 12·2	22 3·8	17·0	57·5	14·1
1803	59	0 29·1	3 12·4	22 6·4	17·1	57·4	14·3
1804	56	0 29·2	3 13·8	22 10·7	15·7	57·6	13·8
1805	54	0 27·9	3 11·2	22 7·0	15·8	57·3	14·8
1806	57	0 30·3	3 12·4	22 7·2	14·4	57·5	14·5
1807	57	0 31·3	3 6·7	22 7·7	13·5	57·3	14·6
1808	55	0 32·5	3 10·1	22 7·2	12·7	57·3	14·1
1809	59	0 29·2	3 10·6	22 11·0	12·0	57·2	14·7
1810	54	0 29·3	3 10·7	22 10·2	11·6	57·7	15·1
1811	60	0 29·8	3 7·2	23 0·4	11·1	57·3	14·3
1812	53	0 27·8	3 6·7	23 1·1	11·3	57·4	14·0
1813	59	0 27·7	3 5·3	22 9·9	12·8	57·1	14·4
1814	59	0 29·0	3 6·3	22 10·5	13·5	57·5	14·4
1815	59	0 29·5	3 5·9	22 7·0	15·0	57·1	14·5
1816	57	0 27·9	3 5·1	22 9·2	15·2	57·2	13·8
1817	57	0 29·2	3 3·4	22 7·8	16·6	57·1	14·3
1818	57	0 28·4	3 3·5	22 9·4	17·5	57·4	14·5
1819	55	0 28·7	3 3·6	22 6·7	17·0	57·3	14·1
1820	58	0 30·2	3 2·7	22 8·7	17·3	57·3	14·2
1821	58	0 31·1	3 2·1	22 9·5	17·5	57·0	14·1
1822	59	0 30·7	3 3·6	22 8·1	17·2	57·2	14·4
1823	57	0 29·7	3 7·1	22 10·5	15·7	57·2	14·1
1824	61	0 29·6	3 4·1	22 11·3	15·3	57·3	14·1
1825	63	0 30·4	3 7·0	22 10·7	14·2	57·3	14·9
1826	59	0 28·0	3 9·5	22 11·2	13·0	57·3	14·5
1827	53	0 29·6	3 6·1	22 11·3	12·4	57·4	14·9
1828	61	0 31·5	3 5·5	23 2·0	11·5	57·1	14·6
1829	56	0 29·9	3 6·0	23 2·0	11·3	57·3	15·1
1830	57	0 31·9	3 2·2	23 1·1	11·6	57·2	14·7
1831	57	0 31·3	3 5·1	23 0·5	12·1	57·1	14·3
1832	55	0 30·0	3 8·5	22 5·0	13·5	57·2	15·3
1833	60	0 29·1	3 11·6	22 9·3	13·5	57·4	14·4
1834	57	0 27·8	3 11·0	22 8·6	15·5	57·4	14·9
1835	53	0 28·0	3 11·1	22 8·3	15·5	57·5	13·7

Table XXIX. (*m*.)

Interpolated from Table XXVIII. by reducing each quantity to Moon's Transit (B) $0^h\,30^m$, and correcting for deviation from Mean Declinations and Parallax.

Year.	Interval.	Height.
	h m	ft.
1802	3 11·8	22·27
1803	3 12·0	22·51
1804	3 13·3	22·79
1805	3 10·4	22·54
1806	3 12·2	22·48
1807	3 6·8	22·52
1808	3 10·6	22·44
1809	3 10·3	22·76
1810	3 10·1	22·58
1811	3 6·7	22·80
1812	3 5·9	22·85
1813	3 4·6	22·67
1814	3 5·7	22·70
1815	3 5·7	22·56
1816	3 4·4	22·71
1817	3 3·1	22·68
1818	3 2·9	22·77
1819	3 3·0	22·57
1820	3 2·6	22·75
1821	3 2·4	22·88
1822	3 3·7	22·72
1823	3 7·0	22·85
1824	3 3·8	22·88
1825	3 6·9	22·80
1826	3 8·8	22·77
1827	3 5·8	22·75
1828	3 5·8	23·03
1829	3 5·8	22·97
1830	3 2·6	22·93
1831	3 5·5	22·90
1832	3 8·4	22·32
1833	3 11·2	22·63
1834	3 10·1	22·65
1835	3 10·3	22·66

Table XXX. (*n*.)

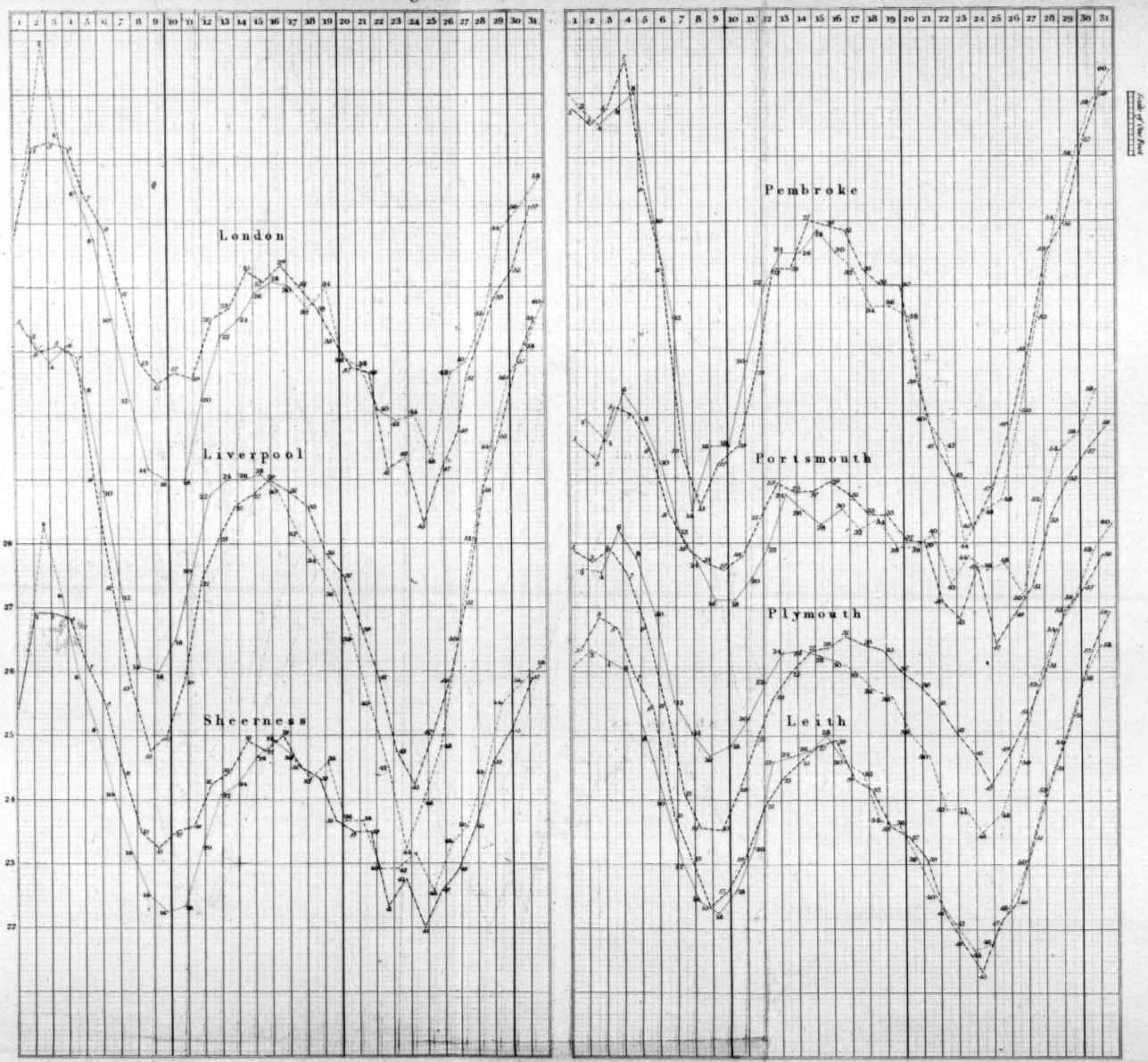
Showing the *Establishment* of the Port of London since 1802, obtained from Table XXIX. by altering the argument from Transit B to Transit F, and reducing it to $0^h\,0^m$ from $0^h\,30^m$. Moon's Hor. Par. 57′, and Decl. 15°.

Moon's Transit F = $0^h\,0^m$.

Year.	Interval*.	Height.	Year.	Interval.	Height.
	h m	ft.		h m	ft.
1802	2 5·9	21·90	1819	1 57·1	22·20
1803	2 6·1	22·14	1820	1 56·7	22·38
1804	2 7·4	22·42	1821	1 56·5	22·51
1805	2 4·5	22·17	1822	1 57·8	22·35
1806	2 6·3	22·11	1823	2 1·1	22·48
1807	2 0·9	22·15	1824	1 57·9	22·51
1808	2 4·7	22·07	1825	2 1·0	22·43
1809	2 4·4	22·39	1826	2 2·9	22·40
1810	2 4·2	22·21	1827	1 59·9	22·38
1811	2 0·8	22·43	1828	1 59·9	22·66
1812	2 0·0	22·48	1829	1 59·9	22·60
1813	1 58·7	22·30	1830	1 56·7	22·56
1814	1 59·8	22·33	1831	1 59·6	22·53
1815	1 59·8	22·19	1832	2 2·5	21·95
1816	1 58·5	22·34	1833	2 5·3	22·26
1817	1 57·2	22·31	1834	2 4·2	22·28
1818	1 57·0	22·40	1835	2 4·4	22·29

* i. e. *Establishment*.

Heights of High Water in May 1836 — See Table XXXI.



All the Heights of Tides depending upon { Upper Transits A. A.M are thus connected ——— } and are marked
{ A.P.M ——— } by odd numbers
{ Lower Transits A.P.M ——— } and are marked
{ A.A.M ——— } by even numbers

The Moons declination is north from May 1st to May 16th The Heights caused by the same Tide are marked with the same figures

Table XXXI.

Observations of High Water in May 1836.

Date.	Plymouth.		Portsmouth.		London Docks.		Pembroke.		Liverpool. Salthouse Dock.		Leith.	
	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.
1836.	h m	ft. in.	h m	ft. in.	h m	ft. in.	h m	ft. in.	h m	ft. in.	h m	ft. in.
May 1. A.M.	5 12	17 10½	11 28	18 6·9	1 40	22 10	6 20	22 9	11 6	18 5	2 12	16 1
P.M.	5 45	17 7½	11 48	18 11·8	2 10	23 8	6 40	22 9	11 25	18 3	2 24	16 4
2. A.M.	6 5	17 8½	.....	.....	2 20	24 2	7 0	22 6	11 50	18 0	2 46	16 4
P.M.	6 22	17 6½	0 5	18 3·9	2 45	25 9	7 15	22 6	.....	.....	3 4	16 10
3. A.M.	6 44	17 10½	0 35	18 7·2	2 55	24 3	7 35	22 9	0 5	17 10	3 23	16 2
P.M.	7 6	18 2½	0 55	19 1·5	3 25	24 4	8 0	22 9	0 30	18 1	3 54	16 8
4. A.M.	7 28	17 6	1 15	19 4·9	3 40	24 2	8 0	23 6	0 50	18 1	4 18	16 0
P.M.	7 54	17 9½	1 35	19 0·2	4 0	23 6	8 25	23 0	1 12	17 11	4 43	15 11
5. A.M.	8 13	16 8	1 58	18 11·4	4 25	23 5	8 50	21 6	1 26	17 5	5 8	15 0
P.M.	8 22	16 10½	2 30	18 5·2	4 50	22 9	9 10	21 0	2 0	16 0	5 35	15 5
6. A.M.	9 1	15 5½	2 50	18 3·2	5 15	22 11	9 30	20 3	2 20	15 10	6 5	14 0
P.M.	9 32	15 6½	3 25	17 5·5	5 35	21 6	10 10	19 6	2 50	14 4	6 27	13 9
7. A.M.	9 56	14 1½	3 45	17 2·0	6 10	21 11	10 45	17 5	3 15	14 2	7 0	13 0
P.M.	10 17	15 1½	4 30	16 11·0	6 20	20 3	11 10	16 6	3 49	12 9	7 40	13 1
8. A.M.	11 2	13 6½	5 15	16 8·6	7 15	20 10	11 55	16 7	4 29	13 1	8 18	12 5
P.M.	11 37	14 8½	5 45	16 8·7	7 35	19 2	.....	.....	5 6	11 9	9 6	12 4
9. A.M.	.....	.....	6 15	16 1·2	8 35	20 6	12 25	17 6	5 50	13 0	9 30	12 3
P.M.	0 32	13 6½	7 0	16 7·0	9 20	19 0	12 55	17 3	6 40	12 0	10 18	12 7
10. A.M.	12 55	14 10	7 20	16 1·3	10 5	20 8	1 10	17 6	7 16	13 6	10 34	12 7
P.M.	1 47	14 2½	8 5	16 9·6	10 30	19 0	2 30	17 6	7 50	12 10	11 28	13 1
11. A.M.	2 19	15 3½	8 43	16 5·7	11 15	20 7	2 55	18 10	8 25	14 7	11 36	13 3
P.M.	2 58	14 11½	9 8	17 4·7	11 30	20 3	3 45	18 8	8 55	14 5	.....	.....
12. A.M.	3 15	15 10	9 42	16 11·5	.....	.....	4 5	20 0	9 14	15 9	0 10	13 11
P.M.	3 40	15 7½	10 5	17 11·0	12 10	21 6	4 20	20 3	9 42	15 1	0 22	14 7
13. A.M.	3 53	16 3½	10 20	17 9·0	12 25	21 3	4 50	20 6	9 57	16 0	0 58	14 4
P.M.	4 23	16 0	10 46	17 9·5	1 0	21 8	5 10	20 3	10 16	15 7	1 2	14 8
14. A.M.	4 43	16 4	10 55	17 6·0	1 10	21 6	5 30	20 6	10 34	16 0	1 32	14 7
P.M.	5 2	16 3½	11 15	17 9·8	1 25	22 3	5 30	21 0	10 48	15 9	1 38	14 9
15. A.M.	5 19	16 3	11 32	17 3·5	1 45	21 11	5 45	20 10	11 10	16 1	1 55	14 10
P.M.	5 38	16 4½	11 51	17 11·2	2 10	22 1	6 10	20 11	11 24	16 0	2 12	15 0
16. A.M.	5 47	16 2	.....	.....	2 20	22 1	6 35	20 6	11 45	15 10	2 29	14 11
P.M.	6 4	16 6½	0 14	17 6·2	2 45	22 4	6 45	20 10	11 58	15 10	2 57	14 7
17. A.M.	6 26	16 0	0 24	17 8·7	3 5	22 0	7 15	20 3	.....	.....	3 13	14 4
P.M.	6 35	16 5	0 39	17 2·8	3 15	22 0	7 35	20 3	0 16	15 2	3 32	14 5
18. A.M.	6 47	15 8½	0 58	17 5·3	3 30	21 8	7 50	19 8	0 30	15 6	3 50	14 2
P.M.	7 4	16 4	1 20	17 4·5	3 50	21 8	8 0	20 0	0 46	14 9	4 10	13 8
19. A.M.	7 31	15 7½	1 35	17 5·0	4 10	22 0	8 25	19 8	1 4	14 10	4 20	13 7
P.M.	7 36	16 0	1 50	16 11·3	4 20	21 2	8 35	20 0	1 22	14 3	4 38	13 7
20. A.M.	7 53	15 0½	2 0	17 0·5	4 35	20 11	8 45	19 6	1 35	14 6	4 58	13 5
P.M.	8 1	15 9½	2 20	16 11·3	4 45	20 9	9 0	18 6	2 0	13 6	5 26	13 1
21. A.M.	8 36	14 8½	2 37	17 0·0	5 10	20 9	9 30	17 11	2 15	13 8	5 41	13 1
P.M.	8 45	15 6	3 0	17 1·6	5 15	20 8	9 45	17 6	2 40	12 6	6 8	12 6
22. A.M.	9 6	13 10½	3 14	16 1·0	5 55	20 1	10 10	17 6	3 0	12 11	6 37	12 3
P.M.	9 29	15 1	3 30	16 4·5	6 0	19 2	10 30	17 0	3 19	11 6	7 5	12 1
23. A.M.	9 53	13 10½	3 56	15 9·8	6 35	19 11	11 0	16 0	3 50	11 9	7 36	11 9
P.M.	10 36	14 8½	4 27	16 9·8	7 0	19 4	11 25	16 3	4 15	10 3	8 14	11 8
24. A.M.	11 2	13 6	5 18	16 7·5	7 35	20 0	.....	.....	4 55	11 3	8 36	11 4
P.M.	11 48	14 3	5 43	16 7·0	7 55	18 4	12 15	16 6	5 40	11 0	9 13	11 9
25. A.M.	.....	.....	6 18	15 5·1	8 55	19 4	12 35	16 10	6 20	12 0	9 53	12 1
P.M.	0 34	13 9½	6 50	16 8·2	9 20	19 2	1 30	16 8	7 0	11 10	10 16	12 4
26. A.M.	1 8	14 9½	7 40	15 10·5	10 10	20 8	1 55	17 10	7 20	12 10	10 46	12 5
P.M.	1 44	14 7½	8 12	16 2·0	10 45	19 9	2 30	18 0	7 58	13 6	11 13	13 0
27. A.M.	2 16	15 4½	8 38	16 4·0	11 20	20 10	3 10	19 0	8 28	14 1	11 33	13 5
P.M.	2 49	15 9½	9 5	17 8·0	11 35	20 7	3 40	19 6	9 46	15 1	.....	.....
28. A.M.	3 6	16 1½	9 21	17 4·6	.....	.....	3 50	20 6	9 11	15 10	0 5	14 2
P.M.	3 37	16 7½	9 56	18 5·1	12 5	21 7	4 20	21 0	9 35	16 6	0 27	14 6
29. A.M.	4 0	16 11½	10 12	18 0·2	12 30	21 10	4 45	21 0	9 55	16 8	0 52	14 11
P.M.	4 27	17 1½	10 37	18 8·5	12 50	22 11	4 45	22 0	10 16	17 7	1 17	15 4
30. A.M.	4 47	17 3½	11 4	18 5·2	1 10	22 3	5 10	22 3	10 40	17 10	1 34	15 11
P.M.	5 26	17 10½	11 23	19 4·6	1 35	23 3	5 45	22 10	11 0	18 1	2 8	16 4
31. A.M.	5 42	17 10	11 42	18 9·5	1 50	23 3	6 10	23 0	11 23	18 6	2 24	16 5
P.M.	6 5	18 3½	.....	.....	2 20	23 8	6 40	23 4	11 45	18 9	2 58	16 11

Index to the Tables.

Results deduced from Observations made at LIVERPOOL.

These observations were made at Liverpool by Mr. HUTCHINSON, Dockmaster at that place: they are now in the possession of the Liverpool Lyceum, and they were granted with the greatest kindness and liberality to the author for the purposes of this inquiry by the Committee of that Institution.

The *intervals* in Tables I. II. IV. V. and VI. should be increased by 36 hours to give the real *interval* between the moon's transit A and the time of high water.

I have concluded that Mr. HUTCHINSON's observations are given in *apparent solar* time.

Table I. (*a.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A in each month of the year, from 13,391 observations made at the Liverpool Docks, between the 1st of January 1774 and the 31st of December 1792.

Table II. (*b.*) (Interpolated from Table I.), showing the Interval between the Apparent Solar Time of the Moon's Transit A, and the Time of High Water at the Liverpool Docks for each month in the year.

Table III. (*c.*) (Interpolated from Table I.), showing the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, in each month of the year.

Table IV. (*d.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, for every minute of her Horizontal Parallax.

Table V. (*e.*), Interpolated from Table IV., and reduced to Moon's Declination 15°.

Table VI. (*f.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Upper and Lower Transit A, A.M. and P.M.

Table VII. (*g.*), showing the Diurnal Inequality at Liverpool, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table II., and the Difference between the Height of High Water and the Height in Table III.

Table VIII. (*h.*), showing a Comparison between the Semimenstrual Correction at Liverpool in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables II. and III.

Table IX. (*i.*), showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from BERNOULLI's theory and from the results of observation contained in Tables II. and III. See Plate I.

Table X. (*j.*), showing the Moon's Parallax Inequality in the Interval and in the

Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Table V. See Plate II.

Table XI. (*k.*), showing the Diurnal Inequality in the Interval and in the Height for the first six months of the year for the Moon's Transit A, P.M. See Plate III.

Table XII. (*l.*), showing the Interval and Height of High Water at the Liverpool Docks, with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax (for the mean of all the Moon's Transits A occurring between 0^h and 1^h) for every year from 1774 to 1792.

Table XIII. (*m.*), interpolated from Table XII. by reducing each quantity to Moon's Transit A ($0^h\ 30^m$), and correcting the quantities for deviation from mean Declinations and Parallax.

Table XIV. (*n.*), showing the *Establishment* of the Port of Liverpool obtained from Table XIII. by altering the argument from Transit A to Transit D, and reducing it to $0^h\ 0^m$ from $0^h\ 30^m$. Moon's Hor. Par. 57′, and Decl. 15°.

Results deduced from Observations made at London.

These observations were made at the London Docks under the direction of the late Mr. Peirse, and they are now in the possession of the Royal Society.

The intervals in Tables XV. XVI. XVIII. XIX. and XX. must be increased by 48 hours to give the real interval between the moon's transit B and the time of high water.

I have concluded that these observations are given in *mean solar* time.

Table XV. (*a.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B in each month of the year, from 24,592 observations made at the London Docks, between the 1st of September 1801 and the 31st of August 1836.

Table XVI. (*b.*) (Interpolated from Table XV.), showing the Interval between the Apparent Solar Time of the Moon's Transit B, and the Time of High Water at the London Docks for each month in the year.

Table XVII. (*c.*) (Interpolated from Table XV.), showing the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, in each month of the year.

Table XVIII. (*d.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, for every minute of her Horizontal Parallax.

Table XIX. (*e.*), Interpolated from Table XVIII.

Table XX. (*f.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, A.M. and P.M.

Table XXI. (*g*.), showing the Diurnal Inequality at London, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table XVI., and the Difference between the Height of High Water and the Height in Table XVII.

Table XXII. (*h*.), showing a Comparison between the Semimenstrual Inequality at London in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables XVI. and XVII.

Table XXIII. (*i*.), showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Tables XVI. and XVII. See Plate I.

Table XXIV. (*j*.), showing the Moon's Parallax Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Table XIX. See Plate II.

Table XXV. (*k*.), showing the Diurnal Inequality in the Interval and in the Height of High Water for the first six months of the year, for the Moon's Transit B, P.M. See Plate III.

As the London discussion contained in this paper has been made with reference to transit B, and the discussion of the Liverpool observations has been made with reference to transit A, it was necessary to pay attention to this circumstance in the comparisons on the Plates. This has been done for the present roughly, by placing the London corrections more to the left by half an hour. The interval corrections would strictly require, in extreme cases, a slight alteration, which may be obtained from Tables XXIII. XXV. and XXVII., given in a former paper*.

Table XXVI., showing that part of the Diurnal Inequality in the Height of High Water depending on the Moon, calculated from the expression $\mathrm{d}\,h = B \sin 2\,\delta'$, assuming for Parallax $57'$, $B = 0{\cdot}5$ feet.

Table XXVII. contains the part of the Diurnal Inequality in the Height depending upon the Sun's Declination, calculated from the expression

$$(A)\ B \sin 2\,\delta \cos \varphi \qquad \log (A) = 9{\cdot}56965.$$

Table XXVIII. (*l*.), showing the Interval and Height of High Water at the London Docks with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax, for the Mean of all the Moon's Transits B occurring between 0^h and 1^h for every year from 1802 to 1835.

Table XXIX. (*m*.), interpolated from Table XXVIII. by reducing each quantity to Moon's Transit (B) $0^h\ 30^m$, and correcting for deviation from Mean Declinations and Parallax.

Table XXX. (*n*.), showing the *Establishment* of the Port of London since 1802, obtained from Table XXIX. by altering the argument from Transit B to Transit F, and reducing it to $0^h\ 0^m$ from $0^h\ 30^m$. Moon's Hor. Par. $57'$, and Decl. $15°$.

Table XXXI. Observations of High Water in May 1836. See Plate IV.

* Philosophical Transactions, 1836, p. 255.

X. *Further Observations on Voltaic Combinations. In a Letter addressed to* Michael Faraday, *Esq. D.C.L. F.R.S., Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal & Imp. Acadd. of Sciences, Paris, Petersburgh, &c. By* J. Frederic Daniell, *F.R.S., Prof. Chem. in King's College, London, &c.*

Received March 30,—Read April 6, 1837.

My dear Faraday,

I HAD intended, ere this, to have addressed you upon the subject of the *measure of affinity* which the *constant battery* may be made to supply, as indicated by the concluding experiment of my last letter; but my attention has been diverted, and the whole of my leisure occupied by what I found to be a necessary preliminary investigation of the effects of changes of temperature upon the voltaic action. In the course of my experiments upon this principal subject, I have also been led to observe some curious disturbances and diversions of the battery current, from secondary combinations; and I now submit the results of the whole inquiry to your judgment, not without a hope that you may consider them of sufficient interest and importance to be communicated to the Royal Society.

You may perhaps recollect that the standard charge, which I finally adopted in the use of the constant battery, was a mixture of eight parts of water with one of oil of vitriol on the side of the zinc, and a saturated solution of sulphate of copper in contact with the copper; and that the average amount of its work, as measured by the voltameter, was 11 cubic inches of mixed gases per five minutes. It occurred to me that the resistance to the current might again be reduced by dissolving the salt in the standard acid instead of water; and upon making the experiment I found the action increased from 11 cubic inches to 13 cubic inches, at which rate it steadily maintained itself; the following being the result of one series of observations.

Time.		Interval.	Voltameter.
h	m	′	Cubic inches.
11	0		
11	5	5	13
11	10	5	26
11	15	5	39
11	20	5	52

Upon one occasion I prepared the charge by adding one part of oil of vitriol to eight parts of the saturated solution of the sulphate, and poured it into the cells

whilst of the high temperature produced by the disengagement of heat during the mixture, which was about 110°; and the following series of experiments will show the great increase of action which followed this accession of heat, and its rapid decline with the temperature. The observations were made at intervals of two minutes; but I have added the calculated rate per five minutes to facilitate comparison with former experiments.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
12 52			
12 54	2	8·8	22·
12 59			
12 61	2	8·	20·
1 3			
1 5	2	7·3	18·2
1 7			
1 9	2	7·	17·5
1 11			
1 13	2	6·5	16·2
1 14			
1 16	2	6·	15·
1 17			
1 19	2	5·7	14·2
1 21	2	4·8	12·

Wishing to follow up this indication of the influence of temperature, I caused a tub to be made which would just receive the ten cells of the battery standing upon small blocks of wood, between which there was room for the syphon tubes to pass. In this situation it was fresh charged with the acid solution; and henceforth I carefully noted the temperature, which in this instance was 68°. Its action was steady for three quarters of an hour at 13·8 cubic inches per five minutes. Two fluid ounces of fresh standard acid were then poured into the inner cells, and the tub was filled with hot water of the temperature of 130° to nearly the top of the cells; the action was now found to be 20 cubic inches per five minutes. The temperature of the exterior solution of sulphate was 106°, and of the interior acid 100°. The experiment was terminated by the bursting of all the membranes which had been exposed for five weeks to the acid solution.

Having remounted the battery, I proceeded to ascertain the effect of different charges in connection with this decided influence of temperature; and, in the first place, having poured a plain aqueous solution of sulphate of copper into the exterior cells, I filled the interior with pure water, making use at the same time of fresh amalgamated zinc rods. At the commencement of the experiment the temperature was 74°; and there was no appreciable decomposition in the voltameter. The tub

was then filled with water at 120°. When the temperature of the interior cell had reached 100° a slow action commenced, which became steady at 0·6 cubic inches per five minutes. The voltameter itself being now also heated to 115°, the following results were obtained:

Time. h m	Interval.	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
11 21			
11 26	5	0·8	0·8
11 31	10	1·6	0·8
11 51	30	5·0	0·8

At the expiration of the experiment the rods were found to be only slightly tarnished, and no copper had penetrated to the interior cells.

The next charge which I made trial of was a solution of muriate of ammonia, in the proportion of two pounds of the salt to a gallon of water for the interior cells, and aqueous solution of sulphate of copper for the exterior. The results were as follow:

	per 5 min. Cubic inches.
At 74° out of water	5·5
At 94 in water	8·8
At 124 in water	12·5

Upon this occasion I first observed that a portion of the current was discharged by the water in which the battery was immersed.

The battery plunged in water of the atmospheric temperature of 74° had been working steadily for twenty minutes at the rate of 3·1 cubic inches per five minutes: when the water was drawn off from the tub the rate immediately rose, and was maintained at 4·2 cubic inches per five minutes.

That a discharge may take place from the copper of one cell to the copper of the next, when the regular circuit is interrupted between the two, I had many opportunities of observing in the powerful currents with which I had been experimenting; for I have frequently seen it pass in the form of a spark when the cells had been too much approximated in the air; and when in water it was indicated by the frothing between the two from the disengagement of gas. In such a case there is no doubt that one of the zinc rods is thrown out of action, and the copper of that cell merely acts as an electrode to the antecedent zinc. I shall hereafter point out to you how readily a portion of a current may be diverted from its principal course to by-paths, if I may so express myself, which may be open to it.

The decomposition of the secondary electrolytes, and the course of the ions in this combination, are worthy of some remark. In several instances when the battery had been in action for a considerable period, the zinc rods were found thickly studded with beautiful, large, transparent crystals of sulphate of zinc. The solution of this

salt was abundantly precipitated by nitrate of baryta and by potassa, the precipitate in the latter case being redissolved by an excess of the alkali. It was not in the slightest degree affected by nitrate of silver; proving that no muriate or chloride existed in it. There were no indications of free ammonia in the exterior cell: the precipitated copper however did not exhibit the beautiful bright pink hue which it ordinarily presents, but was of a dull, greyish, earthy appearance, resembling that of copper over which ammoniacal gas has been passed at a red heat, and probably contained some combined nitrogen. I had not, however, time to examine a product which is worthy of further investigation.

I tried one more experiment with a view to complete the inquiry into the probable advantage of changing the battery charge. I placed a saturated solution of common salt in contact with the zinc, and filled the exterior division of the cells with a saturated aqueous solution of sulphate of copper. With a temperature of 70° the following series of observations was made:

Time. h	Time. m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
11	11			
11	15	4	3·5	4·3
11	19	8	5·9	3·
11	23	12	8·0	2·6
11	27	16	9·9	2·3
11	31	20	11·6	2·1
11	34			
11	54	20	4·	1·0

A fluid ounce of muriatic acid was added to each interior cell, and the action was only brought up to 2·7 cubic inches per five minutes. It thus appeared that the substitution of solutions of the muriates for dilute sulphuric acid, was in every way disadvantageous; and it was moreover found that when the circuit was broken the copper became seriously injured by their action and the formation of a submuriate of that metal.

Wishing now to extend the inquiry into the influence of high temperatures upon the voltaic current, and finding that the membranous tubes would not be able to resist the action of the acid under these circumstances, I endeavoured to find some substitute for the partition of the cells which would not be liable to injury from this cause. After several trials I found that porous earthenware, of the same texture as that of which wine coolers are commonly made, would answer my purpose sufficiently; and the arrangement which I ultimately adopted simplifies the construction of the battery to such a degree as to render it advantageous even under circumstances of ordinary temperature.

The interior cells consist of tubes of this earthenware, closed at the lower end, of the diameter of $1\frac{1}{2}$ inches, and of the same height as the copper cells. The bottoms of the latter are fitted with sockets, in which the tubes are placed, and which confine

them in their proper positions; the perforated copper plates, or colanders, for the reception of the solid sulphate of copper, pass over their upper ends. The tubes can be easily removed and instantly replaced; and the facility of emptying and refilling them renders the addition of syphon tubes unnecessary, except in very particular circumstances. Previously to their use they require to be thoroughly soaked in dilute acid; and I have not found them liable to injury, provided the sulphate of zinc be not allowed to crystallize within them; in which case they become disintegrated from the expansive force of crystallization. When the battery is out of action they may always be removed and emptied, and preserved in a state for immediate use by immersing them in very dilute acid. Liquid conduction is not carried on quite so perfectly through their substance as through the membranes, on account of the less perfect communication between the liquids on their opposite sides; but when the acid sulphate of copper is used the amount of action at ordinary temperatures is from 7 to 8 cubic inches per five minutes, which is quite sufficient for all ordinary purposes, and is moreover perfectly constant and steady.

Having thus prepared the battery, I caused a circular steam vessel to be made of tin plate, round which the cells could be placed upon blocks of wood, and closed in with a cover, in which there was a socket which could at pleasure be connected with the steam pipe of a boiler. Two other sockets were also conveniently placed, which were stopped with corks, through which the electrodes of the battery could pass, when the proper connections were made. I intended to have experimented with a series of ten cells, but owing to a mistake only nine could be conveniently arranged in the steamer.

The general result of numerous experiments was, that, the working rate of the battery having been ascertained at the ordinary atmospheric temperature, when steam was admitted it immediately began to rise; and at the full temperature of 212° was more than doubled, provided no *secondary action* interfered with it. The following series will be sufficient to illustrate the progress of the current: the temperatures were taken by a thermometer immersed in the steam.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.	Temperature. °
10 0				
10 5	5	7·5	7·5	58
10 10	10	15·	7·5	58
		Steam admitted		110
10 25				
10 30	5	9·	9·	170
10 35	10	20·5	11·5	195
10 40	15	37·	16·5	200
10 41				
10 46	5	19·	19·	205
10 51	10	39·	20·	206

Wishing to ascertain whether any portion of this increased effect were owing to a simple increase of conducting power in the electrolyte, or whether it were wholly dependent upon the electro-motive force, or increased energy of affinity, I set the battery in action with a voltameter included in the circuit, the body of which was immersed in water, which could be conveniently raised to the boiling temperature by means of a lamp. The observations were commenced at the temperature of 58°.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
11 25			
11 30	5	6·5	6·5
11 35	10	13·	6·5
Voltameter heated to 130°.			
11 46			
11 51	5	6·5	6·5
Voltameter heated rapidly to 212°.			
11 56	10	14·	7·5
12 1	15	21·5	7·5
12 6	20	29·	7·5

The battery itself was then heated by steam to 135°.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
2 10			
2 15	5	13	13

A cell was then included in the circuit charged with cold standard acid, and without any sulphate of copper.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
2 20			
2 25	5	8	8
Cold cell removed.			
2 30	10	20	12
Cold cell restored with acid sulphate of copper.			
2 35	15	29·5	9·5

The temperature of the battery was then increased to 160°.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
2 38			
2 43	5	11	11
2 47	4	20·5	14

A small portion only of the increased effect would thus appear to depend upon the simple conducting power, which alone could have any influence in the voltameter with platinum electrodes; whilst the increased energy of the electro-motive force

was checked by including in the circuit a cell whose powers had not been exalted by the increased temperature which had been communicated to the others.

At this period of my investigation some unexpected and highly interesting phenomena occurred, which turned out to be of a very complicated nature, and the unravelling of which cost me much time and labour. After careful consideration of the subject, I have come to the conclusion, that the clearest mode of giving you an account of these will be to adhere nearly to the order in which I followed them out.

In heating the battery in the steamer, it frequently happened (and indeed in the very first series of experiments which I made with it) that when the thermometer had nearly reached the boiling point, and the action of the battery was at its maximum, a sudden cessation would take place; the decomposition of water in the voltameter, which was proceeding at the rate of 18 or 20 cubic inches per five minutes, would stop as suddenly as by the lifting of one of the connecting wires; and this suspension of power would continue for hours, provided the high temperature were maintained. Upon turning the steam off, and quickly cooling the steamer, the action would return as suddenly as it had ceased, though generally not to the full amount; falling mostly from about 20 cubic inches to 14 or 15. Upon turning the steam on, it would again stop, and again be renewed by cooling. Upon closely examining the voltameter upon these occasions, it was found that the current was not wholly stopped, but that there was a small residual action amounting to $\frac{1}{4}$ cubic inch per five minutes.

These experiments were often repeated with the same general results; and yet there were times when every care had been taken not to vary any of the circumstances of the arrangement, when they could not be reproduced.

In seeking for the cause of these phenomena, there were two which naturally suggested themselves as probable: the first was the unequal action of heat upon the different elements of the battery, exciting thermo-electric currents; and the second the possible excitement of opposing currents from the metallic steam case.

In heating and cooling the battery by the means which I have described, it is obvious that the temperature of its different parts would be unequally affected; and, in fact, I found by the thermometer that a difference of ten or twenty degrees would occasionally exist between the liquid in contact with the zinc and that in contact with the copper: these differences I attempted to increase and modify in numberless experiments. On one occasion I charged the battery with a cold acid solution of sulphate of copper on the outside, and poured boiling standard acid into the interior tube: the inequality did not long exist, but its action was steady at $8\frac{1}{2}$ cubic inches per five minutes. I heated the connecting wires to different degrees, and ultimately placed spirit-lamps under them, to maintain them at a low red heat; but the working rate of the battery, and its steadiness as measured by the voltameter, were not sensibly affected thereby. In short, I convinced myself, that though heat was obviously connected with the phenomena, heat alone was not their exciting cause.

With the view of ascertaining whether the residual current, after the sudden cessa-

tion of the greater action, were in the same direction as the original battery current, I included a galvanometer of a coarse construction, and consisting of a single large needle, in the circuit with the voltameter. I found that the needle was strongly deflected to the E., and that the deflection increased as the action rose with the rising temperature. Upon the sudden cessation of the action in the voltameter, the needle was suddenly released from its coercing force, and swang violently backwards and forwards W. and E. till it finally rested a few degrees to the W.; proving that the residual current which passed through the voltameter was in the opposite direction to the original current, and was the excess of a counter force which had power enough to arrest its course.

The cover of the steamer was now removed, and it was found that the bottom was covered with a considerable quantity of condensed water, which was slightly acid, from the leakage of one or two of the cells.

Upon taking out some of the connecting wires between the cells, some action still remained both upon the needle and voltameter, which was not wholly stopped till four wires were removed; proving the existence of a current in some circuit which was different from the main circuit of the battery.

I now tested the direction of the current in each separate cell of the battery, by leading extra wires from the zinc and copper to a second galvanometer, and found all in the normal direction (causing a permanent deflection of the needle 50° E.) except one, which presented a contrary current, and deflected the needle 30° W. Upon another occasion I found the current in six separate cells E., and three in the opposite direction, or W.; and once again one cell was found to be neutral, two reverse, and six direct. I proceeded to investigate with great care a phenomenon which promised at first to afford the explanation of the stoppage of the battery current. Let fig. 1. (Plate VIII.) represent the section of the voltaic combination: *a b c d* will be the steam-vessel, and the nine cells, with their connecting wires, will be represented by the nine smaller circles 1, 2, 3, 4, 5, 6, 7, 8, 9. In the experiment which I am about to describe, the electrodes *e* and *f* were unconnected, but all the short connecting wires were in their places. Each cell was tested by the galvanometer by means of extra wires, as at *g*. The cells 1, 2, 3, 4, 5, 6, 7, separately deflected the needle in the normal direction, or from 30° to 40° E. No. 8. deflected it in the opposite direction, or 40° W. The connexion between 8 and 9 was broken, and the same cell deflected the needle 35° E. The connection was then broken between 8 and 7, and the deflection was 30° E.; and when both wires were replaced, the needle returned to 40° W.

The whole circuit was then completed by the connection of 1 and 9 by a short wire, when, notwithstanding a path was open for the circulation of the battery current, the deflection caused by the single cell No. 8. increased to 55° W., which was contrary to the direction of the main circulation. While the circuit was thus complete, the other cells were again tested, with the following result:—

Fig. 1.

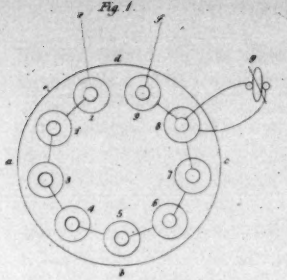


Fig. 2.

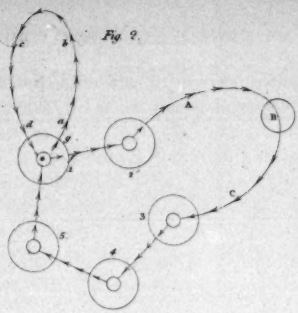


Fig. 3.

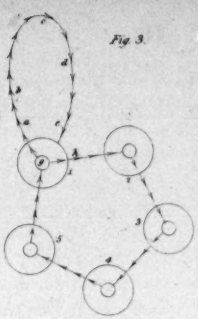


Fig. 5.

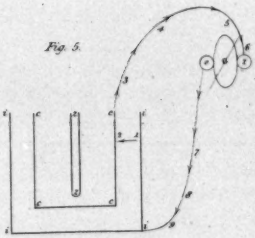


Fig. 6.

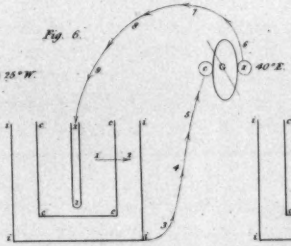


Fig. 7.

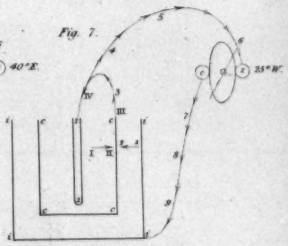


Fig. 4.

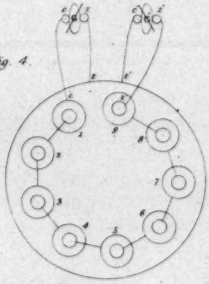


Fig. 8.

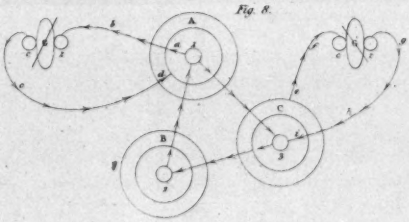


Fig. 9.

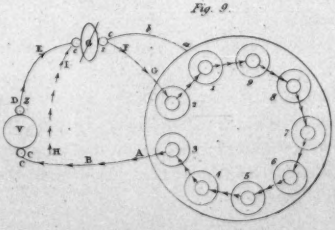
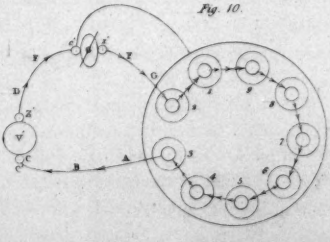


Fig. 10.



No. 8.—55° W.
7.—45 E.
6.—35 W.
5.—10 E.

With No. 4. the needle oscillated from one side of the coil to the other in the most extraordinary manner, first striking with considerable force against the pin on one side and then upon the other. Sometimes it seemed to hesitate between the two, and then to recede in one direction and advance in the opposite by sudden starts and jumps. These oscillations lasted for more than an hour, during which the experiment was continued, with equal force. The other cells were found,

No. 3.—10° W.
2.—20 E.

There was a strong spark upon breaking the connections of all these secondary currents, but they would not pass through the voltameter.

These experiments were frequently repeated with the same general results; sometimes a cell in one position indicating a reverse current, sometimes one in another, the same cell occasionally passing to the normal direction, and at times oscillating violently between the two.

To determine whether the metal steam-vessel had any influence upon these currents, five of the cells were removed to a table, and connected in series with a galvanometer, the needle of which was permanently deflected 90° E. When, in addition to the galvanometer connection, the two extreme cells were connected by a similar wire to those between the other cells, the needle was still deflected 15° E.: so that notwithstanding a shorter path was open to the battery current, and that through a conductor of considerably greater substance than the wires of the galvanometer, a portion still passed by the longer path. Under these circumstances, No. 1. cell was tested by a separate galvanometer, which was deflected 40° W. When the extra short connection of the battery was broken, the deflection from No. 1. fell to 30° W.; and when this cell was totally unconnected with the others, its current was in the normal direction, or 50° E. When the short connections were all restored, it again returned to 40° W. I must here observe, that the galvanometers, not being of the same construction, were not used as accurate measures of the force, but only to indicate the direction of the currents, and occasionally to show that the force was increasing or declining. The other cells were tested in the same way, with the following results:—

No. 2. Battery connected by short wire . . .	30°	E.
Short connection broken . . .	55	E.
No. 3. Battery connected	30	W.
but changed to	E.	
Unconnected	55	E.

No. 4. Battery connected	20	E.
Unconnected	50	E.
Again connected	40	W.
Again unconnected	50	E.
Again connected	40	W.
changed to 30° E.		
and swang violently E. & W.		
No. 5. Battery connected	35	E.
Unconnected	55	E.
Connected . oscillated E. & W.	30	

Hence it appears that this variable current may be produced from the single cells of the battery, under ordinary circumstances, when the whole series is connected by short wires.

I was still desirous of ascertaining whether these currents would be produced in the simplest possible arrangement, viz. when the elements consisted of amalgamated zinc, copper, and dilute sulphuric acid. For this purpose the five cells were thoroughly cleaned, and charged entirely with standard acid, without sulphate of copper. They were connected in series with a voltameter, and each cell was separately tested by the galvanometer, with the following results:

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
1 56				
61	5	$1\frac{1}{4}$	$1\frac{3}{4}$	Cell 1.—45° E.
2 6	10	3	$1\frac{1}{2}$	Cell 2.—45 E.
11	15	4	1	Cell 3.—40 E.
16	20	$4\frac{3}{4}$	$0\frac{3}{4}$	Cell 4.—40 E.
21	25	$5\frac{1}{3}$	$0\frac{3}{5}$	Cell 5.—40 E.

The voltameter was removed, and the circuit completed by a short wire: the separate cells were again tested, with the following result:—

Cell 1.—25° E.
Cell 2.—20 W.
Cell 3.—20 W.
Cell 4.—25 W.
Cell 5.—25 E.

When the cells 2, 3, and 4, whose secondary currents were all W., or in the direction opposite to that of the main current, were all included in the secondary circuit, the deflection of the needle was 30° W.; but the current produced no decomposition in the voltameter when included with them. The deoxidation of the oxide of copper by the hydrogen was thus proved not to have been the exciting cause of the secondary currents.

I was now desirous of making a comparison of all these effects with the similar

phenomena of the battery with its usual charge. The results of the latter are shown in the following Tables:

Time. h m	Interval.	Voltameter.	5 min. rate.	Galvanometer.
3 7				
3 12	5	5	5	Cell 1.—42½° E.
3 17	10	10	5	Cell 2.—45 E.
3 22	15	15	5	Cell 3.—45 E.
3 27	20	19½	4½	Cell 4.—47½ E.
3 32	25	24¼	4¾	Cell 5.—47½ E.

Circuit completed by short wire.

Cell 1.—25° W.
Cell 2.—30 W.
Cell 3.—30 W.
Cell 4.—30 E.
Cell 5.—30 E.

The assisting action of the sulphate of copper was thus found to have increased the decomposing power of the battery current from 1·1 cubic inch to 5 cubic inches, and the force of the secondary current, as measured by the deviation of the galvanometer, from 40° to 47°.

From these experiments we find that when the course of the main battery current is obstructed by causing it to pass through the long wire of a galvanometer, or through the electrolyte of a voltameter, the course of the secondary current from each separate cell is always normal, or in the same direction; but that when the battery current is allowed to flow with the least possible resistance, as by completing the main circuit by a short wire, the secondary current of the separate cells becomes opposite. Hence it might be inferred that the resistance might be so adjusted as that the secondary current should altogether disappear, or vary between the two directions. To ascertain the effect of different degrees of resistance, the following experiments were made.

A galvanometer was included in the main circuit of the battery, formed of a wire one twentieth of an inch in diameter and thirty-two feet in length: this we will designate as No. 1. An extra connection was also made of the circuit by means of a wire of the same thickness, fifty-three feet four inches long. The diameter of the wire forming the battery connections was one seventh of an inch. A secondary circuit was also formed from a single cell through another galvanometer, which we will call No. 2.

The deviation of No. 1. was 70° E.
No. 2. — 40 E.

and both the needles were perfectly steady.

When the extra connection was reduced to half, or twenty-six feet eight inches,

The deviation of No. 1. was 60° E.
No. 2. — 35 E.

and No. 2. wavered a little.

Extra connection again halved, or thirteen feet four inches,

The deviation of No. 1. was 40° E.
No. 2. — 30 E.

and No. 2. began to oscillate.

Extra connection once more halved, or six feet eight inches,

The deviation of No. 1. was 28° E.

and No. 2. oscillated from E. and W. very strongly.

The extra connection was then made by only four inches of the same wire:

The deviation of No. 1. was 15° E.
No. 2. — 30 W.

and the needle was only slightly unsteady.

The extra connection was then entirely removed, and

The deviation of No. 1. was 80° E.
No. 2. — 45 E.

Now with regard to the main battery current, when the extra connection was wholly removed, the whole passed through No. 1. galvanometer with a certain resistance, and was measured by its deflection 80°: when the long wire was added, a portion was diverted into the new channel, and was measured by the decline of the needle to 70°. As the length of the extra wire was shortened, the resistance of this passage decreased, and more and more of the current was diverted from the galvanometer till the deflection of the needle only amounted to 15°, and nearly the whole passed through the extra wire.

The effect of these varying resistances upon the secondary current I think I can explain with the help of the annexed diagram. Let the circles 1, 2, 3, 4, 5, (fig. 2.) represent sections of the five battery cells, and the lines between 3 and 4, 4 and 5, 5 and 1, and 1 and 2, with the arrow heads, the short wires between the zinc of each cell and the copper of the next, with the direction of that conventional current which is supposed to flow from one to the other. Let A B C represent the long wire of a galvanometer, or the electrodes of a voltameter through which the circuit is completed between 2 and 3, and by which the current is resisted. When a secondary connection is formed by the wire *a b c d*, between the zinc *e* and the copper *g* of the cell No. 1, a portion of the main current, which tends to pass through the electrolyte to the copper at *f*, being obstructed in this direction, passes to *g*, and completes its circuit through the wire *a b c d*, and the *diverted current* obviously will flow in the same direction as the main current, or from the copper through the wire to the zinc.

But if, instead of a resisting communication, the primary circuit be completed between 2 and 3 by a short wire, as between the other cells, and as represented at

fig. 3., and a secondary circuit be formed as before with the cell No. 1. by the wire *a b c d e*, a portion of the main current flowing from the copper of No. 5. to the zinc of No. 1, instead of passing through the electrolyte from *g* to *h*, finds a passage through the wire *a b c d e* to *h*, and consequently will appear to flow in an opposite direction to the primary current or from the zinc to the copper. The resistance of the circuits may be so adjusted that the current may sometimes take one course and sometimes the opposite, and produce those oscillations of the needle from E. to W. which have been just described.

The breaking of the secondary circuit did not affect the galvanometer of the primary current, but the breaking of the primary circuit always turned the secondary current into the normal direction, and increased the deviation of its needle; it reduced it, in fact, to the condition of the direct current from the single cell.

It is however obvious that these *diverted* currents of the complete circuit would not of themselves be sufficient to account for the stoppage of the main battery current of which we are in search; nor has it yet been shown how they were produced when the main circuit was broken, as represented at fig. 1. In search of the explanation of these phenomena I turned my attention to the influence of the metallic steam-vessel upon the voltaic arrangement.

When the battery was connected in the usual way with a galvanometer, and the needle was deflected 80° E., if the zinc electrode were lifted and made to touch the tin case in any part, it would remain deflected in the same direction 30°. If, on the other hand, the main circuit were broken at the copper electrode, and it were brought into contact with the tin, the deflection would be to the same amount in the same direction. The difference of that connection is shown at fig. 4, where *t z* represents the zinc electrode in connection with the tin, and passing to the zinc cup of the galvanometer G and the copper electrode C C in its usual position. On the other hand, *z' z'* represents the zinc electrode in its usual communication with the galvanometer, and the copper electrode *c' t'* in connection with the tin. The most striking result of this experiment is, that notwithstanding the connection of the tin with the galvanometer is reversed, the current is in the same direction; or that the current, which we must conceive to flow from the tin in one connection, must flow to the tin in the other. To simplify the conditions of the experiment, I repeated it with a single cell. At the atmospheric temperature of 52°, when the circuit was completed in the usual way through the galvanometer, the deflection was 57° E.; but no deflection was obtained by making connection with the tin. When steam was admitted, the ordinary battery current increased to 65°, and then contact with the hot tin produced a deflection of 20° E. by either electrode.

Four cells were now placed in the steam case upon plates of glass, and the action of the battery with the usual connections being first tried at the temperature of 54°, was found as follows:

Galvanometer alone	70° E.
Galvanometer with voltameter . . .	50° E.

Voltameter alone . . . . 5 cubic inches per five minutes.
Voltameter with galvanometer 4 cubic inches per five minutes.

At this temperature there was no current from the tin by either connection.

When the steam was turned on, the battery current increased rapidly to 85° E. by the galvanometer, and there was a current from the tin connections notwithstanding the glass insulation. This *extra current* was also found to exist during the flow of the main battery current; and when the two were measured by separate galvanometers,

The battery current was . . . 85° E.
And the extra current . . . . 30 E.

It again made no difference whether the tin took the place of the zinc or the copper in the arrangement; the deflection was always in the same direction.

The breaking of neither current affected the other.

To ascertain whether heat alone, independent of the steam, was the cause of this extra current, a tin plate was placed upon the hot sand of a sand-bath, and the battery cells transferred to glass plates upon it. At first no extra current was detected; but as the temperature rose to 150° and 200°, the second galvanometer was affected to the amount of 50° or 60°; and this whether the battery circuit was complete or not. A deflection was even occasioned by making the contact with any part of the iron stove, however distant, with which the tin plate was in contact.

These experiments were repeated and varied numberless times, but not with uniform results: sometimes the extra current had sufficient intensity to pass through a voltameter, producing slow decomposition of the water; but most frequently, however great the deflection of the needle, it would not pass through this obstacle. At other times, in apparently similar circumstances, the extra current could not be detected at all. Whenever produced, however, it was always observed to flow from the tin to the battery, whether the connection with the latter were made with the zinc or the copper.

I now placed one of the battery cells upon a piece of wood in an iron case made to receive it, of the same height, but having a space all round it of about an inch. When the primary circuit was completed by means of a galvanometer, the deflection was 60° E.; but there was no action upon a second galvanometer included in a secondary circuit between the iron and the zinc or the iron, and the copper of the cell. A little dilute sulphuric acid was then poured into the iron case, and immediately a strong extra current was produced. Under these circumstances

The primary current was . . . . . 60° E.
The extra current copper and iron connection . . 20 E.
Zinc and iron . . . . . 40 E.

The analysis of the phenomena of these extra currents was most satisfactorily performed in the following manner:—in figg. 5, 6, and 7, let *i i i i* represent the section of the iron case, *c c c c* the section of the copper cell, and *z z* the zinc rod: let G also represent the situation of the secondary galvanometer with its different connections with the circuit. In fig. 5. the connection with the iron is made with the copper cup of the galvanometer, and the zinc cup is connected with the copper of the cell; and

we see at once that a current is established, which, setting out from the iron, passes through the electrolyte to the copper, and completes its circuit through the galvanometer in the direction 1, 2, 3, 4, 5, 6, 7, 8, 9. In fig. 6. the connection with the iron remaining the same, the battery cell is connected by its zinc rod with the zinc cup of the galvanometer, and we have a powerful reversed current, which we must suppose to set out from the zinc, and to pass through a portion of the electrolyte to the copper, and from the copper through another portion of electrolyte to the iron, and to complete its course in the direction 1, 2, 3, 4, 5, 6, 7, 8, 9. In fig. 7. the main battery circuit is likewise completed, and the primary current will flow in the direction I., II., III., IV.; while the extra circuit, although apparently connected, as in fig. 6, with the zinc, is in fact connected with the copper, as in fig. 5, by means of the main battery connection, and will convey the extra current from the iron through the electrolyte to the copper, and from the copper through the galvanometer back again to the iron in the direction 1, 2, 3, 4, 5, 6, 7, 8, 9.

It will be seen that the two currents coincide in their direction in that part of their circuits which is common to both, viz. III., IV., and 3, 4.

There is no difficulty at all in understanding how an extra current is established from the iron to the copper in addition to the main current from the zinc to the copper; but I was for a long time puzzled to make out how an extra current could pass from the zinc through the electrolyte to the copper, and from the copper through the electrolyte to the iron: it seemed to me that the interposed copper must act as a retarding plate, upon the opposite surfaces of which hydrogen and oxygen must be evolved; and that the intensity of a single circle could not be sufficient to force this passage. You will, I dare say, remember suggesting an experiment which led to the explanation of the difficulty.

Some dilute sulphuric acid was poured into a basin, and a platinum crucible containing some solution of sulphate of copper was placed in it. An amalgamated zinc rod wrapped in filtering paper moistened with dilute acid, to prevent it from precipitating the copper by its local action, was held in the crucible. A plate of iron was also immersed in the acid in the basin; contact with the platinum being carefully avoided. A metallic communication was then made by means of a wire with the zinc and the iron, and we had thus the exact circumstances of the battery cell in the iron case, except that platinum was substituted for copper. No current, however, was thus formed, and no copper was precipitated upon the platinum from the solution of sulphate. A piece of copper-plate was now placed under the platinum crucible, and in contact with it: the current immediately passed, and copper was precipitated upon the interior surface of the crucible from the sulphate. The current of the single circle could not pass by the retarding plate of platina, when oxygen must have been evolved on one side and hydrogen on the other; but when the oxygen was absorbed by the copper, and the hydrogen by the oxide of copper, these concurring affinities enabled the current affinity to make good its circuit.

To vary the experiment with regard to the metallic part of the combination, three

of the battery cells were placed upon zinc plates with interposed flannel moistened with dilute sulphuric acid; the copper was thus placed between two generating plates of the same metal. When the three were connected together in regular series with a galvanometer, the deflection of the needle was 90° E. When a secondary connection was made from the zinc rod of each cell in succession through another galvanometer with the zinc plate on which it stood, the deflection occasioned by the extra current was 20° W. In this case it must have flowed from the zinc through the electrolyte in the flannel to the copper; from the copper through the electrolyte in the cell to the zinc rod; and from the zinc rod through the wire back to the zinc plate. When the main circuit was broken the extra current changed its direction, and occasioned a deflection of the needle 20° E. Upon restoring the primary current the extra current again returned to its original direction, and so invariably upon many successive repetitions of the experiment.

When the extra connection was made between the copper of the cell and the zinc plate, the deflection of the second galvanometer was always 40° W., or opposite to the main current, and was not disturbed by any interruption of the latter.

Being now satisfied that these extra currents were hydro-electric, and dependent upon the action of a liquid upon the metal which was brought into association with the regular voltaic combination, I examined more carefully the circumstances of the arrangement in which I had supposed that I had insulated the cells, and cut them off from any such influence by placing them upon thick glass plates. I now ascertained that the establishment of the extra current was owing to a thin film of moisture formed upon the glass, either by the condensation of steam, or slight leakage from the cells. At ordinary temperatures no action was thus excited, but when the temperature of the combination was sufficiently exalted very energetic currents were sometimes developed by a quantity of moisture, which might well have escaped ordinary observation. When great care was taken to make the glass plates perfectly bright and dry, the extra current was never produced.

It was now also clear that not only could independent extra currents be established, but that different portions of the main battery current could be diverted into this secondary path, and thus the occasional decomposition of water in the voltameter by the extra current could be accounted for.

There is one more relation of the *battery current*, the *diverted current* and the *extra current*, which it may be worth while to point out when they are all three established at the same moment. Let 1, 2, 3 (fig. 8) represent the section of three battery cells, all standing upon blocks of wood in iron cases, the sections of which are represented by A, B, C, the bottoms of the cases being covered with dilute acid. The main connections of the battery are made, and the principal current flows from 1 to 3, from 3 to 2, from 2 to 1. A diverted current may be led off from 1, and may pass in the direction *a b c d* through the galvanometer G, or sometimes in the opposite direction. At the same time an extra current may be established from C, the iron case of the cell 3, through the galvanometer G′, in the direction *e f g h i*.

The making or breaking of this extra current had no effect upon the diverted current *a b c d*, but the two were always in opposite directions. When the first moved the needle to the E. the second deflected its needle W. When the diverted current was W. the breaking the main current always turned it in the normal direction E, and at the same moment the needle of the extra current changed to the W. Upon restoring the main current both needles returned to their former position.

When the battery current, instead of being allowed to flow through short connections, was led through a separate galvanometer, each of the other currents passing also to separate galvanometers, the diverted current varied in the different cells from 45° to 20°, but was always in the normal direction, and the extra current was opposite. When the short connection was added to the battery, as well as the long one through the galvanometer, the latter fell from 90° E. to about 20° E., and the diverted current oscillated rapidly E. and W., and the needle of the extra current changed with it in the opposite directions W. and E.

I could now have no doubt that the explanation of the sudden stoppage and reversal of the battery current, of which I was in search, was to be found either in this diversion which I have described, or from the opposition of extra currents exalted in their power by heat, or possibly from some combination of the two. I therefore returned to the original combination of the battery in the steamer.

I soon ascertained that the extra current could be produced by a connection from any part of the tin case to any of the cells of the battery standing upon wooden blocks, and that its energy was increased both by acidulating the condensed water and by heat. I found also that by leading this current through the same galvanometer and voltameter as the battery current, that it interfered with it in different degrees, even to its stoppage and reversal. I must not attempt to give you the details of the numerous series of observations which I have made upon the subject, but will content myself with stating as concisely as possible the results of the last combination, which have proved always constant.

At figg. 9 and 10 I have represented, as before, a section of the arrangement: the course of the main current is marked by the arrow heads, and is conducted by the electrodes through the voltameter V and the galvanometer G. The battery was first charged in the usual way, and the cells were placed in the steamer upon blocks of damp wood standing in a little acidulated water. The observation commenced at the temperature of 52°, and the following are the tabulated results.

Time. h	m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.	Galvanometer. °
10	58				
11	3	5	6	6	60 E.
11	5				
11	10	5	12	6	60 E.

A connection was now made between the tin and the zinc cup of the galvanometer, as in fig. 9, by *a b c*, and the action was decreased.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.	Galvanometer. °
11 11				
11 16	5	17	5	55 E.
11 21	5	$21\frac{1}{4}$	$4\frac{1}{4}$	

The cover of the steamer was next put on, and the connections made as before by the electrodes passing through the cocks.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.	Galvanometer. °
11 40				
11 45	5	6·5	6·5	60 E.

Connection was again made from the tin (cover) to the zinc cup.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.	Galvanometer. °
11 50	5	11·5	5·	55 E.
11 52	The steam was turned on.			
11 57	5	19·	7·5	65 E.
12 2	5	27·	8·	67·5 E.
12 7	5	36·	9·	70 E.
12 9				
12 14	5	11·	11·	77 E.
12 19	5	21·5	10·5	

The tin was again connected as before.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.	Galvanometer. °
12 24	5	25·	3·5	40 E.
12 29	5	stopped.		0

Now we may observe that in this arrangement the extra current, which we have already found completing its circuit, according to circumstances, either to the zinc or copper of the battery, has a path open to it by the wire F G to the zinc of No. 2, or through the galvanometer and voltameter by the wire B A to the copper of No. 3, the latter being in opposition to the battery current. That in this series of experiments it tended to pass in the latter direction, is proved by the gradual retardation and ultimate neutralization of the latter. When the resistance to the main current was diminished by throwing the voltameter V out of the circuit by a wire passing from H to I, it again passed in the regular course, turning the needle of the galvanometer to the E.

When the secondary connection was made with the tin by the copper cup C of the galvanometer, instead of the zinc cup, the battery current through the voltameter was again stopped, but the needle of the galvanometer turned 80° E., indicating a powerful current through it in the normal direction.

These experiments were frequently repeated with the same results.

In attempting to account for all the variable phenomena of these extra and diverted

currents, it must not be overlooked that it may be possible for the whole battery, or for a certain number of the series of the battery, to force itself a passage through the electrolyte on each side of the copper to the tin, and thus to discharge itself by what would appear to be a reversed current through the secondary communication. This would be determined by the amount of different resistances in the paths which might be open to it. In this way we can account for the different degrees of power in the extra current, and for its being able to pass through the voltameter at times and not at others.

The only difficulty in now accounting for the original stoppage of the battery current which I observed, consists in my not having been aware that there was any metallic communication between the tin and any of the battery connections. I am now however convinced that, notwithstanding precautions were taken to avoid this, contact must have taken place. Indeed the distance between the connecting wires and the rim of the tin cover was but small, and the jarring occasioned by placing the cover in its place may easily have occasioned sufficient disturbance in the arrangement. This supposition also sufficiently explains why the stoppage could not invariably be produced when desired.

I think that I do not deceive myself in believing that the preceding observations may not be without interest and importance to those who are actively engaged in advancing by experiment our knowledge of one of the most wonderful and widely-diffused agencies with which matter has been endowed. If they should answer no higher purpose, they may very probably prevent the application of much labour and thought in the explanation of phenomena of a very striking but perplexing nature, which are very likely to be observed by those who are working in this field of inquiry, and of which in my own case the preceding pages are a very brief abstract. At the same time they afford an exemplification of the advantages of the *constant battery*; for both the diverted currents and the effects of temperature would have been masked and lost in the variable results of the common voltaic combinations.

The effects of heat upon single voltaic circuits have been ably investigated both by M. Marianini* and by Mr. Rogers†; but although both these gentlemen purposed to extend their observations to compound circuits, or the battery, they have probably been prevented by the cause which I have indicated. It is now, however, apparent that in the exact measures of different effects which an invariable current of electricity will enable experimentalists to undertake, the variations of atmospheric temperature even must not be neglected.

I remain, my dear Faraday,

Yours very faithfully,

J. F. Daniell.

King's College,
March, 1837.

* Annales de Chimie, tom. xxxiii. p. 132.

† Silliman's Journal, January, 1835.

Postscript.

I have just completed a *constant battery* of large dimensions, the effects of which exceed my most sanguine expectations, and open new views of the possible application of the extraordinary powers of the voltaic current to economical purposes. It consists of only ten copper cells 20 inches high, $3\frac{1}{2}$ inches diameter, as in the first battery. The interior partitions are formed by merely tying the open ends of the oxen's gullets to the rings of the colanders for supporting the sulphate of copper, and which are made deeper than before, and suspending them in the cells, to the bottoms of which they reach. These membranous bags contain each rather more than a quart of the dilute acid. The zinc rods are of the diameter of $\frac{1}{4}$th of an inch, well amalgamated, and the connections are made as before described. At the temperature of 67° this battery produces, in the voltameters which I have all along employed in these researches, 12 cubic inches of the mixed gases per minute, or 720 cubic inches per hour. Its powers of ignition are very great; and while it will maintain 6 inches of platinum wire $\frac{1}{100}$th of an inch diameter red hot, it will still decompose water at the rate of 14 cubic inches per five minutes. The permanence of this result is very striking.

When the battery is not in use the rods are taken out and wiped, and the membranous bags carefully lifted out of the cells, emptied of their acid, filled with water, and suspended from a frame placed for their reception. By this treatment I do not find that they are liable to any change of texture or deterioration; and I have now membranes which have been in use for several months and are quite perfect. If the acid be perfectly washed out of them they may even be dried with impunity; but it is better to preserve them in a moist state, as when dry they are liable to crack. The acid solution of sulphate of copper remains in the cells without injury, and in ten minutes the battery, when required, may be brought into action. There is no reason to think that the limits of efficiency have yet been nearly attained, and the gullets could easily be connected together so as to obtain bags of any required length. I scarcely, however, think that in simplicity and cheapness of construction the battery can be further improved.

J. F. D.

King's College,
15*th June*, 1837.

XI. *Analysis of the Roots of Equations. By the Rev.* R. MURPHY, *M.A., Fellow of Caius College, Honorary Member of various Philosophical Societies. Communicated by* J. W. LUBBOCK, *Esq. F.R.S.*

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1. THE object of this memoir is to show how the constituent parts of the roots of algebraical equations may be determined, by considering the conditions under which they vanish, and conversely to show the signification of each such constituent part.

2. In equations of degrees higher than the second the same constituent part of the root is found in several places governed by the same radical sign, but affected with the different corresponding roots of unity as multipliers.

3. The root of every equation, of which the coefficients are rational, contains a rational part, for the sum of the roots could not otherwise be rational.

This rational part, as such, is insusceptible of change in the different roots of the same equation, consequently its value is the coefficient of the second term (with a changed sign) divided by the number of roots, or index of the first term.

4. The supposed evanescence of any of the other constituent parts implies that a relation exists between the roots; if such a relation be expressed by equating a function of the roots to zero, that constituent part will be the product of all such functions, and a numerical factor.

5. The joint evanescence of various constituent parts implies the co-existence of various relations between the roots, and thus an interpretation may be given to each of the constituent parts, riveting the expression of the root in the memory, and beautifully converting the solution of a problem into a condensed enunciation of various theorems.

For simplicity these principles are first applied to equations of lower degrees.

6. Let us take for example the general quadratic equation

$$x^2 + a x + b,$$

the two roots of which are represented by x_1, x_2 in the formulæ,

$$x_1 = -\frac{a}{2} + \surd \alpha$$

$$x_2 = -\frac{a}{2} - \surd \alpha.$$

To find α suppose $\alpha = 0$, a relation is then established between the quantities x_1 and x_2, viz.

$$x_1 - x_2 = 0,$$

$x_1 - x_2$ vanishing with α is a factor of it; and since the roots must be symmetrically involved in α, the other factor is $x_2 - x_1$, whence

$$\alpha = k\,(x_1 - x_2)^2,$$

where k is simply a number.

Put for $-\frac{a}{2}$ its expression in terms of the roots, and we thus have

$$x_1 = \frac{x_1 + x_2}{2} + \frac{x_1 - x_2}{2} \cdot \surd 4\,k,$$

$$x_2 = \frac{x_1 + x_2}{2} - \frac{x_1 - x_2}{2}\,\surd 4\,k,$$

whence

$$x_1 - x_2 = (x_1 - x_2)\,\surd 4\,k, \text{ or } k = \tfrac{1}{4};$$

then

$$\alpha = \tfrac{1}{4}\,(x_1 - x_2)^2,$$

which is a symmetrical function, and therefore easily expressed by the coefficients.

7. From this it follows that $\alpha = 0$ is the condition that the equation may have two equal roots; but if the proposed quadratic be represented by $\varphi = 0$, and its derived equation by $\varphi' = 0$, which is the same as $2x + a = 0$, the condition for two equal roots is obtained by eliminating x between these equations, which by the theory of elimination gives as the sought condition

$$\varphi'(x_1) \cdot \varphi'(x_2) = 0;$$

we have thus

$$\alpha = k' \varphi'(x_1) \cdot \varphi'(x_2),$$

k' being numerical.

Now $\varphi(x) = (x - x_1)(x - x_2)$, therefore $\varphi'(x) = (x - x_1) + (x - x_2)$, whence

$$\varphi'(x_1) = x_1 - x_2, \quad \varphi'(x_2) = x_2 - x_1,$$

consequently $k' = -\frac{1}{4}$; and converting the sum and product of x_1, x_2 into the coefficients, we obtain

$$x_1 = -\frac{a}{2} + \surd\left(\frac{a^2}{4} - b\right) \qquad x_2 = -\frac{a}{2} - \surd\left(\frac{a^2}{4} - b\right).$$

8. The constituent parts in the roots which have been the objects of investigation were $-\frac{a}{2}$ and α, and with respect to their evanescence we have the following theorem.

The vanishing of that part of the root of a quadratic which is under the radical sign implies the existence of two equal roots.

The vanishing of the other part which is unaffected by that sign signifies that the roots are equal, but with contrary signs.

By the aid of this theorem we shall be able to find two of the three constituent parts of the roots of a cubic equation.

9. Let us now extend the same views to equations of the third degree, and let $x_1\, x_2\, x_3$ be the three roots of the cubic $x^3 + a\,x^2 + b\,x + c = 0$. Put

$$x_1 = -\frac{a}{3} + \sqrt[3]{\alpha} + \sqrt[3]{\beta}$$

$$x_2 = -\frac{a}{3} + \theta\sqrt[3]{\alpha} + \theta^2\sqrt[3]{\beta}$$

$$x_3 = -\frac{a}{3} + \theta^2\sqrt[3]{\alpha} + \theta\sqrt[3]{\beta}.$$

The numbers θ, θ^2 are the imaginary cube roots of unity, and we may observe that the formulæ for x_3, x_1 differ only from that for x_2 in having θ^2, θ respectively instead of θ.

10. The quantities α, β, which are obviously similarly involved, are the roots of a quadratic, and of the forms

$$\alpha = \alpha' + \sqrt{\alpha''}$$
$$\beta = \alpha' - \sqrt{\alpha''};$$

the three quantities $-\frac{a}{3}$, α' and α'' are the constituent parts of the roots of the cubic in the sense in which those words have been used; the first is the same as $\frac{x_1 + x_2 + x_3}{3}$, and the other two can be found from the conditions of their evanescence as follows.

11. Suppose $\alpha'' = 0$, the theorem of art. 8. gives us $\alpha = \beta$, whence we find $x_2 = x_3$; now $x_2 - x_3$ vanishing with α'' is a factor of it; and the other symmetrical factors are $x_3 - x_2$, $x_1 - x_3$, $x_3 - x_1$, $x_1 - x_2$, $x_2 - x_1$, k being a number, we must therefore have

$$\alpha'' = k\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_2 - x_3)^2.$$

12. To find α' in like manner, suppose $\alpha' = 0$ the theorem of art. 8 before referred to, in this case makes $\alpha = -\beta$, and the three roots of the cubic accordingly are changed to the following:

$$x_1 = -\frac{a}{3}$$

$$x_2 = -\frac{a}{3} + (\theta - \theta^2)\sqrt[3]{\alpha}$$

$$x_3 = -\frac{a}{3} - (\theta - \theta^2)\sqrt[3]{\alpha};$$

whence we readily find that $2\,x_1 = x_2 + x_3$, therefore $2\,x_1 - x_2 - x_3$ is a factor of α', and the other symmetrical factors are $2\,x_2 - x_1 - x_3$, and $2\,x_3 - x_1 - x_2$; hence if k' be a numerical factor

$$\alpha' = k'\,(2\,x_1 - x_2 - x_3)\,(2\,x_2 - x_1 - x_3)\,(2\,x_3 - x_1 - x^2);$$

these symmetrical functions can be expressed by the given coefficients of the equation.

13. The constants k, k' may be easily found in various ways, perhaps the simplest means is to suppose $\alpha' = 0$ to find k, and $\alpha'' = 0$ to find k'.

If we put $x_3 = 0$ and $x_1 = 2\,x_1$, we have $\alpha' = 0$, and $\alpha'' = 4\,k\,x_1^6$, hence $\alpha = 2\,x_1^3\,\surd k$, $\beta = -\,2\,x_1^3\,\surd k$, and substituting in the formula for x_2, we have (since $a = -\,3\,x_1$)

$$2\,x_1 = x_1 + x_1\,\theta\,\sqrt[3]{2\,\surd k} - x_1\,\theta^2\,\sqrt[3]{2\,\surd k}$$

or

$$1 = \theta\,(1 - \theta)\,\sqrt[3]{2\,\surd k},$$

therefore

$$2\,\surd k = \frac{1}{(-\theta)^3} = \frac{1}{3\,(\theta^2 - \theta)}$$

and

$$4\,k = \frac{1}{9\,(\theta - 2 + \theta^2)},$$

but $1 + \theta + \theta^2 = 0$, therefore

$$k = \frac{-1}{2^2 \cdot 3^3}.$$

14. In like manner to find k' suppose $x_2 = x_3 = 0$, which makes $\alpha'' = 0$, and $\alpha' + 2\,k' \cdot x_1^3$, and $\alpha = \beta = \alpha'$, therefore

$$x_1 = \frac{x_1}{3} + 2\,x_1\,\sqrt[3]{2\,k'}$$

or

$$\frac{1}{3} = \sqrt[3]{2\,k'},$$

whence

$$k' = \frac{1}{2 \cdot 3^3}$$

15. The preceding analysis furnishes the following formula,

$$3\,x_1 = (x_1 + x_2 + x_3) + \sqrt[3]{\left\{\frac{1}{2}\,(2\,x_1 - x_2 - x_3)\,(2\,x_2 - x_1 - x_3)\,(2\,x_3 - x_1 - x_2) + \frac{3}{2}\,\surd -3\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_2 - x_3)^2\right\}} + \sqrt[3]{\left\{\frac{1}{2}\,(2\,x_1 - x_2 - x_3)\,(2\,x_2 - x_1 - x_3)\,(2\,x_3 - x_1 - x_2) - \frac{3}{2}\,\surd -3\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_2 - x_3)^2\right\}}$$

the corresponding formulæ for $3\,x_2$, $3\,x_3$ being obtained by writing θ and θ^2 before the cubic radical signs.

In consequence of the negative multiplier $-\,3$ under the sign of square root it is visible that this formula is not arithmetically applicable when the three roots are real and unequal, which is usually termed the irreducible case.

16. The cubic surds in the formula above given are actually extractible, which verifies the solution.

To this end let $2x_1 - x_2 - x_3 = A$, and $x_2 - x_3 = B$, then

$$2x_2 - x_1 - x_3 = \frac{1}{2}(3B - A)$$

$$2x_3 - x_1 - x_2 = \frac{1}{2} \cdot (-3B - A)$$

therefore

$$\frac{1}{2}(2x_1 - x_2 - x_3)(2x_2 - x_1 - x_3)(2x_3 - x_1 - x_2) = \frac{1}{8}(A^3 - 9AB^2).$$

Again

$$x_1 - x_2 = \frac{1}{2}(A - B)$$

$$x_1 - x_3 = \frac{1}{2}(A + B)$$

therefore

$$\frac{3\sqrt{-3}}{2} \cdot (x_1 - x_2)(x_1 - x_3)(x_2 - x_3) = \frac{3\sqrt{-3}}{8}(A^2B - B^3)$$

the total quantity under the first cubic surd thus becomes

$$\frac{1}{8}\{A^3 + 3A^2 \cdot (B\sqrt{-3}) + 3A \cdot (B\sqrt{-3})^2 + (B\sqrt{-3})^3\},$$

the cube root of which is $\frac{1}{2}\{A + B\sqrt{-3}\}$, and the actual root of the second surd is similarly $\frac{1}{2}\{A - B\sqrt{-3}\}$. But

$$\frac{1}{2}(A + B\sqrt{-3}) = x_1 - x_2 \cdot \frac{1 - \sqrt{-3}}{2} - x_3 \cdot \frac{1 + \sqrt{-3}}{2}$$

$$= x_1 + x^2\theta^2 + x^3\theta.$$

And

$$\frac{1}{2}(A - B\sqrt{-3}) = x_1 + x_2\theta + x_3\theta;$$

the formulæ of this solution become then those of VAUDERMONDE, viz.

$$3x_1 = (x_1 + x_2 + x_3) + (x_1 + x_2\theta^2 + x_3\theta) + (x_1 + x_2\theta + x_3\theta^2)$$
$$3x_2 = (x_1 + x_2 + x_3) + (x_1\theta + x_2 + x_3\theta^2) + (x_1\theta^2 + x_2 + x_3\theta)$$
$$3x_3 = (x_1 + x_2 + x_3) + (x_1\theta^2 + x_2\theta + x_3) + (x_1\theta + x_2\theta^2 + x_3),$$

which contain a complete verification.

17. The two constituent parts $\alpha' \alpha''$ of the roots of a cubic equation have been resolved into factors by observing the relations established between the roots by their evanescence; another mode exists for forming the same quantities by elimination between the proposed equation and its first and second derived equations.

Let the given equation $x^3 + ax^2 + bx + c = 0$ be represented by $\varphi(x) = 0$, and the first derived, viz. $3x^2 + 2ax + b = 0$ by $\varphi'(x) = 0$, the second derived $6x + 2a = 0$ by $\varphi''(x) = 0$.

18. When $\alpha'' = 0$ we have seen that the equation $\varphi(x) = 0$ has two equal roots.

But when $\varphi(x) = 0$ has equal roots, is expressed by making the result of eliminating x between $\varphi(x) = 0$ and $\varphi'(x) = 0$ to vanish, and by the theory of elimination this result is $\varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3)$; hence we have ($h$ being a number),

$$\alpha'' = h\,\varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3).$$

19. Suppose now the joint evanescence of α' and α'', the equation has then three equal roots $x_1 = x_2 = x_3$, the system of equations $\alpha' = 0$ $\alpha'' = 0$, the first of three dimensions relative to the roots, the second of six, are therefore the two conditions necessary for the existence of three equal roots.

The results of the elimination of x between $\varphi(x) = 0$ $\varphi''(x) = 0$ and $\varphi(x) = 0$ $\varphi'(x) = 0$, give also the two conditions for three equal roots of the same dimensions as the above, with which this system is identical, we have thus, h' being numerical,

$$\alpha' = h'\,\varphi''(x_1) \cdot \varphi''(x_2) \cdot \varphi''(x_3).$$

20. Since

$$\varphi(x) = (x - x_1)(x - x_2)(x - x_3)$$

$$\varphi'(x) = (x - x_2)(x - x_3) + (x - x_1)(x - x_3) + (x - x_3)(x - x_2)$$

$$\varphi''(x) = 2(x - x_3) + 2(x - x_2) + 2(x - x_1);$$

therefore

$$\varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3) = -(x_1 - x_2)^2 (x_1 - x_3)^2 (x_2 - x_3)^2$$

$$\varphi''(x_1) \cdot \varphi''(x_2) \cdot \varphi''(x_3) = 8(2x_1 - x_2 - x_3)(2x_2 - x_1 - x_3)(2x_3 - x_1 - x_2);$$

the values of α' α'' are therefore conformable to those before found, and

$$h = -k = \frac{1}{2^4 \cdot 3^3} \qquad h' = \frac{k'}{8} = \frac{1}{2^4 \cdot 3^5}.$$

21. In the elimination of a quantity between two equations into which that quantity enters rationally, it is in general indifferent which of the two equations is selected that its roots may be substituted for such quantity, for the dimensions of the product is the same whether we substitute the n roots of an equation of n dimensions in one of m, or the m roots of the latter in the former of n dimensions, and take the products; these products are not only of the same dimensions but imply the coexistence of the same system of equations, and can only differ from each other by numerical multipliers, when the numerical coefficient in the one differs from that of the other.

In the present instance, if ξ_1 ξ_2 be the roots of the equation $\varphi' = 0$, and X that of $\varphi'' = 0$, the values of α' α'' may also be expressed in the following form,

$$\alpha'' = \mathrm{H}\,\varphi(\xi_1) \cdot \varphi(\xi_2)$$

$$\alpha'' = \mathrm{K} \cdot \varphi(\mathrm{X})$$

the factors H, K being numbers.

22. Now if we observe that by the equation $\varphi'(\xi_1) = 0$,

$$\xi_1^2 = -\frac{2a}{3} \cdot \xi_1 - \frac{b}{3}$$

and therefore

$$\xi_1^3 = -\frac{2a}{3}\cdot\xi_1^2 - \frac{b}{3}\cdot\xi_1 = \left(\frac{4a^2}{9} - \frac{b}{3}\right)\cdot\xi_1 + \frac{2ab}{9}$$

it will follow that $\varphi(\xi_1)$ or $\xi_1^3 + a\xi_1^2 + b\xi_1 + c$, is the same as $A\xi_1 + B$, putting

$$A = -\left(\frac{2a^2}{9} - \frac{2b}{3}\right)$$

$$B = c - \frac{ab}{9}.$$

Hence

$$\varphi(\xi_1)\cdot\varphi(\xi_2) = (A\xi_1 + B)(A\xi_2 + B);$$

and since $\xi_1\xi_2 = \frac{b}{3}$ and $\xi_1 + \xi_2 = -\frac{2a}{3}$, therefore

$$\alpha'' = H\left(A^2\cdot\frac{b}{3} - \frac{2a}{3}\cdot AB + B^2\right)$$

$$= H\left(c^2 - \frac{2ab}{3}\cdot c + \frac{4a^3}{27}\cdot c + \frac{4b^2}{27} - \frac{a^2b^2}{27}\right).$$

Again since $X = -\frac{a}{3}$ therefore

$$\alpha' = K\left(c - \frac{ab}{3} + \frac{2a^3}{27}\right).$$

Suppose a and b to vanish, then $\surd\alpha' = c\surd H$, $\alpha'' = K\cdot c$; therefore

$$x_1 = \sqrt[3]{c\{K + \surd H\}} + \sqrt[3]{c\{K - \surd H\}} = \sqrt[3]{-c}: \text{ hence}$$

$$k + \surd H = -1K - \surd H = 0;$$

therefore

$$K = -\frac{1}{2}, H = \frac{1}{4};$$

thus the transformation is affected from symmetrical functions to given coefficients.

23. Expressions for α', α'' having been found by various methods, we have also the following properties with respect to their evanescence,

$$x_1 = -\frac{a}{3} + \sqrt[3]{(\alpha' + \surd\alpha'')} + \sqrt[3]{(\alpha' - \surd\alpha'')}.$$

When the quantity (α'') under the quadratic and cubic radicals vanishes, the proposed equation has two equal roots.

When the quantity (α') under the cubic but not under the quadratic surd vanishes, two of the three differences of the roots are equal, or one root is one half the sum of the other two.

When both these quantities (α', α'') vanish conjointly, the given equation has three equal roots.

Conversely when the proposed has two equal roots the equation of condition is $\alpha'' = 0$, when it has three equal roots the two equations of condition are $\alpha' = 0$, $\alpha'' = 0$.

When the quantity $\left(-\frac{a}{3}\right)$ in the expression for the root which is unaffected by surds vanishes, one root is equal to the sum of the other two with changed signs.

When $-\frac{a}{3}$ and α'' vanish conjointly, two of the roots are each one half of the third with a changed sign.

When $-\frac{a}{3}$ and α' vanish jointly, the equation has two roots equal, but with contrary signs, and the third root is zero.

When $-\frac{a}{3}$, α', α'' all vanish simultaneously, the three roots are equal to each other and to zero.

24. *Biquadratic Equations.*—Let x_1, x_2, x_3, x_4 be the four roots of the equation

$$x^4 + a x^3 + b x^2 + c x + d,$$

the number 4 being composite allows the subdivision of the sums of the roots taken two and two into three pairs, which have rational sums, thus

$$x_1 + x_2 + \frac{a}{2} = -\left(x_3 + x_4 + \frac{a}{2}\right), \left(x_1 + x_3 + \frac{a}{2}\right) = -\left(x_2 + x_4 + \frac{a}{2}\right),$$

$$x_1 + x_4 + \frac{a}{2} = -\left(x_2 + x_3 + \frac{a}{2}\right),$$

therefore the equation, of which the roots are

$$x_1 + x_2 + \frac{a}{2}, x_1 + x_3 + \frac{a}{2}, x_1 + x_4 + \frac{a}{2}, -\left(x_3 + x_4 + \frac{a}{2}\right), \text{ \&c.}$$

is one of six dimensions, but without terms involving the odd powers of the unknown quantity, and therefore these quantities are the square roots of the roots of a cubic equation. Put therefore

$$x_1 + x_2 + \frac{a}{2} = 2\sqrt{\alpha}$$

$$x_1 + x_3 + \frac{a}{2} = 2\sqrt{\beta}$$

$$x_1 + x_4 + \frac{a}{2} = 2\sqrt{\gamma},$$

therefore

$$2x_1 + (x_1 + x_2 + x_3 + x_4) + \frac{3a}{2} = 2(\sqrt{\alpha} + \sqrt{\beta} + \sqrt{\gamma}),$$

or

$$x_1 = -\frac{a}{4} + \sqrt{\alpha} + \sqrt{\beta} + \sqrt{\gamma},$$

the quantities α, β, γ being the roots of a cubic equation, are of the form

$$\alpha = \alpha' + \sqrt[3]{\beta'} + \sqrt[3]{\beta''}$$

$$\beta = \alpha' + \theta\sqrt[3]{\beta'} + \theta^2\sqrt[3]{\beta''}$$

$$\gamma = \alpha' + \theta^2\sqrt[3]{\beta'} + \theta\sqrt[3]{\beta''},$$

when θ, θ^2 are the imaginary cube roots of unity, and β', β'' being the roots of a quadratic are of the forms

$$\beta' = \alpha'' + \surd \alpha'''$$
$$\beta'' = \alpha'' - \surd \alpha'''.$$

The three other roots of the biquadratic are

$$x_2 = 2 \surd \alpha - \left(x_1 + \frac{a}{2}\right) = -\frac{a}{4} + \surd \alpha - \surd \beta - \surd \gamma$$

$$x_3 = 2 \surd \beta - \left(x_1 + \frac{a}{2}\right) = -\frac{a}{4} - \surd \alpha + \surd \beta - \surd \gamma$$

$$x_4 = 2 \surd \gamma - \left(x_1 + \frac{a}{2}\right) = -\frac{a}{4} - \surd \alpha - \surd \beta + \surd \gamma.$$

The constituent or essentially different parts of the roots are $\frac{a}{4}$, α', α'', α''', which we proceed to analyse by the conditions of their evanescence.

25. Suppose $\alpha''' = 0$, then by art. 23 two of the roots of the cubic are equal, or $\beta = \gamma$, from whence we have $x_3 = x_4$, therefore $x_3 - x_4$ is a factor of α''', and forming all the other symmetrical factors, we have

$$\alpha''' = k (x_1 - x_2)^2 (x_1 - x_3)^2 (x_1 - x_4)^2 (x_2 - x_3)^2 (x_2 - x_4)^2 (x_3 - x_4)^2,$$

k being a numerical multiplier.

26. Next suppose $\alpha'' = 0$, then by the properties of the roots of the cubic already demonstrated we have $2\alpha = \beta + \gamma$, or $\alpha - \beta = \gamma - \alpha$, whence

$$(\surd \alpha + \surd \beta)(\surd \alpha - \surd \beta) = (\surd \gamma + \surd \alpha)(\surd \gamma - \surd \alpha),$$

therefore

$$(x_1 - x_4)(x_2 - x_3) = (x_1 - x_3)(x_4 - x_2);$$

one factor of α'' is found thus to be

$$(x_1 - x_4)(x_2 - x_3) + (x_1 - x_3)(x_2 - x_4).$$

The two remaining symmetrical factors are

$$(x_1 - x_2)(x_3 - x_4) + (x_1 - x_4)(x_3 - x_2)$$
$$(x_1 - x_3)(x_4 - x_2) + (x_1 - x_2)(x_4 - x_3);$$

and α'' is the product of all three multiplied by a numerical factor k'.

27. Again, suppose $\alpha' = 0$, then by the article above referred to $\alpha + \beta + \gamma = 0$; but

$$\beta + \gamma = (x_1 - x_2)^2 + (x_3 - x_4)^2$$
$$\alpha + \gamma = (x_1 - x_3)^2 + (x_2 - x_4)^2$$
$$\alpha + \beta = (x_1 - x_4)^2 + (x_2 - x_3)^2,$$

the sum of which being *per se* symmetrical, shows that α' has no other but a numerical factor; therefore

$$\alpha' = k' \{(x_1 - x_2)^2 + (x_1 - x_3)^2 + (x_1 - x_4)^2 + (x_2 - x_3)^2 + (x_2 - x_4)^2 + (x_3 - x_4)^2\}.$$

28. The numbers k, k', k'' may be found by the values already given for the roots of a cubic, by which we have

$$\alpha = \alpha' + \sqrt[3]{(\alpha'' + \surd \alpha''')} + \sqrt[3]{(\alpha'' - \surd \alpha''')}$$

and

$$\alpha' = \frac{\alpha + \beta + \gamma}{3}$$

therefore

$$K'' = \frac{1}{2 \cdot 3}$$

$$\alpha'' = \frac{1}{2 \cdot 3^3} \cdot (2\alpha - \beta - \gamma)(2\beta - \alpha - \gamma)(2\gamma - \alpha - \beta)$$

and the factors into which k' is multiplied are the same as

$$\frac{1}{2^3} \cdot (2\alpha - \beta - \gamma) \cdot (2\beta - \alpha - \gamma) \cdot (2\gamma - \alpha - \beta)$$

therefore

$$k' = \frac{1}{2^4 3^3}.$$

Lastly

$$\alpha''' = -\frac{1}{2^2 \cdot 3^3} \cdot (\alpha - \beta)^2 (\alpha - \gamma)^2 (\beta - \gamma)^2.$$

But since

$$\alpha + \gamma = (x_1 - x_3)^2 + (x_2 - x_4)^2$$

and

$$\beta + \gamma = (x_1 - x_4)^2 + (x_2 - x_3)^2$$

therefore

$$\alpha - \beta = 2(x_1 x_4 - x_1 x_3 + x_2 x_3 - x_2 x_4)$$
$$= 2(x_1 - x_2)(x_4 - x_3).$$

Similarly $\alpha - \gamma$ and $\beta - \gamma$ are expressed, and comparing the expression for α''' thus arising with that found before, we have

$$-2^2 \cdot 3^3 \alpha''' = 2^6 \cdot \frac{\alpha'''}{k} \text{ or } k = -\frac{2^4}{3^3}.$$

29. Let us next seek the same quantities α', α'', α''', by the theory of elimination.

When $\alpha''' = 0$, the proposed equation which it will be convenient to express by $\varphi(x) = 0$ has two equal roots, the condition for which is also obtained by eliminating x between $\varphi(x) = 0$ and the first derived $\varphi'(x) = 0$, the function of the coefficients arising from this elimination is of the same dimensions, and expresses the same condition as the constituent quantity α''', and therefore only differs from it by a numerical multiplier.

This quantity in a symmetrical form relative to the roots is therefore

$$\alpha''' = h \cdot \varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3) \cdot \varphi'(x_4)$$

and since

$$\varphi'(x_1) = (x_1 - x_2)(x_1 - x_3)(x_1 - x_4)$$

we have the same result as by the former method and $h = k$.

30. When α'' vanishes jointly with α''', then since $\alpha = \beta = \gamma$ we have also

$x_2 = x_3 = x_4$, therefore the equation $\varphi(x) = 0$ has three equal roots, and since $\alpha''' = 0$ denotes the existence of two equal roots, therefore $\alpha'' = 0$ is the additional condition for a third, or the system of equations $\alpha'' = 0$, $\alpha''' = 0$, are equivalent to the system $\varphi(x) = 0$, $\varphi'(x) = 0$, $\varphi''(x) = 0$.

Now $\frac{\varphi''(x)}{2} = 6x^2 + 3ax + b = z + b$ for abridgment, we shall next form an equation in z indicative of two equal roots, and eliminating z by the equation $z + b = 0$ we shall obtain α''.

When $x_1 = x_2$, then $6x_1^2 + 3ax_1 = 6x_1^2 - 3x_1(2x_1 + x_3 + x_4)$

$$= -3(x_1x_3 + x_1x_4)$$
$$= -3(x_1x_3 + x_2x_4)$$

the equation

$$z + 3(x_1x_3 + x_2x_4) = 0$$

expresses that $x_1 = x_2$, and forming a cubic equation by taking all the symmetrical simple equations, a condition in z for the existence of any pair of equal roots is obtained, viz.

$$\mathrm{F}(z) = \{z + 3(x_1x_3 + x_2x_4)\} \cdot \{z + 3(x_1x_2 + x_3x_4)\} \cdot \{z + 3(x_1x_4 + x_2x_3)\} = 0$$

or

$$\mathrm{F}(z) = z^3 + 3bz^2 + 9(ac - 4d)z + 27\{d(a^2 - 4b) + c^2\} = 0;$$

and if z be eliminated between this equation, and $z + b = 0$, the result multiplied by a constant will be α'' or

$$\alpha'' = h'\mathrm{F}(-b).$$

Now one factor of

$$\mathrm{F}(-b) = 2(x_1x_3 + x_2x_4) - (x_1x_2 + x_1x_4 + x_2x_3 + x_3x_4)$$
$$= (x_1 - x_2)(x_3 - x_4) + (x_1 - x_4)(x_3 - x_2);$$

the other two being symmetrical with it gives the same value of α'' as before, and $h' = k'$.

31. If $\alpha' = 0$, $\alpha'' = 0$, $\alpha''' = 0$ simultaneously, then $x_1 = x_2 = x_3 = x_4$. Now α' is a function of two dimensions, as is that arising by eliminating x between $\varphi''(x) = 0$, $\varphi'''(x) = 0$, and gives the same condition, but $\varphi'''(x) = 6(4x + a)$, therefore $\varphi''\left(\frac{-a}{4}\right)$ is the result of this elimination, therefore $\alpha' = h''\varphi''\left(\frac{-a}{4}\right)$. Now

$$\varphi''(x) = 2(6x^2 + 3ax + b)$$

therefore

$$\varphi''\left(\frac{-a}{4}\right) = 2b - \frac{3a^2}{4},$$

and our former value of α' is

$$k''\{\Sigma 3x_1^2 - \Sigma 2x_1x_2\} = k''(3a^2 - 8b)$$

hence

$$h'' = -4k''.$$

32. Collecting the results of the last three articles they give us

$$\alpha' = -\frac{2}{3} \cdot \varphi''\left(\frac{-a}{4}\right)$$

$$\alpha'' = \frac{1}{2^4 \cdot 3^3} \cdot \mathrm{F}(-b)$$

$$\alpha''' = -\frac{2^4}{3^3} \cdot \varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3) \cdot \varphi'(x_4),$$

all of which may be expressed easily in terms of the coefficients.

33. *Theorems deduced.*—The root of the biquadratic $x^4 + a x^3 + b x^2 + c x + d = 0$ being expressed by

$$x_1 = -\frac{a}{4} + \sqrt{\{\alpha' + \sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}\}}$$
$$+ \sqrt{\{\alpha' + \theta \sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \theta^2 \sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}\}}$$
$$+ \sqrt{\{\alpha' + \theta^2 \sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \theta \sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}\}},$$

where θ, θ^2 are the imaginary cube roots of unity, then the condition $\alpha''' = 0$ denotes the existence of two equal roots in the proposed equation.

The condition $\alpha'' = 0$ denotes the following relation of the roots

$$(x_1 - x_4)(x_2 - x_3) + (x_1 - x_3)(x_2 - x_4) = 0.$$

The system of coexisting conditions $\alpha'' = 0$, $\alpha''' = 0$ are necessary and sufficient for the existence of three equal roots.

The condition $\alpha' = 0$ denotes the following relation of the roots,

$$\Sigma (x_1 - x_2)^2 = 0.$$

The simultaneous system of conditions $\alpha' = 0$, $\alpha'' = 0$, $\alpha''' = 0$ essentially and sufficiently express the coexistence of four equal roots.

The rational part of the root $-\frac{a}{4}$ only vanishes with the sum of the roots.

34. We now proceed to determine the constituent parts of the roots of equations of the fifth degree by the conditions of their evanescence.

θ, θ^2 represent the imaginary cube roots of unity.

ω, ω^2, ω^3, ω^4 the imaginary fifth roots of unity.

x_1, x_2, x_3, x_4, x_5 are the five roots of the proposed equation of the fifth degree, viz.

$$x^5 + a x^4 + b x^3 + c x^2 + d x + e = 0.$$

$$x_1 = -\frac{a}{5} + \sqrt[5]{\alpha} + \sqrt[5]{\beta} + \sqrt[5]{\gamma} + \sqrt[5]{\delta}$$

$$x_2 = -\frac{a}{5} + \omega \sqrt[5]{\alpha} + \omega^2 \sqrt[5]{\beta} + \omega^3 \sqrt[5]{\gamma} + \omega^4 \sqrt[5]{\delta}$$

$$x_3 = -\frac{a}{5} + \omega^2 \sqrt[5]{\alpha} + \omega^4 \sqrt[5]{\beta} + \omega \sqrt[5]{\gamma} + \omega^3 \sqrt[5]{\delta}$$

$$x_4 = -\frac{a}{5} + \omega^3 \sqrt[5]{\alpha} + \omega \sqrt[5]{\beta} + \omega^4 \sqrt[5]{\gamma} + \omega^2 \sqrt[5]{\delta}$$

$$x_5 = -\frac{a}{5} + \omega^4 \sqrt[5]{\alpha} + \omega^3 \sqrt[5]{\beta} + \omega^2 \sqrt[5]{\gamma} + \omega \sqrt[5]{\delta};$$

the formulæ for x_3, x_4, x_5, x_1 are derived from the formula for x_2, by writing successively ω^2, ω^3, ω^4, ω^5 instead of ω.

$$\alpha = \alpha' + \sqrt{\beta'} + \sqrt{\gamma'} + \sqrt{\delta'}$$
$$\beta = \alpha' + \sqrt{\beta'} - \sqrt{\gamma'} - \sqrt{\delta'}$$
$$\gamma = \alpha' - \sqrt{\beta'} + \sqrt{\gamma'} - \sqrt{\delta'}$$
$$\delta = \alpha' - \sqrt{\beta'} - \sqrt{\gamma'} + \sqrt{\delta'},$$

such being the forms of the roots of a biquadratic.

Again, the expressions for β', γ', δ' as roots of a cubic, are

$$\beta' = \alpha'' + \sqrt[3]{\beta''} + \sqrt[3]{\gamma''}$$
$$\gamma' = \alpha'' + \theta \sqrt[3]{\beta''} + \theta^2 \sqrt[3]{\gamma''}$$
$$\delta' = \alpha'' + \theta^2 \sqrt[3]{\beta''} + \theta \sqrt[3]{\gamma''}.$$

Lastly, β'', γ'' as roots of a quadratic, are expressed by the following formulæ:

$$\beta'' = \alpha''' + \sqrt{\alpha^{iv}}$$
$$\gamma'' = \alpha''' - \sqrt{\alpha^{iv}}.$$

The quantities $-\frac{a}{5}$, α', α'', α''', α^{iv} are the constituent or essentially distinct parts of the roots x_1, x_2, x_3, x_4, x_5, and the analysis of their formation is to be sought by observing all the conditions under which each may vanish.

35. If for $-\frac{a}{5}$ we put $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$, it is obvious that the system of five equations for the roots is equivalent to one of only four, viz.

$$5\sqrt[5]{\alpha} = x_1 + \omega^4 x_2 + \omega^3 x_3 + \omega^2 x_4 + \omega x_5$$
$$5\sqrt[5]{\beta} = x_1 + \omega^3 x_2 + \omega x_3 + \omega^4 x_4 + \omega^2 x_5$$
$$5\sqrt[5]{\gamma} = x_1 + \omega^2 x_2 + \omega^4 x_3 + \omega x_4 + \omega^3 x_5$$
$$5\sqrt[5]{\delta} = x_1 + \omega x_2 + \omega^2 x_3 + \omega^3 x_4 + \omega^4 x_5$$

the right hand members of which equations differ from each other only by the particular fifth root of unity in use, which considered as ω in the first, will be ω^2, ω^3, ω^4 respectively in the second, third, and fourth.

36. Suppose $\alpha^{iv} = 0$, then by art. 33 two roots of the biquadratic must be equal.

First let $\gamma = \delta$, which can only happen under the five following relations,

$$\sqrt[5]{\gamma} = \sqrt[5]{\delta}, \sqrt[5]{\gamma} = \omega\sqrt[5]{\delta}, \sqrt[5]{\gamma} = \omega^2\sqrt[5]{\delta}, \sqrt[5]{\gamma} = \omega^3\sqrt[5]{\delta}, \sqrt[5]{\gamma} = \omega^4\sqrt[5]{\delta},$$

which furnish five factors, linear functions of the differences of the roots, and since

these differences may be taken either way, we have also from the same equations five other factors equal to the former and with contrary signs.

In this manner ten factors of α^{IV} may be found by equating any pair of the four quantities $\alpha, \beta, \gamma, \delta$, and the number of pairs being six, the whole number of factors of α^{IV} is sixty, these factors are very easily formed, and here we present the first thirty factors, the remaining thirty being formed merely by changing the signs of these, or, which is the same, inverting the order of the differences of the roots in each factor.

37. For greater clearness we shall subdivide these thirty factors into five groups, from the first of which x_1 is excluded, from the second x_2, and so on; each subdivision contains six factors, four of which are of one form, and two of a different form; they are as follow:

$$\left.\begin{aligned}
&(x_2 - x_4) + \omega(x_3 - x_4) + \omega^2(x_3 - x_5) \quad (1.)\\
&(x_3 - x_2) + \omega(x_5 - x_2) + \omega^2(x_5 - x_4) \quad (2.)\\
&(x_4 - x_5) + \omega(x_2 - x_5) + \omega^2(x_2 - x_3) \quad (3.)\\
&(x_5 - x_3) + \omega(x_4 - x_3) + \omega^2(x_4 - x_2) \quad (4.)\\
&(x_2 - x_5) + (\omega^2 + \omega^3)(x_3 - x_4) \quad (5.)\\
&(x_3 - x_4) + (\omega^2 + \omega^3)(x_5 - x_2) \quad (6.)
\end{aligned}\right\} (A.)$$

$$\left.\begin{aligned}
&(x_1 - x_4) + \omega(x_5 - x_4) + \omega^2(x_5 - x_3) \quad (1.)\\
&(x_3 - x_5) + \omega(x_4 - x_5) + \omega^2(x_4 - x_1) \quad (2.)\\
&(x_4 - x_3) + \omega(x_1 - x_3) + \omega^2(x_1 - x_5) \quad (3.)\\
&(x_5 - x_1) + \omega(x_3 - x_1) + \omega^2(x_3 - x_4) \quad (4.)\\
&(x_1 - x_3) + (\omega^2 + \omega^3)(x_5 - x_4) \quad (5.)\\
&(x_5 - x_4) + (\omega^2 + \omega^3)(x_3 - x_1) \quad (6.)
\end{aligned}\right\} (B.)$$

$$\left.\begin{aligned}
&(x_1 - x_2) + \omega(x_4 - x_2) + \omega^2(x_4 - x_5) \quad (1.)\\
&(x_2 - x_5) + \omega(x_1 - x_5) + \omega^2(x_1 - x_4) \quad (2.)\\
&(x_4 - x_1) + \omega(x_5 - x_1) + \omega^2(x_5 - x_2) \quad (3.)\\
&(x_5 - x_4) + \omega(x_2 - x_4) + \omega^2(x_2 - x_1) \quad (4.)\\
&(x_1 - x_5) + (\omega^2 + \omega^3)(x_4 - x_2) \quad (5.)\\
&(x_4 - x_2) + (\omega^2 + \omega^3)(x_5 - x_1) \quad (6.)
\end{aligned}\right\} (C.)$$

$$\left.\begin{aligned}
&(x_1 - x_5) + \omega(x_3 - x_5) + \omega^2(x_3 - x_2) \quad (1.)\\
&(x_2 - x_3) + \omega(x_5 - x_3) + \omega^2(x_5 - x_1) \quad (2.)\\
&(x_3 - x_1) + \omega(x_2 - x_1) + \omega^2(x_2 - x_5) \quad (3.)\\
&(x_5 - x_2) + \omega(x_1 - x_2) + \omega^2(x_1 - x_3) \quad (4.)\\
&(x_1 - x_2) + (\omega^2 + \omega^3)(x_3 - x_5) \quad (5.)\\
&(x_3 - x_5) + (\omega^2 + \omega^3)(x_2 - x_1) \quad (6.)
\end{aligned}\right\} (D.)$$

$$\left.\begin{aligned}
&(x_1 - x_3) + \omega\,(x_2 - x_3) + \omega^2\,(x_2 - x_4) \quad \ldots\ldots\ (1.)\\
&(x_2 - x_1) + \omega\,(x_4 - x_1) + \omega^2\,(x_4 - x_3) \quad \ldots\ldots\ (2.)\\
&(x_3 - x_4) + \omega\,(x_1 - x_4) + \omega^2\,(x_1 - x_2) \quad \ldots\ldots\ (3.)\\
&(x_4 - x_2) + \omega\,(x_3 - x_2) + \omega^2\,(x_3 - x_1) \quad \ldots\ldots\ (4.)\\
&(x_1 - x_4) + (\omega^2 + \omega^3)\,(x_2 - x_3) \quad \ldots\ldots\ (5.)\\
&(x_2 - x_3) + (\omega^2 + \omega^3)\,(x_4 - x_1) \quad \ldots\ldots\ (6.)
\end{aligned}\right\}(\text{E.})$$

The other thirty factors of α^{IV} differ from these only in having ω^4 instead of ω, thus changing the sign of (1.), (A.), to obtain the first of these factors, and writing it in an inverted order it gives

$$\omega^2\,\{(x_5 - x_3) + \omega^4\,(x_4 - x_3) + \omega^3\,(x_4 - x_2)\}$$

differing from (4.), (A.) in having ω^4 for ω, for the numerical factor ω^2 may be rejected since $(\omega^2)^{30} = 1$.

The quantity α^{IV} is the product of these sixty factors and a numerical constant k.

38. Formation of the factors of α'''.

The factors in the group A of the preceding article denote for abridgment by their number placed as a subindex to A, and so for all the others, thus by B_3 is meant the third factor in the group B.

The quantity α''' is composed of the three factors of ten dimensions each. Every such factor is the sum of two parts, each decomposable into ten simple factors.

In the first pair of these simple factors x_1 does not enter, in the second pair x_2 is excluded, and so on.

These three compound factors are found as follows: First,

$$A_1 = \frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \sqrt[5]{\delta}) \qquad A_2 = -\frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\beta} - \sqrt[5]{\delta})$$

$$B_2 = \frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega\sqrt[5]{\delta}) \qquad B_3 = -\frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\beta} - \omega^2\sqrt[5]{\delta})$$

$$C_3 = \frac{5}{(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega^2\sqrt[5]{\delta}) \qquad C_4 = -\frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\beta} - \omega^4\sqrt[5]{\delta})$$

$$D_4 = \frac{5}{\omega^2\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega^3\sqrt[5]{\delta}) \qquad D_1 = -\frac{5}{(\omega - 1)}\,(\sqrt[5]{\beta} - \omega\sqrt[5]{\delta})$$

$$E_1 = \frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega^4\sqrt[5]{\delta}) \qquad E_2 = -\frac{5}{\omega^2\,(\omega - 1)}\,(\sqrt[5]{\beta} - \omega^3\sqrt[5]{\delta})$$

$$A_3 = \frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \sqrt[5]{\gamma}) \qquad A_4 = -\frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \sqrt[5]{\beta})$$

$$B_4 = \frac{5}{(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^2\sqrt[5]{\gamma}) \qquad B_1 = -\frac{5}{(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega\sqrt[5]{\beta})$$

$$C_1 = \frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^4\sqrt[5]{\gamma}) \qquad C_2 = -\frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^2\sqrt[5]{\beta})$$

$$D_2 = \frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega\sqrt[5]{\gamma}) \qquad D_3 = -\frac{5}{\omega^2\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^3\sqrt[5]{\beta})$$

$$E_3 = \frac{5}{\omega^3(\omega-1)}(\sqrt[5]{\alpha} - \omega^3\sqrt[5]{\gamma}) \qquad E_4 = -\frac{5}{\omega^3(\omega-1)}(\sqrt[5]{\alpha} - \omega^4\sqrt[5]{\beta})$$

$$A_5 = \frac{5}{\omega^3(\omega-1)}(\sqrt[5]{\beta} - \sqrt[5]{\gamma}) \qquad A_6 = \frac{5}{\omega^3(\omega-1)}(\sqrt[5]{\alpha} - \sqrt[5]{\delta})$$

$$B_5 = -\frac{5}{\omega-1}(\sqrt[5]{\beta} - \omega\sqrt[5]{\gamma}) \qquad B_6 = -\frac{5}{\omega(\omega-1)}(\sqrt[5]{\alpha} - \omega^3\sqrt[5]{\delta})$$

$$C_5 = \frac{5}{\omega^3(\omega-1)}(\sqrt[5]{\beta} - \omega^2\sqrt[5]{\gamma}) \qquad C_6 = -\frac{5}{\omega-1}(\sqrt[5]{\alpha} - \omega\sqrt[5]{\delta})$$

$$D_5 = -\frac{5}{\omega(\omega-1)}(\sqrt[5]{\beta} - \omega^3\sqrt[5]{\gamma}) \qquad D_6 = \frac{5}{\omega^4(\omega-1)}(\sqrt[5]{\alpha} - \omega^4\sqrt[5]{\delta})$$

$$E_5 = \frac{5}{\omega^4(\omega-1)}(\sqrt[5]{\beta} - \omega^4\sqrt[5]{\gamma}) \qquad E_6 = \frac{5}{\omega^3(\omega-1)}(\sqrt[5]{\alpha} - \omega^2\sqrt[5]{\delta})$$

Therefore

$$\left.\begin{aligned}
\gamma - \delta &= h\,.\,A_1 B_2 C_3 D_4 E_1 \\
\beta - \delta &= -h\,.\,A_2 B_3 C_4 D_1 E_2 \\
\alpha - \gamma &= h\,.\,A_3 B_4 C_1 D_2 E_3 \\
\alpha - \beta &= -h\,.\,A_4 B_1 C_2 D_3 E_4 \\
\beta - \gamma &= h\,.\,A_5 B_5 C_5 D_5 E_5 \\
\alpha - \delta &= h\,.\,A_6\,.\,B_6\,.\,C_6\,.\,D_6\,.\,E_6
\end{aligned}\right\}, \quad h = \left(\frac{\omega-1}{5}\right)^5.$$

39. Now the conditions necessary for the evanescence of α''' are by art. 33.

$$(\alpha-\delta)(\beta-\gamma) + (\alpha-\gamma)(\beta-\delta) = 0$$
$$(\alpha-\gamma)(\delta-\beta) + (\alpha-\beta)(\delta-\gamma) = 0$$
$$(\alpha-\beta)(\gamma-\delta) + (\alpha-\delta)(\gamma-\beta) = 0.$$

Substitute the values of $\alpha-\delta$, $\beta-\gamma$, &c. above found, and including h^2 in the numerical multiplier k of the whole, we shall have

$$\begin{aligned}
\alpha''' = k'\,&(A_5 A_6\,.\,B_5 B_6\,.\,C_5 C_6\,.\,D_5 D_6\,.\,E_5 E_6 - A_2 A_3\,.\,B_3 B_4\,.\,C_4 C_1\,.\,D_1 D_2\,.\,E_2 E_3) \\
&\times (A_5 A_6\,.\,B_5 B_6\,.\,C_5 C_6\,.\,D_5 D_6\,.\,E_5 E_6 + A_4 A_1\,.\,B_1 B_2\,.\,C_2 C_3\,.\,D_3 D_4\,.\,E_4 E_1) \\
&\times (A_2 A_3\,.\,B_3 B_4\,.\,C_4 C_1\,.\,D_1 D_2\,.\,E_2 E_3 + A_4 A_1\,.\,B_1 B_2\,.\,C_2 C_3\,.\,D_3 D_4\,.\,E_4 E_1).
\end{aligned}$$

40. The condition that α'' may vanish is $\Sigma(\alpha-\beta)^2 = 0$ by art. 33. Hence

$$\begin{aligned}
\alpha'' = k''\,&(A_1^2 B_2^2 C_3^2 D_4^2 E_1^2 + A_2^2 B_3^2 C_4^2 D_1^2 E_2^2 + A_5^2 B_5^2 C_5^2 D_5^2 E_5^2 \\
&+ A_3^2 B_4^2 C_1^2 D_2^2 E_3^2 + A_4^2 B_1^2 C_2^2 D_3^2 E_4^2 + A_6^2 B_6^2 C_6^2 D_6^2 E_6^2),
\end{aligned}$$

k'' being a numerical quantity.

41. The terms which compose α' are indicated below by the indices which enter them placed between brackets.

$$[5] = 4\,\Sigma x_1^5$$
$$[4,1] = -5\,\Sigma x_1^4 x_2$$
$$[3,2] = -10\,\Sigma x_1^3 x_2^2$$

$$
\begin{aligned}
[3,1,1] = &-20x_1^3(x_2x_3+x_2x_4+x_3x_5+x_4x_5-4x_2x_5-4x_3x_4)\\
&-20x_2^3(x_1x_4+x_1x_5+x_3x_4+x_3x_5-4x_1x_3-4x_4x_5)\\
&-20x_3^3(x_1x_2+x_1x_4+x_2x_5+x_4x_5-4x_1x_5-4x_2x_4)\\
&-20x_4^3(x_1x_3+x_1x_5+x_2x_3+x_2x_5-4x_1x_2-4x_3x_5)\\
&-20x_5^3(x_1x_2+x_1x_3+x_2x_4+x_3x_4-4x_1x_4-4x_2x_3)\\
[2,2,1] = &-30\{x_1^2x_2^2(x_3+x_5-4x_4)+x_1^2x_3^2(x_4+x_5-4x_2)\\
&+x_1^2x_4^2(x_2+x_3-4x_5)+x_1^2x_5^2(x_2+x_4-4x_3)\\
&+x_2^2x_3^2(x_1+x_4-4x_5)+x_2^2x_4^2(x_1+x_5-4x_3)\\
&+x_2^2x_5^2(x_3+x_4-4x_1)+x_3^2x_4^2(x_2+x_5-4x_1)\\
&+x_3^2x_5^2(x_1+x_2-4x_4)+x_4^2x_5^2(x_1+x_3-4x_2)\}\\
[2,1,1,1] = &-60\,\Sigma x_1^2x_2x_3x_4\\
[1,1,1,1,1] = &\;480\,x_1x_2x_3x_4x_5.
\end{aligned}
$$

The quantity α' is the sum of all these multiplied by a constant k'''.

42. It remains to give the value of the constants k, k', k'', k''', which may be easily found by comparison of the above values with the constituent parts of the roots of a biquadratic; they are as follows:

$$k = -\left(\frac{\omega-1}{5}\right)^{60}\cdot\frac{2^4}{3^5}$$

$$k' = \left(\frac{\omega-1}{5}\right)^{30}\cdot\frac{1}{2^4.3^5}$$

$$k'' = \left(\frac{\omega-1}{5}\right)^{10}\cdot\frac{1}{2.3}$$

$$k''' = \left(\frac{1}{5}\right)^5\cdot\frac{1}{4};$$

and the term uninfluenced by surds is $\frac{x_1+x_2+x_3+x_4+x_5}{5}$.

43. All the constituent parts except the last-mentioned vanish when all the roots are equal, but for their separate evanescence the condition for the equality of roots is insufficient, the requisite conditions are easily seen from the factors of such parts already given; they are of two classes, arising from the different relations of the couples $\{(\omega,\omega^2),(\omega,\omega^3),(\omega^3,\omega^4),(\omega^3,\omega^4)\}$, and of the two couples $\{(\omega^2,\omega^3),(\omega,\omega^4)\}$, each member of which contains the same imaginary part with the other, which is not the case with the former couples.

44. I cannot at present, from the pressure of other engagements, pursue these investigations further; if any additional light to the analyst is furnished by the preceding imperfect reflections on a subject so often treated, the author's object will in a great measure be attained; they at least tend to show the imperfection of our

knowledge with respect to the conditions resulting from elimination, where more than two equations are concerned, and they exhibit in the higher powers relations between pairs of roots which have not been as yet expressed, even by the differential calculus.

London, *April* 1, 1837.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY,

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge.....=83 feet 2½ in.

_____ above the mean level of the Sea (presumed about) ....=95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House.....=79 feet.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR JULY AND AUGUST, 1836.

1836.		9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
		Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
							9 A.M.	3 P.M.	Lowest.	Highest.			
JULY	F 1	30.136	71.2	30.134	74.7	62	68.2	81.2	61.9	77.6		E	A.M. Thick haze. P.M. Fine & cloudless. Evening, Cloudy.
	S 2	30.220	79.8	30.200	79.2	67	75.0	80.6	65.3	76.9		SE	Fine—light clouds and wind. Evening, Fine and clear.
	⊙ 3	30.275	79.7	30.243	77.9	60	69.6	78.8	56.4	79.9		SW	A.M. Cloudy—light wind. P.M. Fine and cloudless. Evening, Fine and clear.
	M 4	30.291	81.3	30.229	79.5	63	74.6	84.6	61.6	85.2		SW	Fine and cloudless—light haze. Evening, Fine and clear.
	T 5	30.170	79.3	30.087	79.9	65	76.2	83.6	63.8	77.8		E	A.M. Fine—light clouds. P.M. Fine and cloudless. Evening, Cloudy, with lightning.
	W 6	30.071	80.0	30.126	81.4	70	72.7	74.2	67.2	77.6	.116	SSW	A.M. Thunder and lightning, with heavy rain and wind. P.M. Fine—light clouds. Evening, Cloudy.
	T 7	30.261	77.4	30.249	78.4	58	66.0	72.2	57.2	73.6		SSW	Fine—light clouds and wind. Evening, Cloudy.
	F 8	30.305	74.4	30.301	75.6	58	64.5	71.2	58.3	73.5		WSW	Fine—light clouds, with light brisk wind. Evening, Cloudy.
	S 9	30.257	75.6	30.220	76.6	59	69.6	75.4	58.9	76.7		SW	A.M. Cloudy—light wind. P.M. Fine—nearly cloudless. Evening, Cloudy.
	⊙ 10	30.158	75.3	30.144	77.3	65	72.0	78.8	61.4	80.7		SW	A.M. Cloudy—light wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	M 11	30.142	81.3	30.041	79.5	65	73.3	80.7	63.8	81.2		SSW	Fine and cloudless—light wind. Evening, Cloudy.
	T 12	29.734	73.8	29.814	77.4	67	67.2	71.5	65.0	74.3		SSW	A.M. Overcast—very light rain. P.M. Cloudy—light shower. Evening, Cloudy.
	● W 13	30.075	77.6	30.004	75.5	53	64.5	71.6	53.8	72.2		SW	A.M. Cloudy—light wind. P.M. Fine and cloudless—light wind. Evening, Overcast—light brisk wind.
	T 14	30.002	75.5	30.006	75.4	59	65.4	70.6	58.6	71.7		SW	Fine—light clouds, with light brisk wind. Evening, Cloudy.
	F 15	29.898	69.2	29.757	71.4	59	61.6	63.8	57.0	66.4		S	A.M. Overcast—light brisk wind. P.M. Overcast—heavy rain, with high wind. Evening, Fine and clear.
	S 16	29.774	69.6	29.828	71.8	53	61.2	68.0	50.4	69.0	.316	SW	Fine—light clouds and wind. Evening, Overcast—light wind.
	⊙ 17	29.911	68.2	29.998	71.2	54	61.8	68.6	58.5	69.8		W	A.M. Lightly overcast—light brisk wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	M 18	30.140	72.4	30.093	71.6	54	62.6	69.6	54.8	70.8		W	A.M. Lightly overcast—light brisk wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	T 19	29.938	66.9	29.786	69.9	54	62.8	64.7	52.7	67.8		SSW	Cloudy—light rain, with light brisk wind. Evening, Overcast—light rain.
	W 20	29.560	62.5	29.453	64.2	54	54.3	57.8	52.8	59.2	.311	E	A.M. Overcast—lt. steady rain & wind. P.M. Cloudy—lt. wind.
	T 21	29.677	67.7	29.665	61.2	51	59.2	61.3	46.2	63.2	.269	SW	A.M. Fine—light clouds and wind. P.M. Overcast—thunder and lightning, with heavy rain. Evening, Fine and clear.
	F 22	29.794	64.9	29.816	65.9	51	58.5	61.4	50.3	65.0	.233	WSW	A.M. Lightly overcast—light rain and wind. P.M. Cloudy—light wind.
	S 23	30.070	65.8	30.069	66.3	52	60.4	64.6	51.7	65.8		SW	Fine—light clouds and wind. Evening, Cloudy.
	⊙ 24	29.835	61.3	30.705	65.3	53	55.8	64.3	51.3	65.0	.047	S	A.M. Overcast—light steady rain & wind. P.M. Cloudy—light wind.
	M 25	29.800	62.3	29.895	67.2	54	57.0	64.7	52.8	65.7	.186	WNW	Overcast—light brisk wind. Evening, very light rain.
	T 26	30.138	67.7	30.134	69.5	55	62.6	68.2	54.3	71.3		S	A.M. Fine—light clouds and wind. P.M. Lightly overcast.
	W 27	30.168	66.3	30.164	71.3	59	64.7	72.9	60.2	73.6		SW	A.M. Overcast. P.M. Cloudy—light wind. Evening, Fine & clear.
	○ T 28	30.091	72.5	30.014	72.8	60	67.5	75.5	55.8	76.8		SSW	A.M. Fine—nearly cloudless. P.M. Fine—light clouds and wind. Evening, Cloudy.
	F 29	29.689	68.0	29.594	72.5	62	63.3	68.2	61.4	69.7	.250	NE var.	Overcast—light rain and wind. Heavy rain, 6½ A.M. Evening, Cloudy—light wind.
	S 30	29.990	67.4	30.152	69.2	54	61.6	61.4	54.9	65.5		SW	Fine—light clouds, with light brisk wind. Evening, Cloudy.
	⊙ 31	30.453	70.3	30.433	69.8	54	59.2	66.2	50.5	66.8	.158	SW	A.M. Fine—light clouds & wind. Evening, Cloudy—light wind.
	MEANS..	30.033	71.8	30.001	72.9	58.2	68.2	70.8	57.1	71.9	Sum. 1.886	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.919 / 3 P.M. 29.883
AUGUST	M 1	30.181	64.7	30.059	69.7	56	60.4	69.0	54.3	69.7		SSW	A.M. Overcast—light rain and wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	T 2	30.080	70.0	30.097	68.9	51	68.9	67.6	51.6	69.0		SW	Fine—light clouds and wind. Evening, Fine and clear.
	W 3	30.029	70.7	29.871	72.3	57	65.3	74.4	54.3	74.6		SE	Fine—light clouds and wind. Evening, Fine and clear.
	T 4	29.859	69.3	29.899	72.4	60	66.3	68.8	57.0	72.2		SW	Lightly overcast—light wind. Evening, Cloudy.
	F 5	30.000	66.7	30.016	71.3	60	61.6	66.8	58.7	70.5		N	A.M. Overcast—light wind. P.M. Overcast—light rain and wind. Evening, Fine and clear.
	S 6	30.119	67.7	30.158	71.5	59	63.6	65.4	57.2	67.5		N	Overcast—light brisk wind. Evening, Fine and clear.
	⊙ 7	30.245	68.2	30.204	70.2	57	59.2	67.5	51.9	68.9		N	A.M. Fine—light clouds and wind. P.M. Overcast.
	M 8	30.202	70.2	30.186	71.0	57	62.0	70.0	54.5	70.5		N	Fine—light clouds and wind. Evening, Fine and clear.
	T 9	30.241	67.0	30.188	68.9	58	60.8	66.4	54.9	69.2		N	A.M. Lightly overcast—light wind. P.M. Fine & cloudless—light wind.
	W 10	30.204	66.4	30.166	69.0	57	59.6	69.5	50.9	69.0		N	Fine—light clouds, with light brisk wind. Evening, Cloudy.
	T 11	30.307	64.4	30.303	68.9	54	62.0	69.0	53.7	68.9		NE	A.M. Cloudy—brisk wind. P.M. Fine—light clouds, with brisk wind. Evening, Cloudy.
	● F 12	30.348	64.0	30.303	67.5	58	59.5	68.8	56.4	69.0		ENE	A.M. Overcast—light brisk wind. P.M. Fine—nearly cloudless.
	S 13	30.225	69.2	30.122	69.9	62	65.4	72.0	54.5	72.0		NE	A.M. Fine—light clouds, with brisk wind. P.M. Fine—light clouds and wind.
	⊙ 14	29.930	66.0	29.887	71.0	62	62.2	71.8	59.5	71.9	.272	N	A.M. Overcast—light rain—thunder and lightning, with heavy rain early. P.M. Cloudy—light wind.
	M 15	29.960	67.0	29.988	71.2	62	62.5	71.8	59.0	72.5		WSW	Overcast—light wind.
	T 16	30.194	65.0	30.158	69.0	59	62.7	68.7	54.5	71.6		NNE	A.M. Overcast—light wind. P.M. Cloudy.
	W 17	30.111	69.5	30.113	72.9	63	66.4	73.2	57.2	73.4		WSW	Fine—light clouds and wind.
	T 18	30.081	68.0	29.994	72.0	61	63.8	71.0	56.7	72.0		SW	Overcast—light wind.
	F 19	30.196	69.0	30.188	69.8	52	59.2	66.5	54.5	67.0	.044	SSW var.	A.M. Cloudy—light brisk wind. P.M. Fine—light clouds & wind.
	S 20	29.956	64.5	29.766	66.5	58	59.0	62.0	52.4	62.4		SSW	Overcast throughout the day.
	⊙ 21	29.954	64.0	29.950	67.0	52	58.5	65.3	51.9	66.8	.075	NNE	A.M. Overcast. P.M. Fine—light clouds.
	M 22	29.794	63.9	29.724	69.0	58	59.0	66.7	53.3	67.4	.013	SW var.	A.M. Overcast. P.M. Cloudy—light wind.
	T 23	29.641	64.5	29.691	67.5	57	60.2	63.6	57.2	63.9	.027	N	Overcast throughout the day.
	W 24	29.976	60.2	30.079	65.0	55	55.4	61.5	49.5	61.5	.125	NE	A.M. Overcast—brisk wind. P.M. Fine—light clouds and wind.
	T 25	30.168	61.2	30.115	64.3	53	58.2	64.4	48.4	65.3		E	Overcast throughout the day.
	○ F 26	29.984	61.5	29.976	64.5	58	59.0	65.2	55.6	65.5		SW	Overcast—light rain and wind.
	S 27	29.936	66.0	29.913	67.6	57	62.0	69.5	52.7	70.2	.088	WSW	Fine—light clouds and wind.
	⊙ 28	29.964	62.9	29.968	66.8	59	59.2	65.0	56.4	65.7	.161	E	A.M. Overcast—light rain. P.M. Cloudy.
	M 29	30.148	64.9	30.130	67.8	54	58.5	67.0	51.0	67.7	.188	WNW	A.M. Fine—light clouds and wind. P.M. Cloudy. Evening, Fine and clear.
	T 30	30.212	64.8	30.176	68.9	55	61.0	68.0	52.5	68.0		SSW	P.M. Overcast—light wind. P.M. Cloudy.
	W 31	30.113	69.0	30.035	68.9	60	63.5	69.8	56.0	70.9		SSW	Fine—light clouds and wind. Evening, Fine and clear.
	MEANS..	30.076	66.1	30.046	69.1	57.5	61.5	67.9	54.5	68.9	Sum. .993	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.978 / 3 P.M. 29.939

*** The observations for this month were not taken by the Assistant Secretary on account of absence.

METEOROLOGICAL JOURNAL FOR SEPTEMBER AND OCTOBER, 1836.

1836.		9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit.		Self-registering.		Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
		Barom.	Attach. Therm.	Barom.	Attach. Therm.		9 A.M.	3 P.M.	Lowest.	Highest.			
	T 1	29.816	69.8	29.794	69.4	57	66.4	68.5	56.2	69.6		SW	Fine—nearly cloudless—light wind. Evening, Cloudy.
	F 2	29.768	65.4	29.724	66.4	57	60.3	59.6	54.4	63.5		SW var.	{A.M. Cloudy—very light rain and wind. P.M. Fine—light clouds. Evening, Fine and clear.
	S 3	29.959	63.3	29.829	66.4	52	57.1	63.3	46.9	64.2	.169	S	A.M. Fine—lt. clds. & wind. P.M. Cloudy. Rain during the night.
	⊙ 4	29.386	61.8	29.421	68.0	55	61.8	66.9	54.0	68.0	.463	SW	{A.M. Overcast—heavy rain early. P.M. Fine—light clouds and wind. Evening, Overcast—light rain and wind.
	M 5	29.560	62.7	29.633	66.2	54	56.4	62.4	53.5	62.5	.113	SW var.	{A.M. Fine—light clouds—light brisk wind. Cloudy, with light wind, during the remainder of the day.
	T 6	29.360	64.3	29.309	65.2	54	56.3	61.4	50.9	61.7	.091	SSE var.	{A.M.Overcast—very lt. rain with high wind. 1 P.M. Thunder, with heavy rain and wind. Evening, Cloudy.
	W 7	29.641	60.2	29.712	63.3	52	53.7	59.8	51.7	60.5	.158	NW	Fine—light clouds & wind throughout the day. Evening, Cloudy.
	T 8	29.780	60.0	29.732	63.6	53	55.9	63.0	52.0	63.6		WNW	Fine—light clouds. Evening, Cloudy.
	F 9	29.746	59.7	29.802	62.2	53	57.0	53.8	51.9	62.5		SSW	{A.M. Lightly overcast—light wind. P.M. Overcast—heavy rain and wind. Evening, Fine and clear.
	S 10	29.792	57.3	29.831	60.9	50	53.6	56.3	46.5	56.8	.272	WNW	{A.M. Fine—light clouds and wind, with light shower. P.M. Fine—light clouds. Evening, Cloudy.
	○⊙ 11	30.025	56.8	29.998	58.8	46	50.6	51.8	42.4	56.7	.050	WSW	{A.M. Fine—very light clouds and wind. P.M.Overcast—light steady rain & wind. Ev., Ovct.—continued rain—h. wind.
	M 12	30.002	56.2	30.033	59.0	50	54.5	55.7	49.5	57.2	.122	NE	Overcast—light rain, with high wind.
	T 13	30.005	57.9	30.069	61.2	51	55.8	56.7	51.7	58.3		NNE	{A.M. Fine—light clouds and wind. P.M. Overcast—light brisk wind. Evening, Fine and clear.
	W 14	30.117	56.6	30.130	60.0	52	55.8	56.6	48.4	59.0	.022	NNE	Cloudy—light wind throughout the day.
	T 15	30.170	57.6	30.182	60.0	52	55.2	59.4	47.4	60.2		NE	{A.M. Fine—light clouds and wind. P.M. Overcast—light wind. Evening, Cloudy.
	F 16	30.075	56.0	30.058	59.3	51	53.7	58.2	50.0	58.4		NE var.	Overcast—light rain and wind. Evening, Cloudy.
	S 17	30.077	57.5	30.045	59.6	50	56.4	58.6	49.5	60.6	.088	NE	A.M. Fine—light clouds and wind. P.M. Overcast—light rain.
	⊙ 18	30.002	57.2	29.998	60.3	52	54.8	56.7	50.5	58.5	.141	NNE	Overcast—light brisk wind. Evening, Cloudy—light wind.
	M 19	29.974	55.3	29.942	58.6	48	51.9	56.8	48.3	58.2		NW	Cloudy—light wind throughout the day.
	T 20	29.989	54.9	29.954	59.4	48	51.6	58.8	48.7	59.0		W	{Overcast—very light rain and wind. Evening, Cloudy—light rain.
	W 21	30.152	53.5	30.176	57.1	45	48.4	51.9	42.6	52.3	.091	W	{A.M. Cloudy—light wind. P.M. Fine—light clouds and wind. Evening, Cloudy.
	T 22	30.277	51.6	30.235	56.2	43	47.4	56.3	40.7	57.8		W	A.M. Light fog. P.M. Cloudy—light wind.
	F 23	29.958	55.3	29.897	58.0	52	57.8	62.6	46.7	63.2	.044	SW var.	Overcast—very light rain with high wind throughout the day.
	● S 24	30.152	59.6	30.107	61.6	54	60.0	66.4	53.9	66.4	.161	WSW	{A.M. Fine—light clouds and wind. P.M. Fine—nearly cloudless. Evening, Fine and clear.
	⊙ 25	30.178	60.6	30.180	63.8	55	59.9	66.3	53.9	66.7		SW	{A.M. Cloudy—light wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	M 26	30.144	61.5	30.053	65.7	57	60.4	67.3	57.9	67.9		SSW	{A.M. Overcast—light wind. P.M. Fine—light clouds and wind. Evening, Cloudy—light wind.
	T 27	29.837	63.4	29.836	66.3	59	62.3	64.3	58.6	65.2		SSW	A.M. Ovct—lt. wind. P.M. Cloudy—lt. brisk wind. Ev. Cloudy.
	W 28	29.683	62.4	29.592	63.9	57	58.6	57.4	52.2	62.6	.047	SSW	{A.M. Fine—light clouds—rain during the night. P.M. Overcast—heavy rain, 1 P.M. Evening, Fine and clear.
	T 29	29.272	59.3	29.316	61.8	54	54.4	59.2	51.9	58.9	.486	NNE	A.M. Overcast—lt. rain & wind. P.M. Overcast—continued rain.
	S 30	29.426	58.3	29.477	59.6	49	52.5	52.3	46.6	54.6	.255	WSW	{A.M. Cloudy—light wind. P.M. Fine—light clouds and wind. Evening, Overcast—light rain.
SEPTEMBER	Means ..	29.877	59.2	29.869	62.1	52.1	56.0	59.6	50.3	61.2	Sum. 2.773	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.....	9 A.M. 29.800 / 3 P.M. 29.783
	S 1	29.356	53.7	29.085	55.9	48	52.3	51.4	41.5	55.3	.125	SE var.	{A.M. Overcast—light brisk wind. P.M. Overcast—light rain, with high wind.
	⊙ 2	29.247	53.0	29.346	55.5	44	47.9	51.2	42.9	52.5	.294	SW	{A.M. Fine—light clouds and wind. P.M. Cloudy—light wind. Evening, Overcast—light steady rain.
	M 3	29.008	51.6	29.322	54.2	44	43.8	50.3	43.4	52.2	.380	SSW var.	{A.M. Overcast—very light rain, with high wind. P.M. Fine—nearly cloudless. Evening, Fine and clear.
	T 4	29.497	48.4	29.523	52.3	41	45.2	53.6	39.3	53.3		SSW	A.M. Foggy—lt. wind. P.M. Fine—lt. clouds & wind. Ev. Foggy.
	W 5	29.829	49.0	29.886	53.0	40	44.6	56.3	40.4	56.2		SSE	A.M. Foggy. P.M. Fine—light clouds & wind. Ev. Overcast.
	T 6	29.853	50.0	29.748	53.8	45	49.3	54.2	43.3	57.2	.036	ESE	Overcast—very light rain throughout the day.
	F 7	29.511	54.8	29.423	56.7	51	56.6	58.8	48.6	59.0	.752	SE	Overcast—light rain and wind throughout the day.
	S 8	29.386	56.0	29.390	58.6	52	54.8	56.6	52.7	57.7	.091	SSE	Overcast—light wind throughout the day.
	⊙ 9	29.360	54.6	29.402	57.4	50	51.7	55.2	47.3	56.3	.283	S	{A.M. Fine & cloudless—lt. wind. Heavy rain early, with h. wind. P.M. Fine—lt. clds. with brisk wind. Ev. Overcast—lt. rain.
	○M 10	29.219	54.4	29.322	58.9	50	54.5	58.7	48.4	60.6	.133	SE var.	Overcast—light rain and wind. Evening, Cloudy.
	T 11	29.247	57.9	29.388	60.0	52	55.7	58.4	53.8	58.8	.080	SSW	{A.M. Fine—lt. clouds with high wind (very h. during the night). P.M. Fine—lt. clouds & wind. Ev. Overcast—very lt. rain.
	W 12	29.606	55.4	29.530	58.4	48	52.4	55.9	46.9	58.7		S	A.M. Fine—light clouds. P.M. Overcast—light rain & wind.
	T 13	29.140	57.5	29.333	59.5	52	58.7	58.9	51.5	59.2	.097	SW var.	{A.M. Ovct.—lt. rain with h. wind (very high wind during the night). P.M. Fine—lt. clds. & wind. Ev. Cldy.—lt. rain.
	F 14	29.782	56.4	29.717	59.2	50	54.5	57.7	51.3	58.5	.172	SW	A.M. Fine—lt. clds. & wind. P.M. Overcast—very lt. rain & wind.
	S 15	29.853	56.2	29.887	59.2	52	55.6	59.3	51.0	59.3		E	{A.M. Overcast—light rain and wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	⊙ 16	30.229	54.7	30.212	57.8	48	48.8	57.5	46.4	57.0	.061	SSW	{A.M. Thick fog. P.M. Fine—light clouds and wind. Evening, Thick fog.
	M 17	30.210	55.7	30.176	57.7	51	53.4	56.2	48.2	56.5		E	Overcast—very light rain and wind. Evening, Foggy.
	T 18	30.174	56.9	30.150	60.3	53	56.2	59.4	52.8	61.2		SSE	{A.M. Overcast. P.M. Fine—light clouds and wind. Evening, Cloudy.
	W 19	30.241	58.5	30.322	60.3	54	57.6	58.8	54.4	59.4	.158	SW	{A.M. Overcast—rain during the night. P.M. Fine and cloudless. Evening, Fine and clear.
	T 20	30.469	53.4	30.418	54.0	43	46.2	50.2	41.8	50.3		SW	Thick fog throughout the day. Evening, Fine and clear.
	F 21	30.332	52.2	30.305	55.5	46	49.2	53.3	44.8	52.6		ESE	{A.M. Cloudy—light wind. P.M. Fine—light clouds & wind. Evening, Fine and clear.
	S 22	30.398	51.2	30.382	53.9	45	49.3	53.6	45.8	53.8		E	A.M. Fine—light clouds and wind. P.M. Cloudy—light wind.
	⊙ 23	30.408	49.3	30.378	52.2	42	44.7	54.6	40.4	55.2		E	A.M. Foggy. P.M. Fine & cloudless. Evening, Fine & clear.
	● M 24	30.394	51.5	30.360	53.6	45	49.5	53.2	43.9	53.2		SSW	A.M. Foggy. P.M. Overcast. Evening, Cloudy.
	T 25	30.371	52.6	30.225	53.2	48	49.6	50.6	51.7	51.3		SW	A.M. Overcast—deposition—light wind. Evening, Light fog.
	W 26	30.196	52.6	30.113	54.4	50	50.9	52.4	48.7	52.7		SSW	A.M. Light fog and wind. P.M. Overcast—light wind.
	T 27	29.663	52.7	29.837	52.5	47	51.8	43.7	48.8	47.7		SW	{A.M. Cloudy—light brisk wind. P.M. Fine and cloudless. Evening, Overcast.
	F 28	29.913	46.3	29.857	47.6	39	40.2	40.8	37.2	40.6	.080	WSW	{A.M. Overcast—light rain. P.M. Fine—light clouds and wind. Evening, Cloudy.
	S 29	29.473	42.3	29.703	41.6	35	35.6	33.8	33.6	34.2		S	{A.M. Overcast—heavy fall of snow during the night. P.M. Overcast—light wind. Evening, Snow.
	⊙ 30	30.019	39.0	30.035	41.0	33	35.2	37.7	32.9	37.7	.288	W	Fine and cloudless—light wind. Evening, Fine and clear.
	M 31	30.089	37.3	30.009	40.0	30	33.2	39.5	30.0	39.3	.033	W	Fine and cloudless—light haze and wind. Evening, Cloudy.
OCTOBER	Means ..	29.822	52.1	29.832	54.5	46.1	49.3	52.6	45.3	53.5	Sum. 3.063	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.766 / 3 P.M. 29.769

METEOROLOGICAL JOURNAL FOR NOVEMBER AND DECEMBER, 1836.

1836.		9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	REMARKS.
		Barom.	Attach. Therm.	Barom.	Attach. Therm.		Fahrenheit.		Self-registering.				
							9 A.M.	3 P.M.	Lowest.	Highest.			
	T 1	30.099	37.6	30.008	40.4	32	35.2	41.6	31.7	37.8		SSW	A.M. Fine—lt. clouds. & wind. P.M. Overcast—lt. rain & wind.
	W 2	29.942	42.5	29.895	44.9	39	47.7	52.5	34.8	53.2	.044	SSW	Overcast—very light rain and wind throughout the day.
	T 3	29.840	44.8	29.679	48.2	39	44.8	51.7	42.3	51.7		SW	Overcast—deposition. 5½ P.M. Heavy storm of hail and rain.
	F 4	29.570	45.4	29.451	47.0	40	44.5	45.2	40.9	46.4	.080	SW	A.M. Overcast—light wind. P.M. Overcast—light rain.
	S 5	29.140	45.6	29.287	47.2	40	45.4	44.9	42.8	46.2	.122	SW	{A.M. Overcast—light rain and wind. P.M. Fine—nearly cloudless. Evening, Fine and clear.
	⊙ 6	29.459	42.1	29.439	44.6	36	36.4	43.8	34.0	43.8		WSW	A.M. Fine and cloudless—light wind. P.M. Cloudy—lt. wind.
	M 7	29.631	40.0	29.722	42.8	35	35.8	42.7	32.3	43.2	.055	WSW	{Fine and cloudless—light wind. (Sharp frost during the night.) Evening, Fine and clear.
	T 8	29.978	38.7	30.049	40.7	33	33.6	40.6	31.3	45.8		SW	{A.M. Light fog and wind. (White frost.) P.M. Fine and cloudless. Evening, Overcast—light rain.
	○ W 9	30.010	41.2	29.909	45.2	36	45.8	48.6	33.5	50.3	.036	SSE	A.M. Lightly oct. P.M. Cldy.—lt. wind. Ev. Oct.—lt. rain.
	T 10	29.614	46.3	29.621	49.0	43	50.4	52.0	45.8	53.8	.211	S	{A.M. Lightly cloudy. (Heavy rain during the night.) P.M. Overcast—rain and hail.
	F 11	29.390	47.4	29.429	49.3	44	50.5	49.4	42.6	50.6	.136	E	Overcast—light rain and wind throughout the day.
	S 12	29.821	46.9	29.863	48.0	41	41.4	46.2	40.2	52.0	.052	SW	{A.M. Thick fog—deposition. P.M. Overcast—light wind. Evening, Overcast—light rain and wind.
	⊙ 13	29.687	49.3	29.627	51.5	45	52.2	54.5	40.7	54.9	.038	SE var.	{A.M. Overcast—light rain with high wind. (Very high wind throughout the night.) P.M. Oct.—lt. rain with h. wind.
	M 14	29.656	49.4	29.699	50.7	44	42.7	46.4	41.0	46.0	.216	SW	Fine & cloudless—lt. wind throughout the day. Ev. Fine & clear.
	T 15	30.011	45.2	30.035	47.6	41	38.4	46.3	35.9	46.7		SW	{A.M. Fine—nearly cloudless—light haze. P.M. Thick haze. Evening, Cloudy.
	W 16	29.964	47.3	29.800	49.4	42	46.7	50.2	37.9	50.2		SW	A.M. Cloudy—lt. wind. P.M. Oct.—very lt. rain. Ev. Cloudy.
	T 17	29.475	48.2	29.434	49.2	43	46.7	47.4	45.2	47.5		SW	{A.M. Fine—lt. clouds & wind. (High wind during the night.) P.M. Fine—nearly cloudless. Ev. Overcast—light rain.
	F 18	29.103	46.7	29.172	48.0	42	40.8	43.7	39.8	44.0	.069	WSW	Fine—nearly cloudless throughout the day. Ev. Fine & clear.
	S 19	29.253	42.5	29.186	44.6	38	37.4	42.0	34.4	38.7		SSW	{A.M. Fine—light clouds and haze. P.M. Cloudy—light rain and wind. Evening, Overcast—light rain.
	⊙ 20	29.784	42.2	29.917	43.7	37	38.7	42.8	36.9	42.7	.180	ENE	{A.M. Fine—light clouds and wind. P.M. Fine and cloudless —light wind. Evening, Fine and clear.
	M 21	30.119	40.0	30.071	41.2	34	36.7	38.2	32.6	41.5		SW	Thick fog throughout the day. Evening, Overcast—light rain.
	T 22	29.960	41.5	29.889	41.8	37	38.8	39.2	33.8	49.5	.069	NW	{A.M. Light fog, with deposition. P.M. Overcast. Evening, Overcast—light rain with high wind.
	● W 23	29.272	43.7	29.377	45.2	38	44.6	44.8	37.9	44.3	.163	SW	{A.M. Overcast—light brisk wind. P.M. Cloudy—light brisk wind. Evening, Fine and clear. (& clear.
	T 24	29.479	41.4	29.528	44.8	36	38.5	43.8	36.6	43.6		SW	A.M. Fine & cloudless—lt. wind. P.M. Cldy.—lt. wind. Ev. Fine
	F 25	29.732	40.2	29.558	41.3	35	35.7	39.8	33.5	39.3		WNW	{Overcast—light wind throughout the day. Evening, Overcast —light fog and rain.
	S 26	29.356	39.8	29.274	41.4	36	39.7	44.0	34.5	50.7	.136	E	Li. fog & rain throughout the day. Ev. Oct.—lt. rain & wind.
	⊙ 27	29.530	44.9	29.534	46.8	40	44.5	51.4	39.0	55.3	.158	SW	{Overcast—light fog and rain, with light brisk wind. Ev. Oct. light high wind, with light rain.
	M 28	29.267	51.8	29.303	54.0	48	55.4	56.2	49.2	56.7	.033	S var.	{A.M. Cloudy—high wind. (Very high wind, with light rain throughout the night.) P.M. Cloudy—high wind.
	T 29	28.954	54.2	29.384	54.3	51	56.8	49.4	52.2	52.6	.183	S var.	{A.M. Oct.—high wind. (High wind, with light rain during the night.) P.M. Fine—lt. clouds & wind. Ev. Fine & clear.
	W 30	29.516	51.6	29.536	51.8	46	48.4	49.3	47.5	49.3	.127	S	Overcast—heavy rain and wind throughout the day.
NOVEMBER	Means..	29.620	44.6	29.626	46.5	39.7	43.1	46.3	38.7	47.6	Sum. 2.108	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.586 / 3 P.M. 29.587
	T 1	29.980	48.0	30.037	48.7	42	41.3	44.9	39.0	48.7	.325	SW	{A.M. Overcast—light wind. Heavy rain during the night. P.M. Fine—light clouds. Evening, Overcast.
	F 2	30.012	47.5	29.938	49.9	42	49.0	51.9	40.4	51.7		S	Overcast—very light rain, with high wind. Evening, Overcast.
	S 3	29.772	49.6	29.796	51.2	44	49.3	54.2	48.5	54.2	.036	SSW	{A.M. Overcast—very light rain and wind. Very high wind during the night. P.M. Overcast—light wind.
	⊙ 4	29.816	51.6	29.812	53.0	47	53.7	55.5	48.6	55.5	.161	SW	{Overcast—light wind throughout the day, with occasional rain. Very high wind during the night.
	M 5	29.867	52.8	29.833	54.2	48	54.2	55.6	51.3	55.7	.125	WSW	Overcast—lt. wind throughout the day. Evening, Fine—lt. clds.
	T 6	29.998	52.6	29.964	54.6	48	51.9	52.6	47.2	53.7		S	{A.M. Overcast—very li. rain with high wind. P.M. Fine and cloudless—lt. wind. Ev. Overcast—lt. rain and wind.
	W 7	29.535	53.3	29.445	53.6	47	50.6	51.7	49.3	52.7	.055	SSW	{Fine—lt. clouds & wind throughout the day. Very high wind during the night. Ev. Fine—lt. clouds, with brisk wind.
	○ T 8	29.229	44.5	29.166	50.2	43	46.2	45.3	42.5	46.5	.380	SW	{A.M. Very heavy rain, with high wind. P.M. Fine—light clouds and wind. Evening, Fine and clear.
	F 9	29.069	45.9	29.017	46.9	41	41.3	42.9	38.2	43.6	.036	SSW	Fine—light clouds throughout the day. Evening, Fine & clear.
	S 10	29.213	42.0	29.235	43.6	37	36.5	42.4	34.6	42.0		SSW	{A.M. Fine—nearly cloudless—light haze. P.M. Cloudy—light rain. Evening, Fine and clear.
	⊙ 11	29.586	41.8	29.584	43.0	36	38.0	40.2	35.0	40.2		SW	Overcast—light wind throughout the day. Ev. Thick fog.
	M 12	29.584	39.9	29.426	42.8	35	38.7	49.4	33.5	50.3	.119	ESE	{Overcast—light rain and wind throughout the day. Evening, Overcast—light rain with high wind.
	T 13	29.241	46.9	29.227	48.4	42	47.7	47.7	38.2	49.3	.066	SSW	{Overcast—light brisk wind throughout the day. Very high wind during the night. Evening, Cloudy.
	W 14	29.295	43.8	29.368	45.8	38	38.6	45.3	37.4	45.2		SSW	A.M. Fine—nearly cldless.—lt. wind. P.M. Fine—lt.clds. & wind.
	T 15	29.967	42.3	30.006	42.8	37	38.0	40.6	37.0	44.6		SW	{A.M. Hazy. P.M. Fine—light clouds and wind. Evening, Overcast—heavy rain with high wind.
	F 16	29.728	42.2	29.853	43.7	36	39.3	43.8	37.0	44.0	.183	S	{Fine—light clouds & wind throughout the day. Heavy rain with high wind during the night. Evening, Cloudy.
	S 17	30.124	41.0	30.087	43.2	37	37.2	44.7	35.6	49.0		SW	Hazy—lt. wind throughout the day. Ev. Overcast—very lt. rain.
	⊙ 18	30.095	45.0	30.099	46.7	39	49.2	50.9	36.4	51.2		SSW	{Cloudy—light wind throughout the day. Evening, Overcast —very light rain.
	M 19	30.168	46.9	30.136	58.6	41	47.6	49.8	45.8	49.9		SSW	Overcast—light wind throughout the day. Evening, Cloudy.
	T 20	30.208	48.3	30.251	48.8	43	46.5	44.0	45.7	46.2	.058	SW	{Foggy—deposition—light wind throughout the day. Evening, Overcast.
	W 21	30.279	46.0	30.285	47.2	41	42.8	46.3	39.3	47.0		SSW	A.M. Foggy—deposition—light wind. P.M. Overcast.
	T 22	30.378	46.4	30.273	47.3	40	42.8	46.4	41.9	48.0		SW	A.M. Light fog, P.M. Fine—lt. clds. & wind. Ev. Overcast.
	● F 23	29.857	46.2	29.766	45.3	39	39.7	37.8	38.9	43.7	.102	N	{A.M. Lightly overcast—lt. wind. Rain during the night. P.M. Cloudy—lt. wind. Evening, Fine and clear.
	S 24	29.711	39.7	29.758	40.4	33	34.3	35.5	32.7	38.2		W	{Cloudy—light wind throughout the day. Sharp frost during the night. Evening, Fine and clear.
	⊙ 25	29.766	34.4	29.693	35.2	27	28.3	32.0	27.4	34.0		N var.	{A.M. Oct.—snow with lt. brisk wind. P.M. Fine—lt. clouds. Ev. Oct.—snow—lt. wind. (Oct.—high wind.
	M 26	29.580	32.9	29.548	33.8	27	29.7	31.6	29.2	32.5		NW	Snow, with high wind nearly the whole of the day. Evening,
	T 27	29.578	33.7	29.641	34.3	28	32.5	31.2	31.9	33.0		NE var.	{A.M. Snow, with high wind—freezing very sharply. P.M. Fine—light clouds & wind. Evening, Overcast—snow.
	W 28	29.885	33.2	29.901	35.2	27	33.3	32.3	29.4	31.8		NW	{A.M. Fine—lt. clouds & wind. P.M. Overcast—lt. brisk wind. Ev. Overcast—slight thaw. (Overcast—air frosty.
	T 29	29.998	32.6	30.018	34.7	28	31.9	33.4	29.7	33.2		NE var.	A.M. Overcast—lt. snow & wind. P.M. Oct.—slight thaw. Ev.
	F 30	30.053	32.7	30.085	34.6	27	30.2	31.7	29.2	29.8		N	{A.M. Overcast—light wind. P.M. Overcast—light snow & wind. Evening, Fine and star-light.
	S 31	30.212	31.8	30.326	33.7	27	30.0	32.3	28.2	32.3		NE	Lightly overcast—lt. wind throughout the day. Ev. Light snow.
DECEMBER	Means..	29.799	43.0	29.793	44.9	37.6	41.0	43.4	38.0	44.4	Sum. 1.646	Mean of Barometer, corrected for Capillarity and reduced to 32° Fahr.	9 A.M. 29.770 / 3 P.M. 29.758